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Daily British Pound and US Dollar Exchange Rates Modelling by Seasonal Box-Jenkins Methods

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***Abstract* – A SARIMA model for forecasting daily exchange rates of the British Pound (GBP) and the United States Dollars (USD) is proposed and fitted. The realization spans from 5th February 2014 to 16th October 2014. The time plot of the series called herein PDER shows an initial upward trend up to Saturday 5th July followed by a downward trend. This means that prior to 5th July the dollar depreciated relatively after which it appreciated. A seven-day differencing of PDER yields a series herein called SDPDER exhibiting a generally horizontal trend. The Augmented Dickey Fuller unit root test adjudges PDER as non-stationary and SDPDER as stationary. However the correlogram of SDPDER shows evidence of non-stationarity in the series. A non-seasonal differencing of SDPDER yields the series DSDPDER which also exhibits a horizontal secular trend. It is certified as stationary by the unit root test. Its correlogram shows an autocorrelation structure of a stationary series. Moreover there is an indication of seasonality of period 7 days and the involvement of seasonal autoregressive (AR) and moving average (MA) components order one each. Taking into consideration the duality of AR and MA models suggestive models include (1) A Sarima (0, 1, 1)x(0, 1, 1)$_7$ and (2) A Sarima (0, 1, 0)x(1, 1, 1)$_7$. On Akaike's Information Criterion (AIC) grounds the latter model is the more adequate. This model further yields a more adequate model: Sarima(0, 1, 0)x(0, 1, 1)$_7$. Therefore the proposed model is the Sarima(0, 1, 0)x(0, 1, 1)$_7$. Its residuals are mostly uncorrelated indicating model adequacy. Forecasting of the exchange rates may be based on this model**

***Keywords* – Foreign Exchange Rates, GBP, SARIMA Models, USD.**

I. Introduction and Literature Review

This work involves the modeling of the daily exchange rates of the British Pound (GBP) and the United States Dollar (USD). A model of the rates could be used for their forecasting. Financial data such as these are known to exhibit some seasonal tendencies. Seasonal autoregressive integrated moving average (SARIMA) modeling could be used to explain the variation within such data.

SARIMA models were proposed to model data that are seasonal in nature. They have been used extensively of recent to model such series. A brief review of some relevant recent literature follows hereunder. Chandran and Pandey [1] fitted a SARIMA(1, 1, 1)x(1, 0, 0)$_{12}$ model to monthly potato prices in Delhi market. Wongkoon *et al.* [2] proposed and fitted a SARIMA(2, 0, 1)x(0, 2, 0)$_{12}$ to monthly DHF incidence in Northern Thailand. Prista *et al.* [3] modeled monthly fish catch off Portugal as a SARIMA(0, 0, 5)x(1, 1, 0)$_{12}$. Padhran [4] observed that the SARIMA(1, 1, 1)x(2, 1, 4)$_{12}$ model he proposed to model tourist footfalls in India outperformed other competing models in forecasting. A SARIMA(1, 0, 0)x(1, 1, 1)$_{12}$ model has been fitted to tomato prices in Turkey (See [5]). Fannoh *et al.* [6] used a SARIMA model of order (0, 1, 0)x(2, 0, 0)$_{12}$ to model Liberian inflation rates. Oduro-Gyimah *et al.* [7] modeled monthly microwave transmission as a SARIMA(1, 1, 1)x(0, 1, 2)$_{12}$ model. Khajavi *et al.* [8] applied a SARIMA(1, 0, 0)x(0, 1, 1)$_{12}$ to model mean monthly temperature for Anzali and Babolsar and a SARIMA(0, 0, 2)x(0, 1, 1)$_{12}$ for the Ramsar region. Hassan *et al.* [9] forecasted coarse rice price in Bangladesh on the basis of an adequate SARIMA(1, 1, 1)x(0, 1, 1)$_{12}$. Li *et al.* [10] modeled outpatient numbers by a SARIMA(0, 1, 1)x(0, 1, 1)$_{12}$ model. Monthly rubber production in India may be forecasted using a SARIMA(2, 1, 2)x(1, 1, 10$_{12}$ model (See [11]). Abdelghani *et al.* [12] proposed a SARIMA(1, 0, 0)x(0, 1, 1)$_{12}$ for monthly incidence of cerebrospinal meningitis in Sudan. Otu *et al.* [13] has modeled Nigerian inflation rates as a SARIMA(1, 1, 1)x(0, 0, 1)$_{12}$ model. Malaria mortality rates have been modeled by a SARIMA(1, 1, 1)x(0, 0, 1)$_{12}$ model [14] and tuberculosis incidence has been modeled by a SARIMA(0, 1, 1)x(0, 1, 1)$_{12}$ model [15].

Foreign exchange rates have been modeled by SARIMA models in the recent past. For example, Appiah and Adetunde [16] modeled monthly Cedi-Dollar exchange rates by an ARIMA(1,1,1)model. Etuk[17] fitted a SARIMA(2,1,0)x(0,1,1)$_7$ model to daily Naira-Dollar exchange rates. A SARIMA(0,1,1)x(1,1,1)$_{12}$ model has been fitted to monthly Naira-Euro exchange rates[18]. An additive SARIMA model is used to model daily Ringgit-Naira exchange rates[19]). Monthly exchange rates of the Naira and Pound have been modeled as a SARIMA(0,1,0)x(2,1,1)$_{12}$ model[20] and finally monthly XAF and USD exchange rates have been modeled as a SARIMA(0,1,1)x(0,1,1)$_{12}$ model[21]. This is just to mention a few cases.

II. Materials and Methods

A. Data

The data analyzed in this work are 254 daily GBP and USD exchange rates from Wednesday 5th February 2014 to Thursday 16th October 2014 obtained from the website www.exchangerates.org.uk/GBP-USD-exchange-rate-history.html. It is to be interpreted as the amount of USD in one GBP.

B. Sarima Models

A stationary time series $\{X_t\}$ is said to follow an *autoregressive moving average model of order p and q* if

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \ldots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \ldots + \beta_q \varepsilon_{t-q} \quad (1)$$

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Model (1) is denoted by ARMA(p, q) and may be written as

$$A(L)X_t = B(L)\varepsilon_t \quad (2)$$

where $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \ldots - \alpha_p L^p$ and $B(L) = 1 + \beta_1\varepsilon_{t-1} + \beta_2\varepsilon_{t-2} + \ldots + \beta_q\varepsilon_{t-q}$ and $L^k X_t = X_{t-k}$. A(L) and B(L) are respectively the autoregressive (AR) and the moving average (MA) operators. The sequence of random variables $\{\varepsilon_t\}$ is a white noise process. For model stationarity and invertibility the zeros of A(L) and B(L) must lie outside the unit circle respectively.

Most real-life time series are not stationary. If $\{X_t\}$ is non-stationary, Box and Jenkins [22] proposed that differencing up to a certain order d could make the series stationary. Suppose that differencing up to the d^{th} order is just enough to render the series stationary. If the d^{th} difference, $\{\nabla^d X_t\}$, of $\{X_t\}$ satisfies (1) then $\{X_t\}$ is said to follow an *autoregressive integrated moving average model of order p, d and q* denoted by ARIMA(p, d, q). The symbol ∇ is the difference operator defined by $\nabla = 1 - L$.
Suppose that $\{X_t\}$ exhibits some seasonality of period s, Box and Jenkins [22] further proposed that it may be modeled by

$$A(L)\Phi(L^s)\nabla^d\nabla_s^D X_t = B(L)\Theta(L^s)\varepsilon_t \quad (3)$$

where $\Phi(L)$ and $\Theta(L)$ are seasonal AR and MA operators. Suppose they are polynomials of orders P and Q resoectively, and $\nabla_s = 1 - L^s$, the model (3) is called a *seasonal autoregressive integrated moving average model of order p, d, q, P, D, Q and s* and denoted by a SARIMA(p, d, q)x(P, D, Q)$_s$. D is the minimum order of seasonal differencing for data stationarity.

C. Sarima Model Fitting

The fitting of the model (3) begins with the determination of the orders p, d, q, P, D, Q and s. Knowledge of the theoretical properties of the components of the model (3) is used to estimate the orders. The seasonal period s may be obvious from the time plot or knowledge of the seasonal nature of the series. This is seldom the case. An inspection of the series could reveal a seasonal nature. The differencing orders d and D are often such that their sum is at most 2. Before and after differencing at each stage the series is tested for stationarity using the Augmented Dickey Fuller (ADF) unit root test. The AR orders p and P are respectively chosen as the non-seasonal and the seasonal cut-off lags of the partial autocorrelation function (PACF). Similarly the MA orders q and Q are respectively chosen as the non-seasonal and the seasonal cut-off lags of the autocorrelation function (ACF).
The parameters of the model are estimated by the application of a non-linear optimization criterion like the maximum likelihood procedure of the least error sum of squares procedure. After model fitting the model is subjected to some diagnostic checking process to ascertain the degree of its goodness-of-fit to the data. This is usually done by residual analysis. If the model residuals are uncorrelated and/or normally distributed, this is an indication of an adequate model.

All the analytical work herein shall be done using the Eviews package. It uses the least squares procedure for model estimation.

III. Results and Discussion

The time plot of the realization PDER in Figure 1 shows the series as made up of two parts: the first part up to 5^{th} July has a positive trend and the later part has a negative trend. Clearly this is an indication that before 5^{th} July the pound was rising in value and then decreased in value comparatively. A 7-day differencing of PDER produces the series SDPDER which exhibits a generally horizontal trend (See Figure 2). The ADF test statistic values for PDER and SDPDER are -0.77 and -3.66 respectively. With the 1%, 5% and 10% critical values at -3.46, -2.87 and -2.57 respectively, the test adjudges PDER as non-stationary and SDPDER as stationary. However the correlogram of SDPDER in Figure 3 reveals an autocorrelation structure of a non-stationary series. Non-seasonal differencing of SDPDER yields the series DSDPDER which is observed to have a horizontal trend in Figure 4. Its correlogram of Figure 5 attests to a stationary nature. Furthermore suggestive is seasonality of period 7 and the presence of seasonal AR and MA components of order one each. Also there is evidence of a SARIMA(0, 1, 1)x(0, 1, 1)$_7$ component, the autocorrelations at lags 6 and 8 in the ACF being comparable.

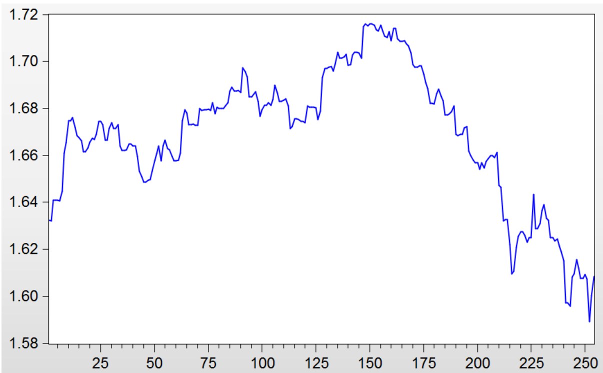

Fig.1. PDER

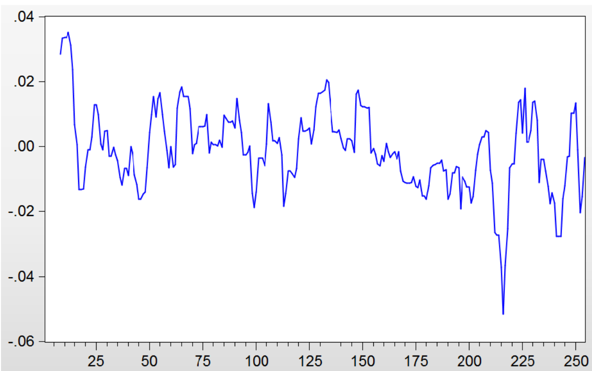

Fig.2. SDPDER

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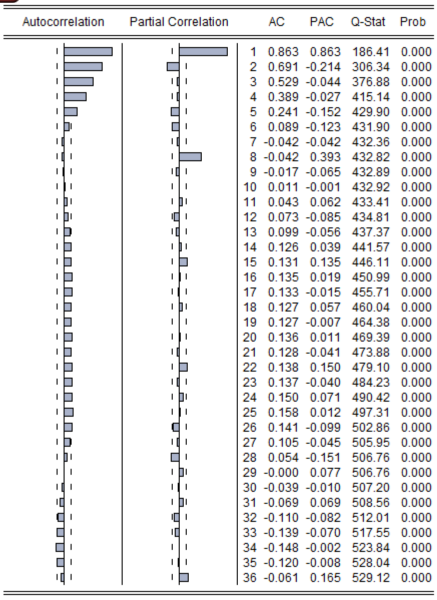

Fig.3. Correlogram of SDPDER

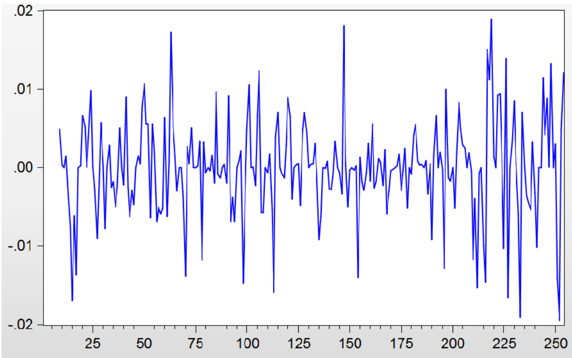

Fig.4. DSDPDER

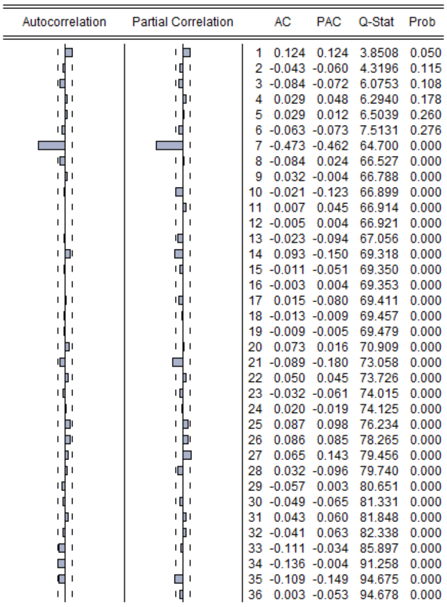

Fig.5. Correlogram of DSDPDER

Suggestive therefore are (1) a SARIMA(0, 1, 1)x(0, 1, 1)$_7$ model and (2) a SARIMA(0, 1, 0)x(1, 1, 1)7 model. Estimated in Table I is the SARIMA(0, 1, 1)x(0, 1, 1)$_7$ model given by

$$X_t - .0748\varepsilon_{t-1} + .9207\varepsilon_{t-7} + .1073\varepsilon_{t-8} = \varepsilon_t \qquad (4)$$

Estimated in Table II is the SARIMA(0, 1, 0)x(1, 1, 1)$_7$ model given by

$$X_t - .0052X_{t-7} + .9309\varepsilon_{t-7} = \varepsilon_t \qquad (5)$$

Both models (4) and (5) point to a SARIMA(0, 1, 0)x(0, 1, 1)$_7$ model because of the non-significance of some of their coefficients. This latest model is estimated in Table III as

$$X_t + .9328\varepsilon_{t-7} = \varepsilon_t \qquad (6)$$

where in (4), (5) and (6) X stands for DSDPDER. Clearly the model (6) is more adequate than either (4) and (5) on minimum AIC grounds. Besides, the correlogram of Figure 6 shows that the residuals are mostly uncorrelated.

Table I: Estimation of SARIMA (0,1,1)X(0,1,1)$_7$

Variable	Coefficient	Standard Error	t statistic	Probability
MA(1)	0.0748	0.0635	1.1779	0.2400
MA(7)	-0.9207	0.0241	-38.2639	0.0000
MA(8)	-0.1073	0.0628	-1.7087	0.0888

AIC = -7.8860

Table II: Estimation of SARIMA (0,1,0)X(1,1,1)$_7$

Variable	Coefficient	Standard Error	t statistic	Probability
AR(7)	0.0052	0.0692	0.0756	0.9398
MA(7)	-0.9309	0.0183	-50.8101	0.0000

AIC = -7.8877

Table III: Estimation of SARIMA (0,1,0)X(0,1,1)$_7$

Variable	Coefficient	Standard Error	t statistic	Probability
MA(7)	-0.9328	0.0164	-56.7210	0.0000

AIC = -7.8911

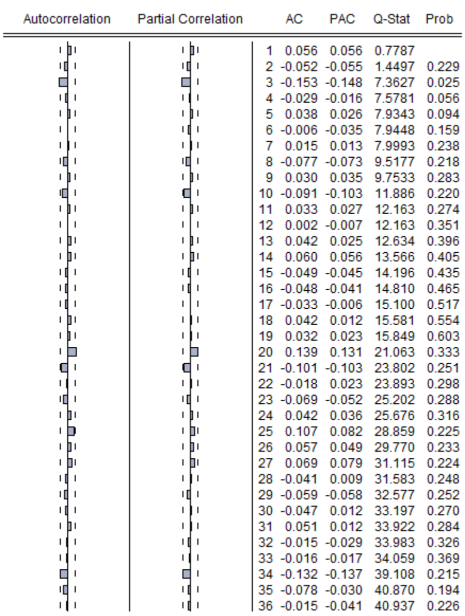

Fig.6. Correlogram of SARIMA (0,1,0x(0,1,1)$_7$ Residuals

IV. Conclusion

It is concluded that daily GBP-USD exchange rates follow the model (6). This means that a current DSDPDER value is dependent exclusively on the one-week-away shocks value. The rates may be forecasted on the basis of the SARIMA (0, 1, 0)x(0, 1, 1)$_7$ model (6).

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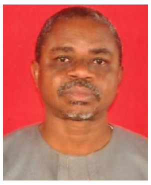

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