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A Note on soEnergy of Cocktailparty and Crown Graphs

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***Abstract* – Let G be finite non trivial connected graph. idegree, odegree, oidegree of a minimal dominating set were introduced and oEnergy of a graph with respect to the dominating set were calculated, an Algorithm to get the soEnergy is being introduced and it is found out for some standard graphs in the earlier papers. In this paper soEnergy of complete graph, cocktail graph, complete bipartite graph and crown graphs with respect to the given minimal dominating sets were found out.**

***Keywords* – oEnergy, soEnergy.**

I. Introduction

Let G = (V, E) be a finite non trivial connected graph. A set D is a dominating set of G if every vertex in V-D is adjacent to some vertex in D. A dominating set D of G is called a minimal dominating set if no proper subset of D is a dominating set.

Cocktail party graph is the graph consisting of two rows of paired nodes in which all nodes but the paired ones are connected with a graph edge [5].A crown graph on 2n vertices is an undirected graph with two sets of vertices u_i and v_i and with an edge from u_i to v_j whenever $i \neq j$[5]. In this paper soEnergy of complete graphs, complete bipartite graphs, cocktail party graphs and crown graphs are being calculated with respect to their minimal dominating sets.

II. Preliminaries

Definition 1.1

Let G be a graph and S be a subset of V(G). Let v∈ V-S, the idegree or indegree of v with respect to S is the number of neighbours of v in V-S and it is denoted by $id_S(v)$.

Definition 1.2

Let G be a graph and S be a subset of V(G). Let v ∈ V-S, the odegree or outdegree of v with respect to S is the number of neighbours of v in S and is denoted as $od_S(v)$.

Definition 1.3

Let G be graph and S be a subset of V(G). Let v ∈ V-S, the oidegree or outindegree of v with respect to S is $od_S(v)-id_S(v)$ if $od_S(v)>id_S(v)$ and it is denoted by $oid_S(v)$.

Definition 1.4

Let G be a graph and S be a subset of V(G). Let v ∈ V-S, the iodegree or inoutdegree of v with respect to S is $id_S(v)-od_S(v)$ if $id_S(v)>od_S(v)$ and it is denoted by $iod_S(v)$.

Definition 1.5

Let G be a graph and D be a dominating set, oEnergy of a graph with respect to D denoted by $o\varepsilon_D(G)$ is the summation of all oid if od > id or otherwise zero.

Definition 1.6

Let G be a graph and D be a minimal dominating set, then energy curve is the curve obtained by joining the oEnergies with respect to D_{i-1} and D_i for $1 \le i \le n$, taking the number of vertices of D_i along the x axis and the oEnergy with respect to the D_i along the y axis.

Definition 1.7

Let G be a graph and D be a minimal dominating set, soEnergy of a graph with respect to D is $\sum_{i=0}^{|V-D|} o\varepsilon_{D_{i+1}}(G)$ where $D_{i+1} = D_i \cup V_{i+1}$, V_{i+1} is a singleton vertex with minimum oidegree of V-D_i and D_0 is a minimal dominating set where $0 \le i \le |V-D|$, it is denoted by $so\varepsilon_D(G)$.

Definition 1.8

Let G be a graph and MDS(G) be the set of all minimal dominating set of G, then Hardihood$^+$ of a graph G is max{ $so\varepsilon_{MDS(G)}(G)$ } is denoted as HD$^+$(G).

Definition 1.9

Let G be a graph and MDS(G) be the set of all minimal dominating set of G, then Hardihood$^-$ of a graph G is min{ $so\varepsilon_{MDS(G)}(G)$ } is denoted as HD$^-$(G).

Given below is the algorithm to find the soEnergy of any given graph with respect to the given dominating set.

Algorithm 1.10

1. Find a minimal dominating set D.
2. Find the idegree, odegree and oidegree for the vertices in the graph induced by <V-D>.
3. If oideree of the vertices are greater than zero, proceed to step 5.
4. If oidegree ≤ 0 then put oenergy = 0 and goto step 6.
5. Shift the vertex with minimum positive oidegree which appears first to the set D.
6. If no such positive oidegree exists, shift a vertex with oidegree zero to the set D otherwise shift a vertex minimum iodegree to the set D.
7. Find the idegree, odegree and iodegree and then oenergy for the vertices in the new <V-D> with respect to new D set.
8. Repeat steps 2 to 7 until |V-D|=0.
9. Find the sum of all the oEnergies at each step to get the soEnergy.

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10. Fix the x axis as the number of vertices of the set D_i and y axis as the oEnergy of the sets at each and every step. Plot the graph.

III. soENERGY OF COMPLETE GRAPHS AND COCKTAIL PARTY GRAPHS

A complete graph with n vertices denoted by Kn is a graph with n vertices in which each vertex is connected to each of the others (with one edge between each pair of vertices). soEnergy of complete graph is calculated below.

Theorem 3.1

Let K_n be the graph and D be a minimal dominating set, then $so\varepsilon_D(k_n) = \sum_{i=\frac{n}{2}}^{n} |V-D_i|[|D_i| - |V-D_i| - 1]$

Proof:

Let D be a minimal dominating set of K_n.

Claim:

For any K_n soEnergy is 0 if $|D_i| < \left\lceil \frac{n}{2} \right\rceil$.

i.e) $|D_i|-(|V-D_i|-1)<1$ if $|D_i| < \left\lceil \frac{n}{2} \right\rceil$

for,

when $|D_i| < \left\lceil \frac{n}{2} \right\rceil$

$|V-D_i|=|n- |\frac{n}{2}||$

$|n- \left\lceil \frac{n}{2} \right\rceil+1|$ or $|n- |\frac{n}{2}||$

$=|n- \left\lceil \frac{n}{2} \right\rceil+1|$ or $\frac{n}{2}$

i.e, $|V-D_i|-1 \geq \left\lceil \frac{n}{2} \right\rceil$ or $\frac{n}{2}$ -1

$|D_i|-(|V-D_i|-1)< \left\lceil \frac{n}{2} \right\rceil - \left\lceil \frac{n}{2} \right\rceil$ or $(\left\lceil \frac{n}{2} \right\rceil - (\left\lceil \frac{n}{2} \right\rceil -1)) <1$

Therefore soEnergy is 0 if $|D_i| < \left\lceil \frac{n}{2} \right\rceil$

After this stage, $|D_i| < \frac{n}{2}$,

oidegree of vertices are $|D_i|-(|V-D_i|-1)$

by definition, oEnergy at ith stage is $(|V-D_i|)\{|D_i|-|V-D_i|-1\}$

$$so\varepsilon_D(K_n) = \sum_{i=\frac{n}{2}}^{n} |V-D_i|[|D_i| - |V-D_i| - 1]$$

Hence the proof.

Energy Curve of K_8

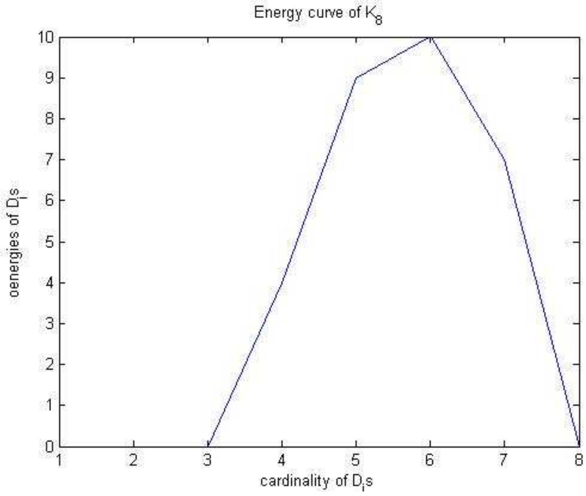

Fig.1.

The cocktail party graph of order n, also called the hyper octahedral graph. It is the graph complement of the Q9 ladder rung graph L_n and the dual graph of the hypercube graph Q_n. [5]

For a cocktail party graph the possible minimal dominating sets are minimum independent dominating set and connected dominating set. Connected dominating set is a dominating set D such that <D> is a connected graph.

Definition 3.2

Cocktail party graph is denoted by K_{nx2}, is a graph having the vertex set $V = \bigcup_{i=1}^{n}(U_i, V_i)$ and the edge set

$E = \{u_i u_j, v_i v_j; i \neq j\} \cup \{u_i v_j, v_i u_j; 1 \leq i \leq n\}$ [9]

Corollary 3.3

Let G be a cocktail party graph K_{nx2},

(i) $so\varepsilon_D(k_{nx2}) = \sum_{i=n+1}^{2n} |V-D_i|[|D_i| - |V-D_i|]$ where D is connected dominating set.

(ii) $so\varepsilon_D(k_{nx2}) = 4 + \sum_{i=n+1}^{2n} |V-D_i|[|D_i| - |V-D_i|]$ where D is minimum independent dominating set.

Proof:

Let G be a cocktail party graph K_{nx2} and D be a minimal dominating set.

i) Let D be the connected dominating set.

Number of vertices in a cocktail party graph is 2n. By replacing n by 2n in theorem 3.1,we get

$$so\varepsilon_D(k_{nx2}) = \sum_{i=n+1}^{2n} |V-D_i|[|D_i| - |V-D_i|].$$

(ii) Let D be minimum independent dominating set.

The cardinality of minimum independent dominating set is 2.

Since the vertices are independent, they gain 2 values for each vertices.

Therefore $so\varepsilon_D(k_{nx2}) = 4 + \sum_{i=n+1}^{2n} |V-D_i|[|D_i| - |V-D_i|]$

where D is minimum independent dominating set.

Corollary 3.4

Let G be a cocktail party graph K_{nx2},

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(i)Hardihood$^+$(K$_{nx2}$) $= 4 + \sum_{i=n+1}^{2n} |V - D_i| [|D_i| - |V - D_i|]$ where D is minimum independent dominating set.

(ii)Hardihood$^-$(K$_{nx2}$) $= \sum_{i=n+1}^{2n} |V - D_i| [|D_i| - |V - D_i|]$ where D is connected dominating set.

Proof:

Let G be a cocktail party graph K_{nx2}.

Minimum independent dominating set and connected dominating set are the minimal dominating sets of cocktail party graphs.

$so\varepsilon_D(k_{nx2})$ of both the dominating sets differ only by 4.

$$4 + \sum_{i=n+1}^{2n} |V - D_i| [|D_i| - |V - D_i|]$$

$$\geq \sum_{i=n+1}^{2n} |V - D_i| [|D_i| - |V - D_i|]$$

Therefore by the definition, Hardihood$^+$(K$_{nx2}$) = $= 4 + \sum_{i=n+1}^{2n} |V - D_i| [|D_i| - |V - D_i|]$. This is the soEnergy when D is minimal independent dominating set.

Hardihood$^-$(K$_{nx2}$) $= \sum_{i=n+1}^{2n} |V - D_i| [|D_i| - |V - D_i|]$ This is the soEnergy when D is connected dominating set.

Hence Hardihood$^+$(K$_{nx2}$) $= 4 + \sum_{i=n+1}^{2n} |V - D_i| [|D_i| - |V - D_i|]$ where D is minimum independent dominating set and Hardihood$^-$(K$_{nx2}$) $= \sum_{i=n+1}^{2n} |V - D_i| [|D_i| - |V - D_i|]$ where D is connected dominating set.

Energy curves of Cocktail party graphs are given below. Energy curve for K_{3x2} and K_{4x2} with respect to minimum independent set and connected dominating sets are traced.

Energy Curves of Cocktail graph with odd number of n

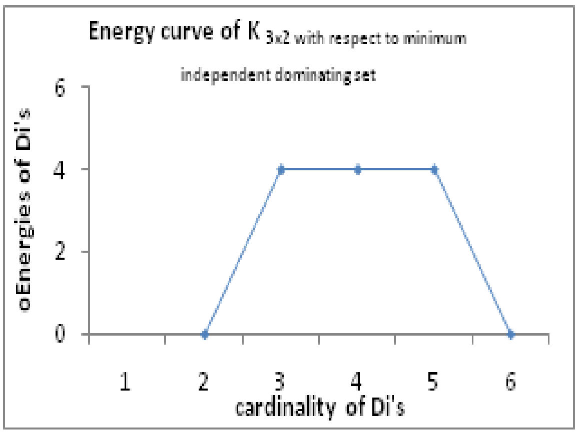

Fig. 2.

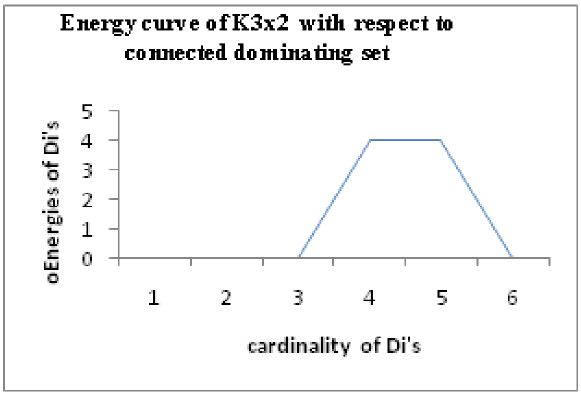

Fig. 3.

Energy curves of cocktail graph with even number of n

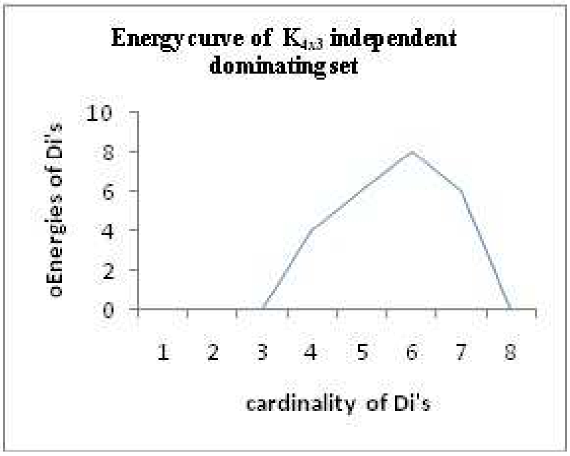

Fig. 4.

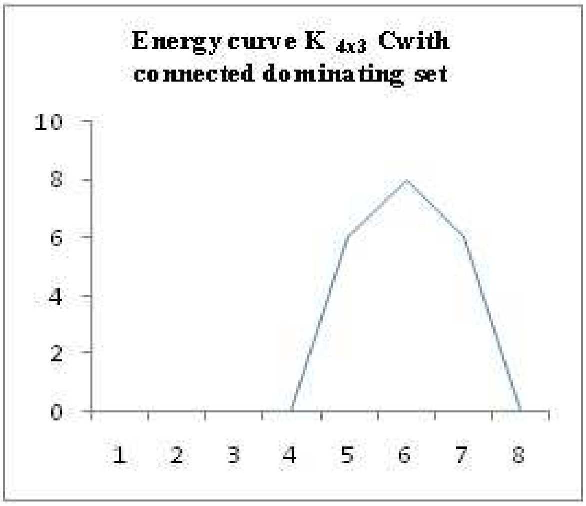

Fig. 5.

IV. soEnergy of Complete Bipartite Graph

A complete bipartite graph is a bipartite graph i.e., a set of graph vertices decomposed into two disjoint sets such that no two graph vertices within the same set are adjacent) such that every pair of graph vertices in the two sets are adjacent. There are three possible minimal dominating sets for a complete bipartite graph.

Definition 4.1

A dominating set of a complete bipartite graph $K_{n,m}$ that contains the vertices in a partite having the cardinality m and cardinality n are called m-partite dominating set and n-partite dominating set respectively.

Theorem 4.2

Let $K_{n,m}$ be the graph, then

(i) $so\varepsilon_D(K_{n,m}) = \frac{nm(m+1)}{2}$ where D is n-partite dominating set.

(ii) $so\varepsilon_D(K_{n,m}) = \frac{nm(n+1)}{2}$ where D is m-partite dominating set.

(iii) $so\varepsilon_D(K_{n,m}) = \sum_{i=n-2}^{n+m-2}(od-id)(D_i)$ where D is minimum independent dominating set.

Proof:

Let G be a complete bipartite graph $K_{n,m}$.

(i) Let D be the n-partite dominating set of the cardinality n, then $id_D=0$ and $od_D=n$ for every v_i in V-D.

Hence $oid_D=n$ for all v_i in V-D.

$o\varepsilon_D(K_{n,m}) = nm, n(m-1), n(m-2),\ldots n,0$

respectively

Therefore $so\varepsilon_D(K_{n,m}) = \frac{mn(n+1)}{2}$ where n>m.

(ii) Let D be the m-partite dominating set of the cardinality m, then $id_D=0$ and $od_D=m$ for every v_i in V-D.

As in case (i) $so\varepsilon_D(K_{n,m}) = \frac{mn(n+1)}{2}$ where D is m-partite dominating set.

(iii) Let D be minimum independent dominating set, taking one vertex from each partite, then the cardinality of the dominating set is 2.

For any $K_{n,m}$ soEnergy is 0 if $|D_i| < n-2$.

i.e) $o\varepsilon_{D_i}(K_{n,m}) = 0$ for i=1,2,..(n-2).

for $id_{Di} > od_{Di}$ when i= 1,2,...(n-2),

By definition, $so\varepsilon_D(K_{n,m}) = \sum_{i=n-2}^{n+m-2}(od-id)(D_i)$

Hence the proof.

If n>m, then Hardihood$^+$($K_{n,m}$) occurs with n-partite dominating set and Hardihood$^-$($K_{n,m}$) occurs for minimum independent dominating set.

Theorem 4.3

(i) Hardihood$^+$($K_{n,m}$) $= \frac{mn(m+1)}{2}$ when D is n-partite dominating set.

(ii) Hardihood$^-$($K_{n,m}$) $= \sum_{i=n-2}^{n+m-2}(od-id)_{D_i}$ where D is minimum independent dominating set.

Proof:

Let G be a complete bipartite graph $K_{n,m}$,n>m.

soEnergies of $K_{n,m}$ are $\frac{nm(m+1)}{2}, \frac{nm(n+1)}{2}, \sum_{i=n-2}^{n+m-2}(od-id)_{D_i}$,they are $\sum_{i=n-2}^{n+m-2}(od-id)_{D_i} \le \frac{mn(n+1)}{2} \le \frac{nm(m+1)}{2}$

Hence Hardihood$^+$($K_{n,m}$)= $\frac{nm(m+1)}{2}$. This is the soEnergy of $K_{n,m}$ when D is n-partite dominating set and Hardihood$^-$ $(K_{n,m}) = \sum_{i=n-2}^{n+m-2}(od-id)_{D_i}$ is the soEnergy when D is minimal dominating set.

Energy Curve of $K_{3,2}$

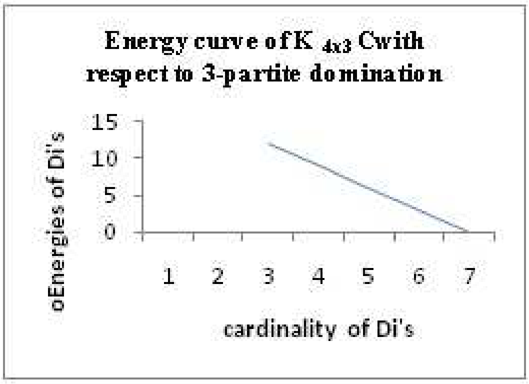

Fig. 6.

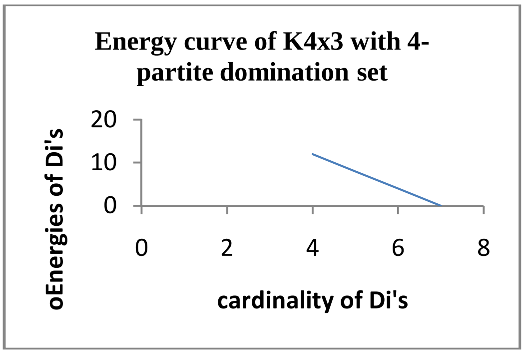

Fig. 7.

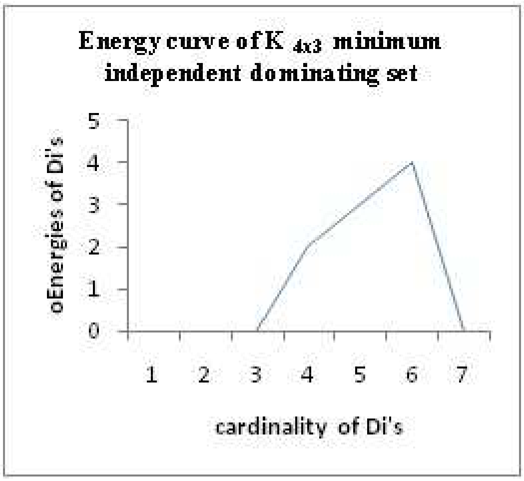

Fig. 8.

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V. soENERGY OF CROWN GRAPHS

The crown graph can be viewed as a complete bipartite graph from which the edges of a perfect matching have been removed, Biggs (1993) [4]. For a crown graph the possible minimal dominating sets with crown graphs are minimum and maximal independent dominating set with cardinality of dominating set as 2 and n. The number of edges of crown graph is n(n-1).

*Definition:*5.1

Crown graph S_n^0 for an integer n≥2 is the graph with vertex set $\{u_1, u_2, ...u_n, v_1, v_2, ..., v_n\}$ and the edge set $\{u_i v_j; 1 \le i, j \le n; i \ne j\}$.[8]

A dominating set D of G is called an independent dominating set if the vertices in D are independent. A dominating set D of G is called a maximum or minimum independent dominating set if D is an independent dominating set with maximum or minimum cardinality.

Theorem: 5.2

Let G be a S_n^0 for an integer n≥2

(i) $so\varepsilon_D(S_n^0) = \frac{n(n^2-1)}{2}$ where D is maximal independent dominating set of cardinality n.

(ii) $so\varepsilon_D(S_n^0) = \sum_{i=n}^{2n}(od - id)_{D_i}$ where D is minimum independent dominating set.

Proof:

Let G be a Crown graph S_n^0 for an integer n≥2.

Let D be the minimal dominating set.

(i) Let D be the maximal independent dominating set with cardinality n.

Then id=0 and od=n-1 for all the n vertices in the set V-D.

Therefore $o\varepsilon_{D_1}(S_n^0) = n(n-1)$

By theorem 14 n(n-1)+(n-1)(n-1)+(n-2)(n-1)+...+(n-1)$

$= \frac{(n-1)(n)(n+1)}{2} = \frac{n(n^2-1)}{2}$

Therefore $so\varepsilon_D(S_n^0) = \frac{n(n^2-1)}{2}$

(ii) Let D be the minimal independent dominating set with cardinality 2.

For any S_n^0, soEnergy is 0 if $|D_i| < n$.

i.e) $o\varepsilon_{D_i}(S_n^0) = 0$ for i=1,2,..(n-1).

for $id_{D_i} > od_{D_i}$ when i= 1,2,...(n-1),

By definition of $so\varepsilon_D(S_n^0) = \sum_{i=n}^{2n}(od - id)_{D_i}$.

Hence the proof.

Theorem: 5.3

Let G be a S_n^0 for an integer n≥2

(i) Hardihood$^+$(S_n^0)=$\frac{n(n^2-1)}{2}$ where D is maximal independent dominating set.

(ii) Hardihood$^-$(S_n^0)=$\sum_{i=n}^{2n}(od - id)_{D_i}$ where D is minimal independent dominating set.

Proof:

Let G be a S_n^0 for an integer n≥2.

soEnergies of a S_n^0 for an integer n≥2 are $\frac{n(n^2-1)}{2}$ and $\sum_{i=n}^{2n}(od - id)_{D_i}$. It is obvious that $ $\sum_{i=n}^{2n}(od - id)_{D_i} \le \frac{n(n^2-1)}{2}$.

Hence Hardihood$^+$(S_n^0)=$\frac{n(n^2-1)}{2}$ which is the soEnergy when D is n-partite dominating set.

And Hardihood$^-$(S_n^0)=$\sum_{i=n}^{2n}(od - id)_{D_i}$ which is the soEnergy when D is minimal independent.

Energy Curves of Crown graph

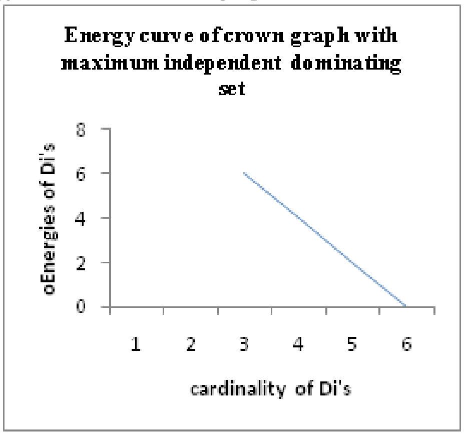

Fig. 9.

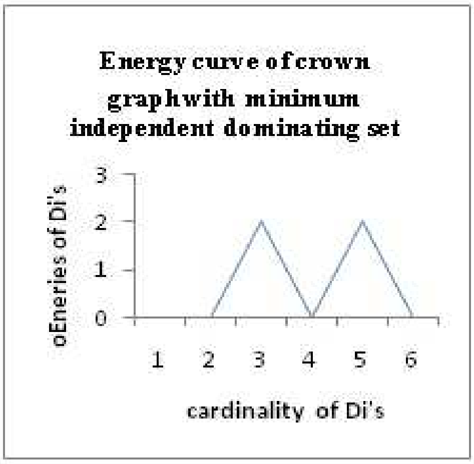

Fig. 10.

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