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Impulse-Response Function Analysis: An Application to Macroeconomics Data of Albania

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***Abstract* – In the analysis of the time series, the vector's model auto regression VAR had a special place. The construction of VAR models is very efficient in the economic analysis. The VAR models help us to discover in empirical way the ties that could exist in short-terms or mid-terms periods. A special place during this analysis, it has also the finding and interpretation of impulse-responses. In this study it will be realized the presentation of VAR model, its evaluation for the ties between the interest rate of credit and interest rate in Albania for the period 1996-2014. Likewise with the help of VAR models, will be found mathematically impulse-response and later will be implemented in the case of interest rate of credits and deposits.**

Keywords **– VAR, Impulse-Response, Causality.**

I. Introduction

The concept of impulse response analysis has been made popular by Sims (1980, 1981) and can be used to study causality between variables. It defines a variable y1 to be causal for another variable y2 if new (unpredicted) information in y1,t leads us to change our forecast of y2,t+h for at least one h ≥ 1 (see Hamilton (1994, p. 319)). The new information is commonly modeled as a one–time exogenous shock which occurs in one variable of the system, say y1. If variable y1 is causal for another variable y2, the one–time shock in y1 should change the path of y2. In other words, y2 should respond to the impulse in y1, henceforth the name impulse response analysis.

Granger Causality (Granger 1969) is a concept that belongs to the field of econometrics, which refers to the ties between two time series. Granger (1969) defined causality as a term of predictability based on fact that the effect cannot come before the cause. Later, Goebel (Goebel, Roebroeck et al. 2003) implemented Granger causality to discover the direction of information flow between the zones of the brain.

Formally, we consider a time series k-dimensional $\mathbf{y}_t$

$$\mathbf{y}_t = [y_{1t} \quad y_{2t} \quad \dots \quad y_{kt}]'$$

Composed from k time series in time t. The identification of Granger causality is based on determination of prediction of future values of series Yt, by using the data of a set of series values with time delay until the lag p, $(y_{t-1}, y_{t-2}, \dots, y_{t-p})$. In this way, we take an auto regression vector model k-dimensional (VAR) of order p, defined from:

$$\mathbf{y}_t = \mathbf{v} + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \dots + \mathbf{A}_p\mathbf{y}_{t-p} + \mathbf{u}_t,$$

where $\mathbf{u}_t$ is vector of cause of remnant with average zero and matrix of covariance given from:

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_{11}^2 & \sigma_{21} & \cdots & \sigma_{k1} \\ \sigma_{12} & \sigma_{22}^2 & \cdots & \sigma_{k2} \\ \sigma_{13} & \sigma_{23} & \cdots & \sigma_{k3} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1k} & \sigma_{2k} & \cdots & \sigma_{kk}^2 \end{bmatrix}$$

And $\mathbf{v}$ and $\mathbf{A}_i$ $(i=1,2,\dots,p)$ are matrixes of coefficients given from:

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix} \qquad \mathbf{A}_i = \begin{bmatrix} a_{11i} & a_{21i} & \cdots & a_{k1i} \\ a_{12i} & a_{22i} & \cdots & a_{k2i} \\ a_{13i} & a_{23i} & \cdots & a_{k3i} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1ki} & a_{2ki} & \cdots & a_{kki} \end{bmatrix}$$

The model VAR usually is a way for the identification of Granger causality. One important result of model VAR is that series y_{jt} does not cause y_{lt}, then and only then when coefficients $a_{jli}=0$ for every i. With other words, the written values on y_{jt} are used to predict the future values of y_{lt}.

For this reason, the Granger causality can be identified simply as a simple analysis of VAR models, and the direction of causality can be interpreted as the direction of the flow of information.

Furthermore, Granger causality relationship is not necessarily reciprocal, for example, y_{jt} may Granger cause the signal y_{lt}, without any implication that y_{lt} Granger causes y_{jt}.

There are two widely used approaches to assigning significance to the elements of matrices A_i. The first is based on a Wald test for the statistical significance of the causality coefficients of a VAR model (Lutkepohl 1993). The second one is based on the computation of F statistics by considering the ratio of residual variances and is described in detail by Geweke (Geweke, 1982).

II. VAR Analysis

Consider a two-variable VAR(1) with k=2.

(1) $y_t = b_{10} - b_{12}z_t + c_{11}y_{t-1} + c_{12}z_{t-1} + \varepsilon_{yt}$

(2) $z_t = b_{20} - b_{21}y_t + c_{21}y_{t-1} + c_{22}z_{t-1} + \varepsilon_{zt}$

with $\varepsilon_{it} \sim i.i.d(0, \sigma_{\varepsilon i}^2)$ and $\operatorname{cov}(\varepsilon_y, \varepsilon_z) = 0$

In matrix form:

(3) $\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$

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More simply:

(4) $\boxed{BX_t = \Gamma_0 + \Gamma_1 X_{t-1} + \varepsilon_t}$

The structural form of Var model

By multiplying matrixes equations 4 side by side with the vice-versa matrix B, we take:

$B^{-1}BX_t = B^{-1}\Gamma_0 + B^{-1}\Gamma_1 X_{t-1} + B^{-1}\varepsilon_t$, so:

(5) $\boxed{X_t = A_0 + A_1 X_{t-1} + e_t}$

VAR Model in Standard Form

The above model can be simply merged as:

(6) $$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

These error terms are composites of the structural innovations from the primitive system.
What are their characteristics/moments?

$e_t = B^{-1}\varepsilon_t$ where

$$B^{-1} = \frac{1}{|B|} B^a = \frac{1}{|B|}(B^*)^T = \frac{1}{(1-b_{21}b_{12})} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix}$$

B^* = cofactor of B and $(B^*)^T$ = transpose.

Thus

(7) $$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \frac{1}{(1-b_{21}b_{12})} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

Or $e_{1t} = \dfrac{\varepsilon_{yt} - b_{12}\varepsilon_{zt}}{\Delta}$ where $\Delta = 1 - b_{21}b_{12}$

$$e_{2t} = \frac{-b_{21}\varepsilon_{yt} + \varepsilon_{zt}}{\Delta}$$

ε's are white noise, thus e's are $(0, \sigma_i^2)$:

$E(e_{it}) = 0$

$Var(e_{1t}) = E(e_{1t}^2) = \dfrac{E(\varepsilon_{yt}^2 + b_{12}^2\varepsilon_{zt}^2)}{\Delta^2} = \dfrac{\sigma_y^2 + b_{12}^2\sigma_z^2}{\Delta^2}$ is time independent, and the same is true for $Var(e_{2t})$.

But covariances are not zero:

Covar $(e_{1t}, e_{2t}) = E(e_{1t}e_{2t}) =$

$$\frac{E[(\varepsilon_{yt} - b_{12}\varepsilon_{zt})(\varepsilon_{zt} - b_{21}\varepsilon_{yt})]}{\Delta^2} = \frac{-(b_{12}\sigma_z^2 + b_{21}\sigma_y^2)}{\Delta^2} \neq 0.$$

So the shocks in a standard VAR are correlated. The only way to remove the correlation and make the covar = 0 is if we assume that the contemporaneous effects are zero: $b_{12} = b_{21} = 0$.

The var/covar matrix of the VAR shocks:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}.$$

III. Impact Multipliers

They trace the impact effect of a one unit change in a structural innovation. Ex: find the impact effect of $\varepsilon_{z,t}$ on y_t and z_t:

$$\frac{dy_t}{d\varepsilon_{z,t}} = \Phi_{12}(0) \qquad \frac{dz_t}{d\varepsilon_{z,t}} = \Phi_{22}(0)$$

Let's trace the effect one period ahead on and z_{t+1}

$$\frac{dy_{t+1}}{d\varepsilon_{z,t}} = \Phi_{12}(1) \qquad \frac{dz_{t+1}}{d\varepsilon_{z,t}} = \Phi_{22}(1)$$

Note that this is the same effect on y_t and z_t of a structural innovation one period ago:

$$\frac{dy_t}{d\varepsilon_{z,t-1}} = \Phi_{12}(1) \qquad \frac{dz_t}{d\varepsilon_{z,t-1}} = \Phi_{22}(1)$$

Impulse response functions are the plots of the effect of $\varepsilon_{z,t}$ on current and all future y and z. IRs show how $\{y_t\}$ or $\{z_t\}$ react to different shocks.

Impulse response function of y to a one unit change in the shock to z

$= \Phi_{12}(0)$, $\Phi_{12}(1)$, $\Phi_{12}(2)$, ...

Cumulated effect is the sum over IR functions: $\sum_{i=0}^{n}\Phi_{12}(i)$.

Long-run cumulated effect: $\lim_{n\to\infty}\sum_{i=0}^{n}\Phi_{12}(i)$

In practice we cannot calculate these effects since the SVAR is under identified. So we must impose additional restrictions on the VAR to identify the impulse responses. If we use the Cholesky decomposition and assume that y does not have a contemporaneous effect on z, then $b_{12} = 0$. Thus the error structure becomes lower triangular:

(a) $$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \begin{bmatrix} 1 & -b_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

The ε_y shock doesn't affect z directly but it affects it indirectly through its lagged effect in VAR.

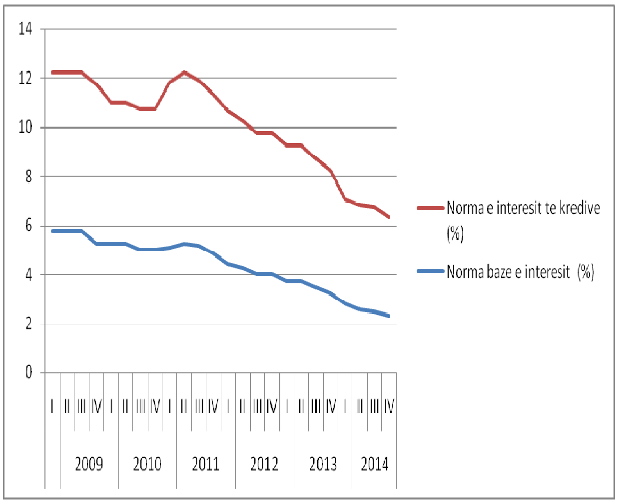
Granger Causality: If the z shock affects e1, e2 and the y shock doesn't affect e2 but it affects e1, then z is causally prior to y.

IV. Empiric Analysis

Due to their importance, the interest rates form a very well-studied field in economics. In our study we have selected to treat them in another point of view, the study of mutual influence of both interest rates, basic rates and credits' interest rates. Our goal is to discover the relationship that exists between them, and this will be realized by following the methodology and the process of an econometric model of a simple regression.

The data over the interest rates and credits' interest rates are taken from the publications of the Albanian Bank. The interest rates (basic rates) represent the rates of agreements of one- week repurchase (REPO), meanwhile the credits' interest rate are the new one day credits' interest rate. In the graph 1 it is presented their performance during years 2009-2014.

Graph 1. The performance of interest rate in Albania , 2009-2014



Source: Publication of the Bank of Albania

The basic rates of interest decreased from 2009 in 2010, in the first period of 2011 they increased, but in the following years has had constant decrease. In the same direction have moved also the credits' interest rates, but with a higher degree. This, certainly is understandable because the credits' interest rate is found from basic rate plus one percent (currently is decreased in historical levels).

The construction of VAR model

VAR model constructed in our case is a model VAR (1) two dimensional which is presented in the below table.

Vector Auto regression Estimates Included observations: 230 after adjustments Standard errors in ( ) & t-statistics in [ ]		
	NI	NK
NI(-1)	1.019505 (0.01085) [93.9718]	0.225510 (0.05052) [4.46416]
NK(-1)	-0.014932 (0.00641) [-2.33041]	0.860265 (0.02983) [28.8354]
R-squared	0.992547	0.875551

Let's compute the function of the impulse response for two series taken in the study $\{y_t\}$, $\{z_t\}$ in VAR (1) model two dimensional evaluated above. We mark yt= Ni and Zy=NK, the evaluated model is:

$$y_t = 1.02 y_{t-1} - 0.01 z_{t-1} + e_{1t}$$

$$z_t = 0.22 y_{t-1} + 0.86 z_{t-1} + e_{2t}$$

$\sigma_1^2 = \sigma_2^2$ and $\rho_{12} = 0.89$.

For this, we must get the estimates of the primitive function (SVAR) from the estimated coefficients:

Assume Cholesky decomposition $\Rightarrow b_{21} = 0$.

$$\rho_{12} = \frac{Cov_{1,2}}{SE_1 SE_2} = \frac{-b_{12}\sigma^2}{\sigma^2} = 0.89 \Rightarrow b_{12} = -0.89$$

Although this information is sufficient to calculate the impulse responses in this simple model, we can extract all of the coefficients of the primitive system as follows:

$$a_{10} = a_{20} = 0 \Rightarrow b_{20} = 0$$

$$b_{10} - b_{12} b_{20} = 0 \Rightarrow b_{10} = 0$$

$a_{22} = c_{22} = 0.86$ and $a_{21} = c_{21} = 0.22$

From $a_{11} = 1.02 = c_{11} - b_{12} c_{21}$ we get $c_{11} = 0.82$.

From $a_{11} = 1.02 = c_{11} - b_{12} c_{21}$

and

$$a_{12} = -0.01 = c_{12} - b_{12} c_{22} = c_{12} + 0.89(0.86) \Rightarrow c_{12} = -0.77$$

Substitute b_{12} into (a) to get:

$$e_{1t} = \varepsilon_{yt} + 0.89 \varepsilon_{zt}$$

$$e_{2t} = \varepsilon_{zt}$$

A 1 unit ε_{zt} shock is instantaneously absorbed 100% by z and 89% by y.

Impact multipliers:

(11) $y_t = 1.02 y_{t-1} - 0.01 z_{t-1} + \varepsilon_{yt} + 0.89 \varepsilon_{zt}$

$z_t = 0.22 y_{t-1} + 0.86 z_{t-1} + \varepsilon_{zt}$

At t = 0: $\frac{dy_t}{d\varepsilon_{z,t}} = 0.89$ $\quad \frac{dz_t}{d\varepsilon_{z,t}} = 1$

At t = 1: forward (b) by one period:

$$\frac{dy_{t+1}}{d\varepsilon_{z,t}} = 1.02 \frac{dy_t}{d\varepsilon_{z,t}} - 0.01 \frac{dz_t}{d\varepsilon_{z,t}} = 0.897$$

$$\frac{dz_{t+1}}{d\varepsilon_{z,t}} = 0.22 \frac{dy_t}{d\varepsilon_{z,t}} + 0.86 \frac{dz_t}{d\varepsilon_{z,t}} = 1.06$$

At t = 2: forward (b) by two periods:

$$\frac{dy_{t+2}}{d\varepsilon_{z,t}} = 1.02 \frac{dy_{t+1}}{d\varepsilon_{z,t}} - 0.01 \frac{dz_{t+1}}{d\varepsilon_{z,t}} = 0.9$$

$$\frac{dz_{t+2}}{d\varepsilon_{z,t}} = 0.22 \frac{dy_{t+1}}{d\varepsilon_{z,t}} + 0.86 \frac{dz_{t+1}}{d\varepsilon_{z,t}} = 1.1$$

Long-run multipliers: both variables go back to zero.

Cumulative multipliers:

$$\sum_{i=0}^{n} \frac{dy_{t+i}}{d\varepsilon_{z,t}} = 0.89 + 0.897 + 0.90 + ..$$

$$\sum_{i=0}^{n} \frac{dz_{t+i}}{d\varepsilon_{z,t}} = 1 + 1.06 + 1.1 + ...$$

VAR Lag Order Selection An important preliminary step in model building and impulse response analysis is the selection of the VAR lag order.

In appendix you can notice the graphical representation of impulse response of the evaluated model.

Determination of Lag Length: The determination of lag length is a trade-off between the curse of dimensionality and abbreviate models, which are not appropriate to indicate the dynamic adjustment. If the lag length is to short, autocorrelation of the error terms could lead to apparently significant and inefficient estimators. Therefore, one would receive wrong results. With the so called curse of dimensionality we understand, that even with a relatively small lag length a large number of parameters if required. On the other hand, with increasing number of parameters, the degrees of freedom decrease, which could possibly result in significant of inefficient estimators.

Information Criteria: The idea of information criteria is similar to the trade-off discussed above. On the one hand, the model should be able to reflect the observed process as precise as possible (error terms should be as small as possible) and on the other hand, to many variables lead to inefficient estimators. Therefore, the information criteria are combined out of the squared sum of residuals and a penalty term for the number of lags. In detail, for T observations we chose the lag length p in a way that the reduction of the squared residuals after augmenting lag p+1, is smaller than the according boost in the penalty term.

In this thesis we use some commonly used lag-order selection criteria to choose the lad order, such as AIC, HQ, SC and FPE.

Using Akaike Information Criterion to choose lag order.

$$AIC = -2\frac{\log L}{n} + \frac{2k}{n}$$

Using Schwartz Criterion to choose lag order.

$$SC = -2\frac{\log L}{n} + \frac{k\log n}{n}$$

Using Hannan-Quinn to choose lag order

$$HQ = -2\frac{\log L}{n} + 2k\frac{Ln(\log n)}{n}$$

Statistical Tests: An alternative to the global preparation are local statistic tests. For example, one could use Log Likelihood-Ratio Test (LR) to specify the lag length. Based on a VAR-Model with lag length p, one can check if the explanatory power of the model increased after taken lag p+1 into consideration. With the support of the LR test, one can control if the change in the power is significant or if it has only a power comparable to a random variable. Formally the LR Test is defined as,

$$\lambda = \frac{L(\beta_r)}{L(\beta_{ur})}$$

where r stands for restricted and ur for unrestricted model. Hence, it is tested whether it is possible to restrict all p+1 lags to zero. Additional lags cannot increase the power of estimation if λ is close to one, but if λ is close to zero. Therefore, the usual Wald-Test, which is adequate for an x^2 - distribution can be used. Unfortunately, the LR-test is not sufficient for models with missing lag structures.

V. Conclusion

In this study was described one model VAR(p) and especially one model two dimensional VAR(1). The criteria of selection of models are very important in the definition lag of model VAR. Mathematically from one model two dimensional VAR (1) are extracted the impulse responses. The usage of this technique was implemented in the case of Albania for two very important macroeconomics variables as interest rate of deposits and credits' interest rates. The impulse response are very important for the economists and politics makers because they show the moment of shock that may happen to every variable when we have changes in lag at the other variable. In the case of our two variables results se they are needed six months that the changes in interest rate of deposit to appear significantly in the credits' rate. So, a decrease of the interest rate today, gives its impact after 6 months.

References

[1] Stešević, I, 2008. "Econometric Model of Interest Rates on Deposits in Montenegro", *PANOECONOMICUS*, 2008, 3, str. 383-398

[2] Sinaj, V, 2014, "*Ekonometri*"

[3] Banka e Shqipërisë, publikimet për normat bazë të interesit dhe të kredisë http://www.bankofalbania.org/web/Tregjet_dhe_kursi_11_1.php

[4] Brigo, D, Mercurio, F. 2006, "*Interest Rate Models - Theory and Practice. With Smile, Inflation and Credit*". Springer, second edition edition, 2006.

[5] AKINLO, A. 2004 "Foreign direct investment and growth in Nigeria: An empirical investigation," Journal of Policy Modeling 26, 627–639

[6] ARELLANO, M., AND BOND, S. R. (1991). "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations," Review of Economic Studies, 58, 277-297.

[7] BAHRAUMAHAH, A.Z AND M.A. THANOON (2006) "Foreign Capital flow and economic growth in East Asian Countries" *China Economic Review*,17 (2006) 70-83.

[8] Davidson, R., and MacKinnon, J.G. (1993) Estimation and Inference in Econometrics. Oxford: Oxford University Press.

[9] Dickey, D. A. and Fuller, W. A. (1981) "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root." 1057-1071.

[10] Da Engle, Robert F. and Clive W. J. Granger (1987) "Co-integration and Error Correction: presentation 55(2): 251-276.