International Journal of Applied Science and Mathematics
Volume 1, Issue 2, ISSN (Online): 2394-2894

The Problem Solving Strategy of Solving Mathematical Problems: The Case Study of Esaase Bontefufuo Senior High Technical School, Amansie West District of Ghana

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***Abstract* – This paper is an action research paper which involves a sample of thirty five (35) third year students of Esaase Bontefufuo Senior High Technical School. The study was aimed at using the Problem Solving Strategy to help improve students' performance in mathematics. Before the study, it was observed that the students were not able to understand and solve mathematical problems properly. The problem solving strategy of teaching and learning was employed as the intervention strategy and was carried out in a series of activities. The pre – test and post – test scores obtained by the students were analyzed quantitatively based on the research questions that preceded the study. The results obtained from the pre – test and post – test was comparatively analyzed and showed a significant improvement on the students' ability in solving mathematical problems using the problem solving strategy. It was then concluded from the findings that the problem solving strategy of teaching and learning employed during the intervention processes improved the students' academic performance. The problem solving strategy of teaching and learning promoted students participation in the teaching and learning process and environment, and it must be encouraged by all.**

***Keywords* – Problem Solving Strategy, Mathematical Problems, Problem Solving Skills, Mathematics Education.**

I. INTRODUCTION

The importance of mathematics can be seen from its application in our daily lives and its role in technology. No other subject forms a strong binding force among various branches of science than mathematics and without it; knowledge of science often remains superficial [14].

Mathematics is among one of the most important subjects within the list of foundation subjects that make up the core curriculum for basic education in most countries throughout the world including Ghana.Reference [11] are also of the view that, mathematics can enable students achieve deeper understanding of science concepts by providing ways to quantify and explain science relationships.It is part of life and without it man cannot function [15].

Mathematics is the means of sharpening the mind, shaping reasoning ability and developing personality, hence its immense contribution to the general and basic education of many countries [2].

Regardless of the fact that mathematics is one of the basic subjects to build on for higher education, most students shy away from it. Most of them sometimes wish they could be excluded from mathematics lessons.In the authors' years of experience asclassroom teachers', several comment and utterances like: "Mathematics is a difficult subject"; 'It is preserved for students with higher intellectual ability'; 'Females cannot do mathematics'; "It is full of rules"; "It is inhuman" has been heard among a greater majority of students.These indicate that students have raised mental barriers against mathematics and these accounts for the massive drift towards the study of the social sciences such as the general arts by many potential science and mathematics students at higher educational levels [15]. It has also been observed that students' performance in mathematics has not been impressive by National Education Assessment and Trends in International Mathematics and Science Study (TIMSS) reports [1].

The trend therefore calls for the need to introduce problem solving strategy as a component of the mathematics curriculum to help apply mathematics to solve everyday life problems. Problem solving mathematics methodologies emphasize the teaching of mathematics content through processes that students encounter in real life situations. The Education Reform Review Committee in Ghana recommended a problem solving curriculum for pre-university education and this curriculum took effect in 2007. The current Senior High School (SHS) in Ghana syllabus specifically recommends the application of appropriate mathematical problem solving strategies in the teaching and learning of mathematics [13].Problem solving is not an isolated topic to be treated but rather a method with which students' may acquire the techniques to solve problems. The teacher's role in the problem solving mathematics classroom is drastically transformed from his or her traditional one as a source of knowledge and authority to a guide and facilitator.

In the book, '*How to Solve It*', by [17], he suggests that: "One of the first and foremost duties of the teacher is not to give his students the impression that mathematical problems have little connection with each other, and no connection at all with anything else....The teacher should

encourage the students to imagine cases in which they could utilize again the procedure used, or apply the result obtained"(p.15-16).

In his book, Polyasuggests four areas of problem-solving, which have become a widely used framework often recommended for teaching and assessing problem-solving skills by some researchers. These are:

1. Understanding the problem,
2. Devising a plan to solve the problem,
3. Implementing the plan, and
4. Reflecting on the problem.

The first step in Polya's model is to understand the problem. In order for a problem-solver to find a solution, they must first understand what they are being asked to find out.

Polya further suggested that teachers should ask students questions similar to the ones listed below:

- Do you understand all the words used in stating the problem?
- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or a diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

Teachers should decide which strategies to use based on students' answers to the questions above. For example, if a student does not understand the meaning of all of the words in the problem, he needs a vocabulary strategy, but if he can restate the problem, teaching him to paraphrase would not be beneficial.

According to [7], modelling and some form of interpretation must be existent for actual or authentic mathematical problem solving to occur. They further proposed that some form of reasoning must take place which ultimately promotes meaningful learning.

References [10] also outline some benefits of teaching mathematics through problem solving which include:

1. Helping students to understand that mathematics develops through sense-making process;
2. Deepening students' understanding of underlying mathematical ideas and methods, and
3. Engaging students' interest (p. 20)

The purpose of this research was to encourage students to participate in their mathematics classrooms in order to build self-confidence, improve their performance and to use mathematics appropriately in their daily lives. For instance, in mathematics classrooms, allowing students to work in groups in problem solving situations help them to analyse situations, to pose questions, check for accurate results and try different strategies.

II. Statement of the Problem

Concept-representation of all mathematical ideas needs to be taught with the use of practical activities and guidance from the teachers so that students can develop the concepts for themselves. However, mathematics topics have its' own fundamental concept in our everyday life experiences. The ultimate goal of mathematics teaching and learning is to help develop students' ability to solve a wide variety of complex mathematics problems and also to apply mathematics ideas to real world situations. The mathematics syllabus in Ghana recommends the use of mathematics in our daily life by recognizing and applying appropriate mathematics problem-solving strategies [13].

According to [12] more emphasis on teaching are based on the theoretical aspects while equal attention is not focused on the practical aspects of teaching and that problem solving is unpopular among Ghanaian students. Upon a careful examination of their end of term examination scripts and the solutions provided to assignments given to the students, the authors realized that most of the students could not solve non-routine mathematical problems. Most students in Esaase Bontefufuo Senior High Technical School remember the routine techniques without understanding why and how it works and therefore find problems in interpreting their answers. Mathematical skills in students are most often lacked because students are not given enough problem solving strategies.

Thus, this study was aimed at using the problem solving strategy to improve students' performance in solving mathematical problems.The authors believe that students can develop firm conceptual understanding in mathematics when problem solving strategy is employed in the mathematics classroom.Moreover, effective use of problem solving strategy of teaching and learning will promote cooperative learning among students, students' confidence and enhance their effective mathematical confidence in the classroom.

III. Research Questions

The study is intended to provide answers to the following questions:

1. How can students' performance in mathematics be improved through the use of problem solving strategy?
2. To what extent will the use of problem solving strategy of teaching and learning help students improve their competence in solving mathematical problems?

IV. Review of Related Literature

A. Problem Solving Strategies

[16] ascertained that problem-solving strategies are various methods that can be used to solve different types of problems. They further stressed that making a model, picture, or diagram; looking for a pattern; guessing and checking; making assumptions; creating an organized list; making a table or chart; solving a simpler problem; working backwards; and using logical reasoning are common problem-solving strategies.

Reference [18] also suggests that there is no chance of being able to solve a problem unless you understand the problem first. Understanding the problem requires not only knowing what to do but also the key pieces of information that somehow need to be put together to obtain the answer. As one will often not be able to absorb all the important information of a problem at once, Polya

advised that it is always necessary to read a problem several times both at the start and during working. Hence [18] propose a four-stage problem solving model which includes: understand and explore the problem, find a strategy, use the strategy to solve the problem, look back and reflect on the solution. Although these four stages of Problem Solving are listed in order of progression, for problems which are difficult it may not be possible to simply move through them consecutively to produce an answer. It is obvious that students move backwards and forwards between and across the steps in a problem solving process. During the solution process, one may find it necessary to look back at the original question from time to time to make sure that they are on the right path.

In a research study by [19], he suggests that besides the notion that problem solving skills are promoted when students transfer background skills, and are exposed to contextually rich and relevant problems, problem solving skills has to do with self-awareness and regulations. These include:

- Making sure that you understand what a problem is all about before you attempt a solution;
- Plan how to solve a problem;
- Monitoring, or keeping track of how well things are going during a solution; and
- Allocating resources, or deciding what to do, and for how long, as you work on the problem

The view is that during the problem solving process, students consciously interpret a problem, analyse the important attributes and decide on a solution [20]. [6]explains that problem solving skills entails more than drawing on one's background knowledge, instead information must be effectively applied to new problem situations.

[21] designed, implemented and evaluated a learning environment that emphasized students' acquisition of an overall metacognitive strategy for solving mathematical problems involving five stages. These are:

1. Build a mental representation of the problem
2. Decide how to solve the problem
3. Execute the necessary calculations
4. Interpret the outcome and formulate an answer
5. Evaluate the solution

Taking these stages into account during instruction is helpful and can improve students' problem solving ability.

According to [5], the success in solving a problem is directly related to the choice of the appropriate strategy. The most frequent solution strategies are:

1. Making systematic list
2. Drawing diagram
3. Looking for pattern
4. Making a table
5. Writing equation
6. Reasoning and simplifying the problem and
7. Guess and check

Additionally, The Institute for Advanced Study/Park City Mathematics Institute International Seminar [8] also outlined the process of solving mathematical problems to include:

1. Understanding and defining the problem,
2. Exploring the problem,
3. Hypothesizing,
4. Testing and formalizing,
5. Reflecting/evaluating

It is the authors' belief that these are feasible strategies to be implemented among students to assess whether problem solving strategy will improve performance and enhance students' competence in relation to solving mathematical problems.

V. METHODOLOGY

A. Research Design

The research design of this paper is an Action Research. The Action Research is structured in the form of pre-test, intervention and post-test to examine how the performance of the third year students is improved using problem solving strategy at Esaase Bontefufuo Senior High Technical School. In the view of [4], an Action Research uses pre-test and post-test data from the teaching of two instructional units to identify students controlled factors which promote or inhibit pupils' academic achievement. It also involves the application of scientific methods to solve classroom problems [9].

Reference [3] stated that action research helps teachers to be equipped with knowledge on the various approaches or methods that they can use to suit the children they teach. It also helps the teachers to evaluate the effectiveness of his/her teaching. Educational researchers use Action Research because it seeks to identify peculiar problem in the educational field especially in the classroom and suggest possible rectification of the problem by offering suitable intervention strategies and recommendations for use by other educators.The authors agreed on using action research as a research design for this paper because it deals with small scale intervention which is appropriate for a one classroom situation in the context of which the study was carried out.

B. Population and Sampling

To address the objective of the paper, Esaase Bontefufuo Senior High Technical School, Esaase was selected. The school is located in the Amansie West District of Ashanti Region of Ghana.The school runs four academic programs, namely, General Arts, Home Economics, Technical Studies and Business Studies. The school has a population of eight hundred and twenty – nine (829) students. The school is a mixed institution with three hundred and ninety – seven (397) males and four hundred and thirty-two (432) females. Two hundred and ninety six (296) are in the third year whiles the rest are in the second and first year.

The research was conducted with thirty - five (35) third year students of which twenty – four (24) were males and the remaining eleven (11) were females. Purposive sampling technique was used in this research and this is because an action research was adopted for this research paper.

C. Instrumentation

Tests were used as instruments for the collection of data for the research. Tests are a set of questions, exercises or

practical activities to measure someone's skill, ability and knowledge. It is used to determine the weakness or strength of students in a lesson [3]. The tests were used in two folds, that is, pre-test and post-test. The pre-test and the post-test were task given to students to carry out in order to know their level of performance. These tests also served as bases for evaluating the students.

VI. Data Analysis and Results

The results of the study obtained by the students were analyzed and discussed in relation to the objectives of this research. The pre – test was focused on finding out the students' understandingand level of finding solutions to various mathematical problems. A total of ten (10) questions which were marked out of fifty (50) marks was conducted for the thirty five (35) students and was administered in a period of 50 minutes. Table I shows a frequency distribution in percentages of the scores obtained by the students in the pre – test.

Table I. Frequency Distribution of Pre-test Scores in Percentages

Score	Frequency	Percentage (%)
1-10	7	20
11-20	21	60
21-30	7	20
31-40	0	0
41-50	0	0
Total	35	100

After administering the pre – test, the authors observed a generally low performance of the students in terms of solving mathematical problems involving the use of problem solving strategy, which was below the average score. The indication was that, students were not able to understand the mathematical problems and were not able to use appropriate strategies in finding solution to the mathematical problems.

To address these challenges of the students, a series of intervention activities using the problem solving strategy of teaching and learning were organized by the authors for the students and a post – test was administered to them. This was aimed at ascertaining whether the intervention had gone down well with the students.

The post – test, involving ten (10) questions, the students were required to find solution to mathematical problems by using problem solving strategy approach. It was also administered in a period of 50 minutes and was marked out of fifty (50) marks. Table II is the frequency distribution in percentages of the marks obtained by the students in the post – test.

Table II. Frequency Distribution of Post – test Scores in Percentages

Score	Frequency	Percentage (%)
1-10	0	0
11-20	7	20
21-30	14	40
31-40	7	20
41-50	7	20
Total	35	100

The post – test scores showed a change in the performance of the students as compared to that of the pre –test scores. The authors attributed the improvement in the students' performance to change in the teaching strategy adopted during the intervention. With the introduction of the problem solving strategy, the students' were exposed to numerous activities during the intervention processes.

The inferential analysis of the pre – test and the post – test was undertaken by the authors. The data used for this analysis were the pre – test and post – test scores obtained by the students. Table III indicates the mean, standard deviation and standard error mean of the paired samples.

Table III. Descriptive Statistics of Pre – test and Post – test Scores

N	Mean	Std. Deviation	Std. Error	Mean
Pretest	35	16.14	5.451	.921
Posttest	35	30.43	8.883	1.501

Upon comparing, a conclusion can be drawn from the mean scores of both the pre – test scores and the post – test scores. The mean scores showed a significant improvement in students' performance in solving mathematical problems. The intervention process using the problem solving strategy of teaching and learning has helped the students in finding solution to various mathematical problems.

VII. Discussion

The improved performance of the students on the post – test demonstrated much on what was suggested in the literature review. This success came as a result of the authors continuous encouraging of different problem solving skills adopted on using problem solving strategy in solving mathematical problems as well as cooperative learning among the students during the intervention activities.

Considering the scores obtained by the students in the pre – test and post – test, as shown in Tables 1 and Table 2 respectively, it can be confirmed that the performance of the students before the intervention was very poor. The respective frequency distribution tables of the pre – test and post – test (Tables I and Table II) clearly indicated the difference in the scores obtained by the students. In the pre – test (see Table 1) for instance, twenty - eight (28) out the thirty-five (35) students scored less than half of the 50 marks for the test representing 80% of the total number of students. This poor performance of the students in the pre – test was due to the teaching strategy used in teaching the students previously. With this teaching strategy, the students did not have the chance of using their experience to create their own understanding, since lessons were delivered to them in an organized form. The students were used to only rote (traditional) learning method and imitating teachers, this did not give the students the needed wisdom to survive independently, applying problem solving strategy in solving mathematical problems and as well as real - world situations. As a result of this, students were not able to understand and find

solution to mathematical problems. The authors also identified that the students lacked cooperative learning abilities and hence the average students could not help thelow performing colleagues. The students could not analyze simple mathematical problems and come out with its solution pattern.

Results from the post – test scores by the students, as indicated in Table II, clearly showed that the students performed much better as compared to the pre – test scores. This suggests that they had improved upon their ability to find solution to mathematical problems through the use of problem solving strategy. From the frequency distribution of the post – test scores (Table II), out the thirty-five (35) students who took part in the test, twenty-eight (28) of them obtained more than half of the total mark of 50 for the test, representing 80%of the total students number. The results indicated an upwards trend in the post – test scores, which showed that the intervention activities were effective in assisting the students overcome their problems and helping them in their learning process.

The improvement in the performance of the students, which was shown in the post – test scores they obtained, was not by chance, but through the problem solving strategy of teaching that the authors employed during the intervention activities. With the problem solving strategy of teaching approach, the authors designed a well – planned intervention activity in the lessons with the students. The problem solving strategy approach of teaching enabled the students to participate actively in the lessons and also encouraged cooperative learning among the students. And in effect, each student in the group was not only responsible for learning what was being taught alone, but also helped their colleagues who were still having problems and thus created a good learning atmosphere.

VIII. Revisiting the Research Questions

From the intervention activities initiated by the authors' with the thirty-five (35) third year students' of Esaase Bontefufuo Senior High Technical School, as well as the findings in the data analysis, the research questions were revisited and analyzed. The findings from the research study were related to the research questions and assessed whether they had answered the research questions undoubtedly.

Research Question 1: How can students' performance in mathematics be improved through the use of problem solving strategy?

Table III showed the result of the intervention by comparing the pre-test scores of the individual students' with their respective post test scores. It revealed a significant improvement in students' performance after the introduction of the problem solving strategy of teaching and learning by the authors. The post-test mean score of 30.43 (Standard deviation of 8.883) is significantly higher than the pre-test mean score of 16.14 (Standard deviation of 5.451). This indicated that the use of problem solving strategy in teaching and learning mathematics topics brought about a tremendous improvement in students' conceptual understanding and performance.

Research Question 2: To what extent will the use of problem solving strategy of teaching and learning help students improve their competence in solving mathematical problems?

The authors observed that students who previously employed the rote (traditional) learning method of solving mathematical problems were discouraged and refrained from such methods. The observations made it clear that students have putting a stop to their usual 'chew 'and 'pour' (memorizing and imitating) method of solving mathematical problems as well. The problem solving strategy of teaching through the interventional processes has helped the students to now understand the concepts of solving mathematical problems systematically and not relying on the memorized procedures. With the problem solving strategy of teaching and learning, the students can have the chance of using their own experience to create their understanding rather than being delivered to them in an already organized form. Again, students discovered during the intervention that there are often several correct ways of finding a solution to a mathematical problem.

IX. Conclusion

The authors are of the view that if mathematics teachers are able to use the appropriate methods in teaching students such as Problem solving strategy in teaching mathematical concepts which involves:

- Proper understanding and applying of mathematics;
- Sequencing lessons in a way that will placed the student at the centre of learning;
- Seeing pattern throughout mathematics and making connections between concepts;
- Listening to others reasoning mathematically and equally communicate their reasoning as well;
- Developing the capacity and the flexibility of thinking that will help students in approaching new areas of mathematics and new problems;
- Providing examples and non-examples of the topic under study to help for better understanding;
- Enjoying mathematics and being willing to pursue it to the highest level;
- Teachers serving as a facilitators and giving students the opportunity in exploring and constructing their own understanding;
- Viewing themselves as capable of doing mathematics

Then this will bring about learning for better understanding and not rote (traditional) learning as far as the learning of mathematical concepts is concerned. This shows that there is no guaranteed way of making sure students learn but if they are well motivated, and the information is presented in well-structured manner using interesting and appropriate teaching and learning strategies and materials, then student will perform better.

Acknowledgment

The authors acknowledge with appreciation the assistance offered by the third year students of Esaase Bontefufuo Senior High Technical School, selected

purposely for the research. The authors are grateful to the entire staff of the school as well. They also express their gratitude to unknown reviewers for their useful comments.

Competing Interests

Authors have declared that no competing interests exist.

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