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Degree of Approximation of a Function Belonging to W (L_r, ξ (t)) (r >1)-Class by (E, q), (q >0)-Means of Conjugate Fourier Series

Kusum Sharma, S. S.Malik

Abstract – **In this paper, we determine the degree of approximation of the conjugate of 2π-periodic function belonging to $W(L_r,\xi(t))(r>1)$-class by using $(E,q)(q>0)$-Means of its conjugate Fourier series. Our result generalizes the results of Shukla [14] and Mishra et. al [12].**

Keywords – **Degree of Approximation, W (L_r, ξ (t))(r >1)-Class Of Function, (E, q) Summability, Conjugate Fourier Series, Lebesgue Integral.**

I. Introduction

The degree of approximation of functions belonging to $Lip\alpha$, $Lip(\alpha,r)$, $Lip(\xi(t),r)$ and $W(L_r,\xi(t))$ classes using different summability methods has been determined by a number of researchers like Khan [4, 5, 6, 7], Khan and Ram [8], Chandra [1, 2], Leindler [9], Mishra et al. [11] and Mittal, Rhoades and Mishra [13]. Chandra [2] has studied the degree of approximation of a signal (function) belonging to $Lip\alpha$-class by (E,q) means, $q>0$. Generalizing the result of Chandra [2], very interesting result has been proved by Shukla [14] for the signals (functions) belonging to $Lip(\alpha,r)$-class through trigonometric Fourier approximation by applying $(E,q)(q>0)$ summability matrix. Thereafter, Mishra et. al [12] generalized the result of Shukla [14] for the functions belonging to $Lip(\xi(t),r)$-class by $(E,q)(q>0)$ summability. In this paper, a theorem concerning the degree of approximation of the conjugate of a signal (function) f belonging to $W(L_r,\xi(t))(r>1)$-class by (E,q) summability of conjugate Fourier series has been established which in turn generalizes the results of Shukla [14] and Mishra et. al [12].

II. Definition and Notations

Let $f(x)$ be periodic with period 2π and integrable in the sense of Lebesgue . The Fourier series associated with f at a point x is defined by

$$f(x)\sim\frac{a_0}{2}+\sum_{n=0}^{\infty}(a_n\cos nx+b_n\sin nx) \qquad (2.1)$$

with n[th] partial sums $s_n(f;x)$.The conjugate series of Fourier series (2.1) of f is given by

$$\sum_{n=1}^{\infty}(a_n\sin nx-b_n\cos nx) \qquad (2.2)$$

Throughout this paper, we will call (2.2) as conjugate Four-ier series of function f . L_∞ - norm of a function $f:R\to R$ is defined by

$$\|f\|_\infty=\sup\{|f(x)|:x\in R\} \qquad (2.3)$$

L_r - norm of a function is defined by

$$\|f\|_r=\left(\int_0^{2\pi}|f(x)|^r dx\right)^{\frac{1}{r}},1\le r<\infty \qquad (2.4)$$

The degree of approximation of a function $f:R\to R$ by a trigonometric polynomial t_n of order n under sup norm $\|\ \|_\infty$ is defined by

$$\|t_n-f\|_\infty=\sup\{|t_n(x)-f(x)|:x\in R\}$$ (Zygmund [15]) (2.5)

and $E_n(f)$ of a function $f\in L_r$ is given by

$$E_n(f)=\min_{t_n}\|t_n-f\|_r \qquad (2.6)$$

This method of approximation is called trigonometric Four-ier approximation (TFA).

A function $f\in Lip\alpha$ if

$$|f(x+t)-f(x)|=O(|t^\alpha|)\ for\ 0<\alpha\le 1 \qquad (2.7)$$

$f(x)\in Lip(\alpha,r)$ for $0\le x\le 2\pi$, if

$$\left(\int_0^{2\pi}|f(x+t)-f(x)|^r dx\right)^{\frac{1}{r}}=O(|t^\alpha|),0<\alpha\le 1,r\ge 1 \qquad (2.8)$$

(Definition 5.38 of Mc Fadden [10])Given a positive increasing function $\xi(t)$ and $r\ge 1$, $f(x)\in Lip(\xi(t),r)$, if

$$\left(\int_0^{2\pi}|f(x+t)-f(x)|^r dx\right)^{\frac{1}{r}}=O(\xi(t)) \qquad (2.9)$$

If $\xi(t)=t^{\alpha}$ then $Lip\ (\xi(t), r)$ class reduces to $Lip(\alpha,r)$ and if $r\to\infty$ then $Lip(\alpha,r)$ classreduces to the $Lip\alpha$ class.and that $f\in W(L_r,\xi(t))$

$$\left(\int_0^{2\pi}\left|\{f(x+t)-f(x)\}\sin^{\beta}x\right|^r dx\right)^{\frac{1}{r}}=O(\xi(t)),\quad \beta\ge 0 \qquad (2.10)$$

If β = 0, then $W(L_r,\xi(t))$ reduces to the class $Lip\ (\xi(t), r)$ and if $\xi(t)=t^{\alpha}$ then $Lip\ (\xi(t), r)$ class coincides with the class $Lip(\alpha,r)$ and if $r\to\infty$ then $Lip(\alpha,r)$ class reduces to the class $Lip\alpha$.

We observe that $Lip\alpha\subseteq Lip(\alpha,r)\subseteq Lip(\xi(t),r)\subseteq W(L_r,\xi(t))$ for $0<\alpha\le 1,\ r\ge 1$. Let $\sum_{n=0}^{\infty}u_n$ be a given infinite series with sequence of its nth partial sums $\{s_n\}$.

If (E,q) transform is defined as the nth partial sum of (E,q) summability and is given by

$$(E,q)=E_n^q=\frac{1}{(1+q)^n}\sum_{k=0}^{n}\binom{n}{k}q^{n-k}s_k\to s \text{ as } n\to\infty \qquad (2.11)$$

then the infinite series $\sum_{n=0}^{\infty}u_n$ is said to be summable (E,q) to the definite number s (Hardy [3]).

We note that E_n^q is also trigonometric polynomial of degree (or order) " n".

We use the following notations:

$$\psi(t)=f(x+t)-f(x-t)$$

$$\tilde{K}_n(t)=\frac{1}{2\pi(1+q)^n}\sum_{k=0}^{n}\left\{\binom{n}{k}q^{n-k}\frac{1}{(1+k)}\sum_{v=0}^{k}\frac{\cos\left(k+\frac{1}{2}\right)t}{\sin\left(\frac{t}{2}\right)}\right\}$$

III. Known Result

Dealing with the trigonometric approximation of a signal $f\in Lip(\xi(t),r)$ by (E,q) means of conjugate Fourier series Mishra et al. [12] proved the following theorem:

A. Theorem 3.1

Let $\bar{f}$, conjugate to a 2π-periodic signal (function) f belonging to $Lip(\xi(t),r)$-class, then its degree of approximation by (E, q) means of conjugate series of Fourier series is given by

$$\left\|\tilde{E}_n^q-\tilde{f}\right\|_r=O\left\{(n+1)^{\frac{1}{r}}\ \xi\left(\frac{1}{n+1}\right)\right\} \qquad (3.1)$$

provided positive increasing ξ (t) satisfies the following conditions:

$$\left(\int_0^{\frac{\pi}{n+1}}\left(\frac{|\psi_x(t)|}{\xi(t)}\right)^r dt\right)^{\frac{1}{r}}=O\ (1) \qquad (3.2)$$

and

$$\left\{\int_{\frac{\pi}{n+1}}^{\pi}\left(\frac{t^{-\delta}|\psi_x(t)|}{\xi(t)}\right)^r dt\right\}^{\frac{1}{r}}=O\left\{(n+1)^{\delta}\right\} \qquad (3.3)$$

where δ is an arbitrary number such that $s(1-\delta)-1>0$, $\frac{1}{r}+\frac{1}{s}=1, 1\le r\le\infty$, conditions (3.2) and (3.3) hold uniformly in x and $\tilde{E}_n^q$ is the nth (E,q) means of the conjugate series of its Fourier seriesand the conjugate function $\bar{f}$ is defined for almost every x by

$$\bar{f}(x)=-\frac{1}{2\pi}\int_0^{\pi}\psi(t)\cot\left(\frac{t}{2}\right)dt \qquad (3.4)$$

IV. Main Theorem

If $\bar{f}$, conjugate to a 2π-periodic signal (function) f belongs to $W(L_r,\xi(t))$-class, then its degree of approximation by (E, q) means of its conjugate Fourier series is given by

$$\left\|\tilde{E}_n^q-\tilde{f}\right\|_r=O\left\{(n+1)^{\beta+\frac{1}{r}}\ \xi\left(\frac{1}{n+1}\right)\right\} \qquad (4.1)$$

provided ξ (t) satisfies the following conditions:

$$\left(\int_0^{\frac{\pi}{n+1}}\left(\frac{t\,|\psi(t)|}{\xi(t)}\right)^r \sin^{\beta r}t\ dt\right)^{\frac{1}{r}}=O\left(\frac{1}{n+1}\right) \qquad (4.2)$$

$$\left\{\int_{\frac{\pi}{n+1}}^{\pi}\left(\frac{t^{-\delta}|\psi_x(t)|}{\xi(t)}\right)^r dt\right\}^{\frac{1}{r}}=O\left\{(n+1)^{\delta}\right\} \qquad (4.3)$$

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and

$$\left\{\frac{\xi(t)}{t}\right\} \text{ is non increasing in t,} \qquad (4.4)$$

where δ is an arbitrary number such that $s(1-\delta)-1>0$, $\frac{1}{r}+\frac{1}{s}=1, 1\le r\le\infty$, conditions (4.2) and (4.3) hold uniformly in x and $\tilde{E}_n^q$ is the n[th] (E,q) means of the conjugate Fourier series and

$$\bar{f}(x)=-\frac{1}{2\pi}\int_0^{\pi}\psi(t)\cot\left(\frac{t}{2}\right)dt \qquad (4.5)$$

Note.4.1

$\xi(\pi/n+1)\le\pi\xi(1/n+1),\ for\ (\pi/n+1)\ge(1/n+1).$

Note. 4.2 For $\beta=0$, our theorem reduces to theorem 3.1 of Mishra et. al [12].

V. Leema

Following lemma is required for the proof of our theorem:

A. Lemma 5.1. [14]. For $0\le t\le\pi$, we have

$$\left|\tilde{K}_n(t)\right|=O\left(t^{-1}e^{-2snt/(\pi(1+s))^2}\right) \qquad (5.1)$$

VI. Proof of Main Theorem

Let $\tilde{s}_n(x)$ denote the partial sum of the series (2.2), then we have

$$\tilde{s}_n(x)-\tilde{f}(x)=\frac{1}{2\pi}\int_0^{\pi}\psi(t)\frac{\cos\left(n+\frac{1}{2}\right)t}{\sin\left(\frac{t}{2}\right)}dt$$

Therefore the transform $\tilde{E}_n^q$ of $\tilde{s}_n(f:x)$ is given by

$$\tilde{E}_n^q-\tilde{f}(x)=\frac{1}{2\pi(1+q)^n}\int_0^{\pi}\psi(t)\left\{\sum_{k=0}^{n}\binom{n}{k}q^{n-k}\frac{\cos\left(k+\frac{1}{2}\right)t}{\sin\left(\frac{t}{2}\right)}\right\}dt$$

$$=\int_0^{\pi}\psi(t)\,\tilde{K}_n(t)\,dt$$

$$=\left[\int_0^{\frac{\pi}{n+1}}+\int_{\frac{\pi}{n+1}}^{\pi}\right]\psi(t)\,\tilde{K}_n(t)\,dt$$

$$=I_1+I_2\quad(say) \qquad (6.1)$$

We consider, $|I_1|\le\int_0^{\frac{\pi}{n+1}}\psi(t)\left|\tilde{K}_n(t)\right|dt$

Using Hölder's inequality and the fact that $\psi(t)\in W(L_r,\xi(t))$, due to the fact that $f\in W(L_r,\xi(t))$, condition (4.2) and Lemma 5.1, we have

$$|I_1|\le\left(\int_0^{\frac{\pi}{n+1}}\left\{\frac{t\,|\psi(t)|\sin^{\beta}t}{\xi(t)}\right\}^r dt\right)^{\frac{1}{r}}\left[\int_0^{\frac{\pi}{n+1}}\left\{\frac{\xi(t)\left|\tilde{K}_n(t)\right|}{t\sin^{\beta}t}\right\}^s dt\right]^{\frac{1}{s}}$$

$$=O\left(\frac{1}{n+1}\right)\left[\int_0^{\frac{\pi}{n+1}}\left\{\frac{\xi(t)\left|\tilde{K}_n(t)\right|}{t^{1+\beta}}\right\}^s dt\right]^{\frac{1}{s}}$$

$$=O\left(\frac{1}{n+1}\right)\left[\int_{0+}^{\frac{\pi}{n+1}}\left\{\frac{\xi(t)e^{-2snt^2/(\pi(1+s))^2}}{t^{2+\beta}}\right\}^s dt\right]^{\frac{1}{s}}$$

$$=O\left(\frac{1}{n+1}\right)\left[\int_{0+}^{\frac{\pi}{n+1}}\left\{\frac{\xi(t)}{t^{2+\beta}}\right\}^s dt\right]^{\frac{1}{s}}$$

Since $\xi(t)$ is a positive increasing function so using condition (4.4), we have $\xi(\pi/n+1)\le\pi\xi(1/n+1),\ for\ (\pi/n+1)\ge(1/n+1).$ and Second Mean value Theorem for integrals, we get

$$|I_1|=O\left\{\left(\frac{1}{n+1}\right)\xi\left(\frac{\pi}{n+1}\right)\right\}\left\{\int_{\in}^{\frac{\pi}{n+1}}\left(\frac{dt}{t^{(2+\beta)s}}\right)\right\}^{\frac{1}{s}}$$

$$for\ some\ \ 0<\in<\frac{\pi}{n+1}$$

$$=O\left[\left(\frac{1}{n+1}\right)\xi\left(\frac{1}{n+1}\right)\left\{\frac{t^{-(2+\beta)s+1}}{-(2+\beta)s+1}\right\}_{\in}^{\frac{\pi}{n+1}}\right]^{\frac{1}{s}}$$

$$=O\left[\left(\frac{1}{n+1}\right)\xi\left(\frac{1}{n+1}\right)\left\{(n+1)^{2+\beta-\frac{1}{s}}\right\}\right]$$

$$=O\left\{(n+1)^{\beta+1-\frac{1}{s}}\ \xi\left(\frac{1}{n+1}\right)\right\}$$

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$$= O\left[(n+1)^{\beta+\frac{1}{r}}\xi\left(\frac{1}{n+1}\right)\right] \text{since } \frac{1}{r}+\frac{1}{s}=1, \quad 1\le r\le\infty \tag{6.2}$$

Now we consider,

$$|I_2| \le \int_{\frac{\pi}{n+1}}^{\pi} \psi(t)\left|\tilde{K}_n(t)\right| dt$$

Using Hölders inequality,

$$|I_2| \le \left(\int_{\frac{\pi}{n+1}}^{\pi}\left\{\frac{t^{-\delta}|\psi(t)|\sin^\beta t}{\xi(t)}\right\}^r dt\right)^{\frac{1}{r}}\left[\int_{\frac{\pi}{n+1}}^{\pi}\left\{\frac{\xi(t)\left|\tilde{K}_n(t)\right|}{t^{-\delta}\sin^\beta t}\right\}^s dt\right]^{\frac{1}{s}}$$

$$= O\left\{(n+1)^\delta\right\}\left[\int_{\frac{\pi}{n+1}}^{\pi}\left\{\frac{\xi(t)\left|\tilde{K}_n(t)\right|}{t^{-\delta+\beta}}\right\}^s dt\right]^{\frac{1}{s}} \quad by \ (4.3)$$

$$= O\left\{(n+1)^\delta\right\}$$

$$\left[\int_{\frac{\pi}{n+1}}^{\pi}\left\{\frac{\xi(t)e^{-2snt^2/(\pi(1+s))^2}}{t^{1-\delta+\beta}}\right\}^s dt\right]^{\frac{1}{s}} \quad \text{by Lemma 5.1}$$

$$= O\left\{(n+1)^\delta\right\}\left[\int_{\frac{\pi}{n+1}}^{\pi}\left\{\frac{\xi(t)}{t^{1-\delta+\beta}}\right\}^s dt\right]^{\frac{1}{s}}$$

Now putting $t=\frac{1}{y}$,

$$I_2 = O\left\{(n+1)^\delta\right\}\left[\int_{\frac{1}{\pi}}^{\frac{n+1}{\pi}}\left\{\frac{\xi\left(\frac{1}{y}\right)}{y^{\delta-1-\beta}}\right\}^s \frac{dy}{y^2}\right]^{\frac{1}{s}}$$

Since $\xi(t)$ is a positive increasing function and $\frac{\xi(1/y)}{(1/y)}$ is also increasing function and using Second MeanValueTheorem

$$I_2 = O\left\{(n+1)^\delta\left(\frac{n+1}{\pi}\right)\xi\left(\frac{\pi}{n+1}\right)\right\}\left[\int_{\eta}^{\frac{n+1}{\pi}}\left(\frac{dy}{y^{s\delta-\beta+2}}\right)\right]^{\frac{1}{s}}$$

, for some $\frac{1}{\pi}\le\eta\le\frac{n+1}{\pi}$

$$= O\left\{(n+1)^{\delta+1}\xi\left(\frac{1}{n+1}\right)\right\}\left[\int_{1}^{\frac{n+1}{\pi}}\left(\frac{dy}{y^{s\delta-\beta+2}}\right)dy\right]^{\frac{1}{s}}$$

$$\frac{1}{\pi}\le 1\le\frac{n+1}{\pi}$$

$$= O\left\{(n+1)^{\delta+1}\xi\left(\frac{1}{n+1}\right)\right\}\left[\left\{\frac{y^{-1+\beta-\delta s}}{-1+\beta-\delta s}\right\}_1^{\frac{n+1}{\pi}}\right]^{\frac{1}{s}}$$

$$= O\left\{(n+1)^{\delta+1}\xi\left(\frac{1}{n+1}\right)\right\}\left[(n+1)^{\beta-\delta-\frac{1}{s}}\right]$$

$$= O\left\{(n+1)^{\beta+1-\frac{1}{s}}\xi\left(\frac{1}{n+1}\right)\right\}$$

$$= O\left[(n+1)^{\beta+\frac{1}{r}}\xi\left(\frac{1}{n+1}\right)\right] \text{since } \frac{1}{r}+\frac{1}{s}=1 \tag{6.3}$$

Now combining (6.1), (6.2) and (6.3), we get

$$\left\|\tilde{E}_n^q-\tilde{f}\right\|_r = O\left\{(n+1)^{\beta+\frac{1}{r}}\xi\left(\frac{1}{n+1}\right)\right\}$$

Now using L_r - norm, we get

$$\left\|\tilde{E}_n^q-\tilde{f}\right\|_r = \left\{\int_0^{2\pi}\left|\tilde{E}_n^q-\tilde{f}\right|^r dx\right\}^{\frac{1}{r}}$$

$$= O\left[\int_0^{2\pi}\left\{(n+1)^{\beta+\frac{1}{r}}\xi\left(\frac{1}{n+1}\right)\right\}^r dx\right]^{\frac{1}{r}}$$

$$= O\left\{(n+1)^{\beta+\frac{1}{r}}\xi\left(\frac{1}{n+1}\right)\right\}\left[\left\{\int_0^{2\pi}dx\right\}^{\frac{1}{r}}\right]$$

$$= O\left\{(n+1)^{\beta+\frac{1}{r}}\xi\left(\frac{1}{n+1}\right)\right\}$$

This completes the proof of the main theorem.

VII. Applications

Following corollaries can be derived from our main theorem.

A. Corollary 1

If $\xi(t)=t^\alpha, 0<\alpha\le 1$, then weighted class $W(L_r,\xi(t))(r\ge 1)$, reduces to the class $Lip(\alpha,r)$ and the degree of approximation of a function $\tilde{f}$, conjugate to

2π - periodic function f belonging to the class $Lip(\alpha, r)$, $\frac{1}{r} \leq \alpha \leq 1$ is given by

$$\left\| \tilde{E}_n^q - \tilde{f} \right\|_r = O\left(\frac{1}{(n+1)^{\alpha - \frac{1}{r}}} \right) \quad (7.1)$$

Proof: The result follows by setting $\beta = 0$ in (4.1).

B. *Corollary 2*

If $\xi(t) = t^{\alpha}$, $0 < \alpha < 1$, and $r = \infty$ in corollary 1,then $f \in Lip\alpha$, in this case using (7.1), we have

$$\left| \tilde{E}_n^q - \tilde{f} \right| = O\left(\frac{1}{(n+1)^{\alpha}} \right)$$

Proof: For $r = \infty$,we get

$$\left\| \tilde{E}_n^q - \tilde{f} \right\|_\infty = \sup_{0 \leq x \leq 2\pi} \left| \tilde{E}_n^q - \tilde{f} \right| = O\left(\frac{1}{(n+1)^{\alpha}} \right)$$

that is,

$$\left| \tilde{E}_n^q - \tilde{f} \right| = O\left(\frac{1}{(n+1)^{\alpha}} \right)$$

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Author's Profile

Dr. Kusum Sharma

She was born on 31st July, 1985 in Nawalgarh, Rajasthan. She has done her M.Sc. in Mathematics (Fluid Dynamics) from university of Rajasthan in 2005 and got Ph.D. degree in 2013 from Mody University. Her research area includes summability and approximation theory. Currently, she is working as an assistant professor in the mathematics department, ModyUniversity.
Email: kusum31sharma@rediffmail.com

Dr. S. S. Malik

He was born on 15th Feb., 1981 in Rohtak, Haryana. He got his M.Sc. degree from Guru Jambheshwar University, Hisar in 2005 and was awarded Ph.D. degree in 2012. His area of research is Differential equations. At present, he is working as an associate professor and Head of the department of Mathematics in BIET, Sikar, Rajasthan.
Email: Drssmalik17@gmail.com