International Journal of Applied Science and Mathematics
Volume 2, Issue 6, ISSN (Online): 2394-2894

Domination Number in Edge Semientire Graph

Venkanagouda M. Goudar
Sri Gauthama Research Centre,
Department of Mathematics,
Sri Siddhartha Institute of Technology,
Tumkur, Karnataka, India
Email: vmgouda@gmail.com

Venkatesh S.H
Research Scholar, Sri Gauthama Research Centre, (Affiliated to Kuvempu University), Sri Siddhartha Institute of Technology, Tumkur, karnataka, India
Email: Sh1.venkatesh@gmail.com

Venkatesha
Department of Mathematics,
Kuvempu University, Shankarghatta,
Shimoga, Karnataka, India.
Email: vensprem@gmail.com

Abstract **– A set $D \subseteq V[e_e(G)]$ is dominating set of edge semientire graph,if every vertex not in D is adjacent to a vertex in D.The domination number of the edge semientire graph is the minimum cardinality of domination set of edge semientire graph $e_e(G)$. We also study the graph theoretic properties of $\gamma[e_e(G)]$ and its exact values for some standard graphs. The relation between $\gamma[e_e(G)]$ with other parameters are also investigated.**

Mathematics Subject Classification: **05C69, 05C70**

Keywords **– Connected Dominating Set, Dominating Set, Edge Semientire Graph, Total Dominating Set.**

I. Introduction

All the graphs considered here are simple, finite, non-trivial, undirected and connected. As usual $|V|=p$ and $|E|=q$ denotes the number of vertices and edges of a graph $G(V,E)$ respectively. By the neighborhood of a vertex v of G we mean the set $N_G(v)=\{u \in V(G): uv \in E(G)\}$. The degree of a vertex v, denoted by $d_G(v)$, is the cardinality of its neighborhood. In general $<X>$ to denote the subgraph induced by the set of vertices X and $N_G(v)$.

By a leaf we mean a vertex of degree one, while a support vertex is a vertex adjacent to a leaf. We say that a support vertex is strong(weak respectively) if it is adjacent to at least two leaves(exactly one leaf, respectively). The path on n vertices we denote by P_n.

The notation $\alpha_0(G)(\alpha_1(G))$ is the minimum number of vertices(edges) in a vertex(edge) cover of G. The maximum distance $d(u,v)$ for all u in G is eccentricity of v and maximum eccentricity is the diameter $diam(G)$. We say that a subset of $V(G)$ is independent if there is no edge between every two its vertices. A subset $D \subseteq V(G)$ is a dominating set of G if every vertex of $V(G)-D$ has a neighbor in D. The dominating number of G, denoted by $\gamma(G)$ is the minimum cardinality of a dominating set of G.

A set $F \subseteq E(G)$ is said to be an edge dominating set if every edge in $E(G)-F$ is adjacent to some edge in F. The Edge domination number of G is the cardinality of smallest edge dominating set of G and is denoted by $\gamma'(G)$. This concept was introduced by Arumugam and velammal [1].

A dominating set D is called connected dominating set of G if D is also connected. The connected domination number of a graph G, denoted by $\gamma_c(G)$ =min{$|D|$; D is connected dominating set of G}. The concept of domination in graphs with its many variations of parameters is now well studied in graph theory [4], [5].

A dominating set D is a total dominating set if the induced subgraph $<D>$ has no isolated vertices. The total domination number $\gamma_t(G)$ of a graph G is the minimum cardinality of a total dominating set. This concept was introduced by Cockayne,Dawes and Hedetniemi [2].

The edge semientire graph $e_e(G)$ of a plane graph G is a graph whose vertices can be plot in one to one correspondence with the edges and regions of G,in such a way that two vertices of $e_e(G)$ are adjacent if and only if the correspondence elements of G are adjacent. This concept was introduced by Kulli et.al.,[6].

II. Preliminary Notes

We need the following results to prove further results.

Theorem 2.1: [7] If G is a graph with no isolated vertex then $\gamma(G) \le \frac{p}{2}$.

Theorem 2.2: [8] For any graph G then $\gamma[G] \ge \lceil \frac{p}{1+\Delta(G)} \rceil$

In this next section we discuss results on domination number of edge semientire graph of a graph.

III. Domination Number in Edge Semientire Graph

We now define the domination number of edge semientire graph of a graph. A set $D \subseteq V[e_e(G)]$ is said to be dominating set of $e_e(G)$, if every vertex not in D is adjacent to a vertex in D of $e_e(G)$. The domination number of $e_e(G)$ is denoted by $\gamma[e_e(G)]$ is the minimum cardinality of a dominating set.

Analogously, a dominating set D of edge semientire graph is connected dominating set if the induced subgraph $<D>$ is connected. The connected domination number $\gamma_c[e_e(G)]$ is the minimum cardinality of connected dominating set of $e_e(G)$.

A dominating set D of edge semientire graph is total dominating set if the induced subgraph $<D>$ has no isolated vertices. In this paper we study the graph theoretical properties of $\gamma[e_e(G)]$, $\gamma_c[e_e(G)]$, $\gamma_t[e_e(G)]$ and many bounds were obtained in terms of elements of G. Also the relationship with these domination parameters were found. In fig 3.1 we depicted the graph G and the edge semientire graph. The dominating set of semientire graph is $D_1 = \{r_3, 5\}$.

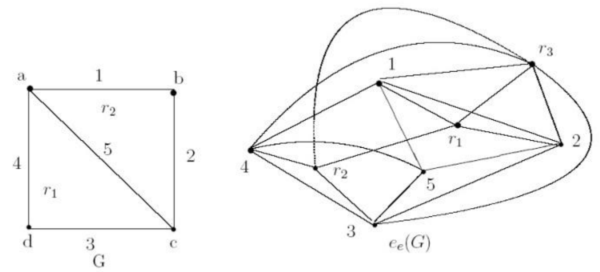

fig 3.1

IV. RESULTS

First we list out the exact values of $\gamma[e_e(G)]$ for some standard graphs.

Observation 4.1

a) For any tree T, $\gamma[e_e(T)] = 1$.

b)For any cycle C_n, $\gamma[e_e(C_n)] = 1$.

c) For any Wheel W_n, $\gamma[e_e(W_n)] = 2$.

Theorem 4.2: For any connected (p,q) graph G, $\gamma[e_e(G)] \le \lceil \frac{p}{2} \rceil +1$.

Proof. Let $E' = \{e_1', e_2', e_3', \ldots e_q'\}$ be the vertices of $e_e(G)$ corresponding to the edge set $E = \{e_1, e_2, \ldots. e_q\}$ of G and $R' = \{r_1', r_2', r_3', \ldots r_m'\}$ be the vertices of $e_e(G)$ corresponding to the region set $R = \{r_1, r_2, \ldots. r_m\}$ of G.

Let $F = \{e_1, e_2, \ldots e_k\}$ for $1 \le k \le q$ be the edge dominating set of G corresponding to the vertex set $F' = \{e_1', e_2', \ldots e_k'\}$ of $e_e(G)$. Let $K = E' - F'$ and let $D_1 = \{v_1, v_2, \ldots. v_d\}$ be the dominating set of $e_e(G)$ such that $D_1 \subseteq F' \cup K \cup R'$. We have the following cases

Case 1. If $v_i \in D_1$ for all i, such that $v_i \in F'$, then $F' = D_1$. It follows that, $\gamma[e_e(G)] = |D_1|$. Hence by Theorem 2.1, $\gamma[e_e(G)] \le \lceil \frac{p}{2} \rceil +1$.

Case 2. Suppose, $v_i \in D_1$ and $v_j \notin F'$ for some $1 \le j \le d$. Let $K' \subseteq K$, $R_1' \subseteq R'$ such that $K' = \cup N(v_k)$ for all $v_k \in F' \cup K \cup R'$ and $R_1' = \cup N(v_l)$ for all $v_l \in F' \cup K \cup R'$ and $v_l \notin K'$. $F' \cup K' \cup R_1' = D_1$. It follows that, $\gamma[e_e(G)] = |D_1|$. Hence by Theorem2.1, $\gamma[e_e(G)] \le \lceil \frac{p}{2} \rceil +1$.

Case 3. If $v_i \in D_1$ and $v_i \in F' \cup R_2'$ for all i and $v_i \notin K$. Let $R_2' \subseteq R'$ such that $R_2' = \cup N(v_k)$ for all $v_k \in F' \cup R'$. Clearly, $F' \cup R' = D_1$ and hence $\gamma[e_e(G)] = |D_1|$. By Theorem2.1, $\gamma[e_e(G)] \le \lceil \frac{p}{2} \rceil +1$.

Case 4. If $v_i \in D_1$ and $v_i \in R_3'$ for all i and $v_i \notin E'$. Let $R_3' \subseteq R'$ such that $R_3' = \cup N(v_t)$ for all $v_t \in R'$. Clearly, $R' = D_1$ and hence $\gamma[e_e(G)] = |D_1|$.

By Theorem 2.1, $|D_1| \le \lceil \frac{p}{2} \rceil +1$, $\gamma[e_e(G)] \le \lceil \frac{p}{2} \rceil +1$.

Theorem 4.3: For any connected (p,q) graph G, $\gamma[e_e(G)] \le$ 2diam(G).

Proof. Let $E' = \{e_1', e_2', e_3', \ldots e_q'\}$ be the vertices of $e_e(G)$ corresponding to the edge set $E = \{e_1, e_2, \ldots. e_q\}$ of G and $R' = \{r_1', r_2', r_3', \ldots r_m'\}$ be the vertices of $e_e(G)$ corresponding to the region set $R = \{r_1, r_2, \ldots. r_m\}$ of G. The set $E_1 = \{e_1, e_2, \ldots. e_k\}$ of G constitute the diametral path in G. Clearly $|E_1| = diam(G)$. Now, without loss of generality let D_1 be the minimum dominating set in $e_e(G)$ and $D_2 \subseteq V[e_e(G)] - D_1$ such that $D_2 \in N(D_1)$. Since $E_1 \subseteq V[e_e(G)] - R'$ and D_1 is a $\gamma[e_e(G)]$ set then the diametral path includes at most $\gamma[e_e(G)] - 1$ edges joining the neighborhood of the vertices of D_1.

Hence $\gamma[e_e(G)] \le$ 2diam(G).

Theorem 4.4: For any connected (p,q) graph G, $\gamma[e_e(G)] \le \alpha_1 + 2$.

Proof. Let $E' = \{e_1', e_2', e_3', \ldots e_q'\}$ be the vertices of

International Journal of Applied Science and Mathematics
Volume 2, Issue 6, ISSN (Online): 2394-2894

$e_e(G)$ corresponding to the edge set $E=\{e_1,e_2,....e_q\}$ of G and $R'=\{r_1',r_2',r_3',...r_m'\}$ be the vertices of $e_e(G)$ corresponding to the region set $R=\{r_1,r_2,....r_m\}$ of G.

Suppose $S=\{e_1,e_2,,,,,e_j\}$ be the set of all end edges in G .Then $S\cup K$ where $K\subseteq E(G)-S$ be the set of edges of G such that $|S\cup K|=\alpha_1(G)$. Further, let $D_1=\{u_i\,|\,u_i\in V[e_e(G)]$ for some i$\}$ be the dominating set in $e_e(G)$. Now if $D_2=\{u_i'\,|\,u_i'\in N(u_i)\forall u_i\in D_1\}$ such that $D_1\cup D_2=V[e_e(G)]$.

Since $V[e_e(G)]=E'\cup R'$ and $E'\subseteq V[e_e(G)]$, we have $S\cup K\subseteq V[e_e(G)]$.

Hence $|D_1|\le \alpha_1+2$.

Theorem 4.5: For any connected (p,q) graph G, $\gamma[e_e(G)]\le q-\Delta(G)$ where $\Delta(G)$ is the maximum degree of G.

Proof. Let $V[e_e(G)]=E'\cup R'$ where $E'=\{e_1',e_2',e_3',...e_q'\}$ be the vertices of $e_e(G)$ corresponding to the edge set $E=\{e_1,e_2,....e_q\}$ of G and $R'=\{r_1',r_2',r_3',...r_m'\}$ be the vertices of $e_e(G)$ corresponding to the region set $R=\{r_1,r_2,....r_m\}$ of G.

Let $V[G]=\{v_1,v_2,,,,v_p\}$ be the set of vertices of G and v_i be any vertex of G such that $\Delta(G)=N(v_i)$. Consider the set $I=\{e_j/1\le j\le q\}$ of edges of G adjacent to v_i correspondence to the vertex set $I'=\{e_j'/1\le j\le q\}$ in $e_e(G)$.

Let $D_1=\{u_i/u_i\in V[e_e(G)]\}$ be the dominating set of $e_e(G)$ and $D_2=\{u_i'/u_i'\in N(u_i)\forall u_i\in D_1\}$ such that $D_1\cup D_2=V[e_e(G)]$. Clearly $D_1\subseteq V[e_e(G)]$ and $D_1\subseteq (E'-I')\cup R_1'$ where $R_1'\subseteq R'$.

Hence $|D_1|\le|E|-|I|$. Since $|E'|=|E|=q$ and $|I'|=|I|=\Delta(G)$,

we have, $\gamma[e_e(G)]\le q-\Delta(G)$.

Theorem 4.6: For any non-trivial connected (p,q) graph G, $\gamma[e_e(G)]+\gamma'(G)\le(p-1)$.

Proof. Let $|V(G)|=p$ and let $A=\{e_1,e_2,,,,e_k\}$ be the edge set of G such that $|A|=\alpha_1$. Let $B=E(G)-A$ be the maximum independent set of G such that $|B|=\beta_1$. Suppose $F=\{e_1,e_2....e_j\}$ be the minimum independent edge dominating set.Then F itself is a edge dominating set of G such that $|F|=\gamma'(G)$. Let D_1 be the dominating set of $e_e(G)$ such that $|D_1|=\gamma[e_e(G)]$. Every vertex of $V[e_e(G)]-D_1$ is adjacent to at least one vertex in D_1. Then D_1 is a minimum dominating set of $e_e(G)$. Clearly, $|D_1|\cup|F|\le|A|\cup|B|$.

Hence $\gamma[e_e(G)]+\gamma'(G)\le\alpha_1+\beta_1+1$, this implies $\gamma[e_e(G)]+\gamma'(G)\le(p+1)$.

Theorem 4.7: For any graph G, the connected domination number of edge semientire graph, $\gamma_c[e_e(G)]\ge\lceil\frac{p}{1+\Delta(G)}\rceil$.

Proof. Let $V[e_e(G)]=E'\cup R'$ where $E'=\{e_1',e_2',e_3',...e_q'\}$ be the vertices of $e_e(G)$ corresponding to the edge set $E=\{e_1,e_2,....e_q\}$ of G and $R'=\{r_1',r_2',r_3',...r_m'\}$ be the vertices of $e_e(G)$ corresponding to the region set $R=\{r_1,r_2,....r_m\}$ of G.

Let $V[G]=\{v_1,v_2,,,,v_p\}$ be the set of vertices of G such that $|V[G]|=p$. Let v_i be any vertex such that $\Delta(G)=N(v_i)$. Let $D_1=\{u_i/u_i\in V[e_e(G)]$ for some i$\}$ be the dominating set of $e_e(G)$ and $D'=\{u_i'\,|\,u_i'\in N(u_i)\forall u_i\in D_1\}$ such that $D_1\cup D'=V[e_e(G)]$. If D_1 is connected then D_1 itself forms a connected dominating set of $e_e(G)$. Otherwise, if D_1 is disconnected then $D_1'\subset D'$ such that $D_1\cup D_1'$ is the minimum connected dominating set of $e_e(G)$.

By the Theorem2.2, it follows that $\gamma_c(e_e(G))\ge\lceil\frac{p}{1+\Delta(G)}\rceil$.

Theorem 4.8: For any (p,q) connected graph G, $\gamma_c[e_e(G)]\le\lceil\frac{p}{2}\rceil+4$, where $\gamma_c[e_e(G)]$ be the connected domination number of $e_e(G)$.

Proof. Let $V[e_e(G)]=E'\cup R'$ where, $E'=\{e_1',e_2',e_3',...e_q'\}$ be the vertices of $e_e(G)$ corresponding to the edge set $E=\{e_1,e_2,....e_q\}$ of G and $R'=\{r_1',r_2',r_3',...r_m'\}$ be the vertices of $e_e(G)$ corresponding to the region set $R=\{r_1,r_2,....r_m\}$ of G. Let $V[G]=\{v_1,v_2,,,,v_p\}$ be the set of vertices of G such that $|V[G]|=p$. Let $D_1=\{u_i/u_i\in V[e_e(G)]\}$ be the dominating set of $e_e(G)$ and $D'=\{u_i'|u_i'\in N(u_i)\forall u_i\in D_1\}$ such that $D_1\cup D'=V[e_e(G)]$. If D_1 is connected then D_1 itself forms a connected dominating set of $e_e(G)$. Otherwise, if D_1 is disconnected then we consider $D_1'\subset D'$ such that $K=D_1\cup D_1'$ is the minimum connected dominating set of $e_e(G)$. Since $K\subseteq V[e_e(G)]$ and by the Theorem2.1, it follows that $|D_1\cup D_1'|\geq\lceil\frac{p}{2}\rceil+4$.

Hence $\gamma_c(e_e(G))\geq\lceil\frac{p}{2}\rceil+4$.

Theorem 4.9: For any connected (p,q) graph G, $\gamma_c[e_e(G)]\leq diam(G)+\alpha_0(G)$.

Proof. Let $A=\{v_i/$ for some i $\}$ be the minimum number of vertices which covers all edges of G such $|A|=\alpha_0$. Let $V[G]=\{v_i/$ for all i $\}$ be the set of vertices in G then there exists two vertices $v_i,v_j\in V(G)$ such that $K_1=d(v_i,v_j)=\{v_1,v_2,,,,,v_k\}$ forms a diametral path in G. Let $V[e_e(G)]=E'\cup R'$ where, $E'=\{e_1',e_2',e_3',...e_q'\}$ be the vertices of $e_e(G)$ corresponding to the edge set $E=\{e_1,e_2,....e_q\}$ of G and $R'=\{r_1',r_2',r_3',...r_m'\}$ be the vertices of $e_e(G)$ corresponding to the region set $R=\{r_1,r_2,....r_m\}$ of G. Let $D_1=\{u_i/u_i\in V[e_e(G)]\}$ be the dominating set of $e_e(G)$ and $D'=\{u_i'/u_i'\in N(u_i)\forall u_i\in D_1\}$ such that $D_1\cup D'=V[e_e(G)]$. We have the following cases.

Case 1. Suppose D_1 itself forms connected dominating edge semi entire graph.Since D_1 includes the diametrical path. Hence $|D_1|\leq|K_1\cup A|$.

$\Rightarrow|D_1|\leq|K_1|\cup|A|$.

$\Rightarrow\gamma_c[e_e(G)]\leq diam(G)+\alpha_0(G)$.

Case 2. Suppose D_1 is disconnected then consider $D_1'\subset D'$ such that $<D_1\cup D_1'>$ is minimally connected dominating set of $e_e(G)$. Since D_1 includes the diametrical path, we have $|D_1\cup D_1'|\leq|K_1\cup A|$

$\Rightarrow|D_1\cup D_1'|\leq|K_1|\cup|A|$

$\therefore\gamma_c[e_e(G)]\leq diam(G)+\alpha_0(G)$.

Theorem 4.10: For any connected (p,q) graph, $\gamma_t[e_e(G)]\leq\lceil\frac{2p}{3}\rceil$, $\gamma_t[e_e(G)]$ be the total domination number of $e_e(G)$.

Proof. Let $E'=\{e_1',e_2',e_3',...e_q'\}$ be the vertices of $e_e(G)$ corresponding to the edge set $E=\{e_1,e_2,....e_q\}$ of G and $R'=\{r_1',r_2',r_3',...r_m'\}$ be the vertices of $e_e(G)$ corresponding to the region set $R=\{r_1,r_2,....r_m\}$ of G. Let $F=\{e_1,e_2,...e_k\}$ be the edge dominating set of G corresponding to the vertex set $F'=\{e_1',e_2',...e_k'\}$ of $e_e(G)$. Let $K=E'-F'$ and let $D_1=\{v_1,v_2,....v_d\}$ be the minimum dominating set of $e_e(G)$ such that $D_1\subseteq F'\cup K\cup R'$. We have the following cases.

Case 1. If D_1 has no isolated vertices, D_1 itself forms total dominating set of $e_e(G)$. By the Theorem2.1, it follows $|D_1|\leq\lceil\frac{2p}{3}\rceil$.

Hence $\gamma_t[e_e(G)]\leq\lceil\frac{2p}{3}\rceil$.

Case 2. If D_1 has isolated vertices then we consider $D_1'\subset(F'\cup K\cup R')-D_1$ and $<D_1\cup D_1'>$ is the minimum total dominating set of $e_e(G)$. By the Theorem2.1, it follows $|D_1\cup D_1'|\leq\lceil\frac{2p}{3}\rceil$.

Hence $\gamma_t[e_e]\leq\lceil\frac{2p}{3}\rceil$.

Theorem 4.11: For any connected (p,q) graph G, $\gamma[e_e(G)]\leq\gamma_t(G)+\Delta(G)$, where $\gamma_t(G)$ be the total domination number of G.

Proof. Let D be any minimum dominating set of G and let $K\subseteq V(G)$ be the minimum set of vertices in G which are adjacent to D. Clearly, $D\cup K$ be the total dominating set of G. By the definition of $e_e(G)$, $V[e_e(G)]=E'(G)\cup R'(G)$.

Further, let $D_1=\{u_i/u_i\in V[e_e(G)]\}$ be the dominating set of $e_e(G)$ and $D_2=\{u_i'/u_i'\in N(u_i)\forall u_i\in D_1\}$ be the set of vertices in $e_e(G)$ adjacent to at least one member of D_1 in $e_e(G)$. Then D_1 is a minimum dominating set of $e_e(G)$. Let $V[G]=\{v_1,v_2,,,,,v_p\}$ be the set of vertices of G such that $|V[G]|=p$. Let v_i

International Journal of Applied Science and Mathematics
Volume 2, Issue 6, ISSN (Online): 2394-2894

be any vertex such that $\Delta(G) = N(v_i)$. Clearly, $|D_1| \leq |D \cup K| + \Delta(G)$.

Hence $\gamma[e_e] \leq \gamma_t(G) + \Delta(G)$.

Theorem 4.12: For any connected (p,q) graph G, $\gamma_c[e_e(G)] \geq \gamma'(G)$, where $\gamma'(G)$ be the edge domination number of G.

Proof. Let $E' = \{e_1', e_2', e_3', \ldots e_q'\}$ be the vertices of $e_e(G)$ corresponding to the edge set $E = \{e_1, e_2, \ldots. e_q\}$ of G and $R' = \{r_1', r_2', r_3', \ldots r_m'\}$ be the vertices of $e_e(G)$ corresponding to the region set $R = \{r_1, r_2, \ldots. r_m\}$ of G. Suppose $F = \{e_1, e_2, \ldots, e_j\}$ be the minimum independent edge dominating set of G. Then F itself is a edge dominating set in G such that $F \subseteq E$ and $|F| = \gamma'(G)$. Every edge in $E - F$ is adjacent to at least one edge of F. Let $D_1 = \{u_i / u_i \in V[e_e(G)]\}$ be the dominating set of $e_e(G)$. Now if $D_2 = \{u_i' / u_i' \in N(u_i) \forall u_i \in D_1\}$ such that $D_1 \cup D_2 = V[e_e(G)]$. We have the following cases.

Case1. If D_1 is connected then D_1 itself forms connected domination of $e_e(G)$. Now $|D_1| \geq |F|$

$\Rightarrow \gamma_c[e_e(G)] \geq \gamma'(G)$.

Case2. If D_1 is not connected , we consider $D_2' \subset D_2$ such that $< D_1 \cup D_2' >$ is the minimum connected domination of $e_e(G)$.

$\Rightarrow |D_1 \cup D_2'| \geq |F|$

$\gamma_c[e_e(G)] \geq \gamma'(G)$.

Theorem 4.13: For any connected (p,q) graph G, $\gamma_c[e_e(G)] \leq \gamma'(G) + \gamma_c(G) + 3$.

Proof. Let $E' = \{e_1', e_2', e_3', \ldots e_q'\}$ be the vertices of $e_e(G)$ corresponding to the edge set $E = \{e_1, e_2, \ldots. e_q\}$ of G and $R' = \{r_1', r_2', r_3', \ldots r_m'\}$ be the vertices of $e_e(G)$ corresponding to the region set $R = \{r_1, r_2, \ldots. r_m\}$ of G.

Let $F = \{e_1, e_2, , , , e_k\}$ be the edge dominating set of G such that $|F| = \gamma'(G)$. Let $K = E(G) - F(G)$. Suppose F is connected then $F = F_1$ itself forms a connected dominating set of G such that $|F_1| = \gamma_c(G)$. Otherwise , suppose F is not connected. We consider $K_1 \subset K$ such that $< F \cup K_1 >$ is a minimum connected dominating set of G such that $|F \cup K_1| = \gamma_c(G)$. By the definition of $e_e(G)$, $V[e_e(G)] = E'(G) \cup R'(G)$.

Let $D_1 = \{u_i / u_i \in V[e_e(G)]\}$ be the dominating set of $e_e(G)$ and $D' = \{u_i' / u_i' \in N(u_i) \forall u_i \in D_1\}$ be the set of vertices in $e_e(G)$ adjacent to at least one member of D_1 in $e_e(G)$. Then D_1 is a minimum dominating set of $e_e(G)$.

Suppose D_1 is connected then $D_1 = H_1$ itself forms a connected dominating set of $e_e(G)$ such that $|H_1| = \gamma_c[e_e(G)]$. Otherwise, Consider $D_2 \subset D'$ such that $< D_1 \cup D_2 >$ is a minimum connected dominating set of $e_e(G)$ such that $|D_1 \cup D_2| = \gamma_c[e_e(G)]$.

We consider the following cases

Case 1. Suppose F is a dominating set of G. Let F_1 is a connected dominating set of G and D_1 is connected dominating set of $e_e(G)$ then $|H_1| \leq |F| + |F_1| + 3$.

Hence $\gamma_c[e_e(G)] \leq \gamma'(G) + \gamma_c(G) + 3$.

Case2. Suppose F is a dominating set of G, $< F_1 \cup K_1 >$ is a connected dominating set of G and D_1 is connected dominating set of $e_e(G)$ then $|H_1| \leq |F| + |F_1 \cup K_1| + 3$.

Hence $\gamma_c[e_e(G)] \leq \gamma'(G) + \gamma_c(G) + 3$.

Case3. Suppose F is a dominating set of G, $< F_1 \cup K_1 >$ is a connected dominating set of G and $< D_1 \cup D_2 >$ is connected dominating set of $e_e(G)$ then $|D_1 \cup D_2| \leq |F| + |F \cup K_1| + 3$.

Hence $\gamma_c[e_e(G)] \leq \gamma'(G) + \gamma_c(G) + 3$.

Case4. Suppose F is a dominating set of G, F_1 is a connected dominating set of G and $< D_1 \cup D_2 >$ is connected dominating set of $e_e(G)$ then $|D_1 \cup D_2| \leq |F| + |F_1| + 3$.

Hence $\gamma_c[e_e(G)] \leq \gamma'(G) + \gamma_c(G) + 3$.

V. Conclusion

In this paper we established some domination results on edge semientire graphs. Many bounds on domination number of edge semientire graph are obtained.

References

[1] Arumugam.S and Velammal.,(1998), Edge domination in graphs, Taiwanese Journal of mathematics, Vol.2, No.2, 173-179.

[2] C.J.Cockayne, R.M.Dawes and S.T.Hedetniemi, Total domination in graphs, networks, 10(1980) 211-219.
[3] F.Harary, Graph theory, adison wesley , reading mass, (1969).
[4] T.W.Haynes, S.T.Hedetniemi and P.J.Slater, Fundamentals of domination in graphs, Marcel Dekker, Newyork, (1997).
[5] T.W.Haynes, S.T.Hedetniemi and P.J.Slater, Domination in graphs:advanced topics, Marcel Dekker, Newyork, (1998).
[6] Kulli V.R and Akka D.G, On semientire graphs, Jour.math.sci.vol.14, No.6, 1980.
[7] O.Ore, Theory of graphs, Amer.Math.Soc.Colloq.Publ., 38, Providence(1962).
[8] H.B.Walikar, B.D.Acharya and E.Sampathkumar, Recent developments in the theory of domination in graphs. In MRI Lecture Notes in Math. Mehta research Instit.,Allahbad No.1(1979).