International Journal of Applied Science and Mathematics
Volume 3, Issue 2, ISSN (Online): 2394-2894

Compactness of the Topological Space

Shaolong Peng
School of Science, Shandong University of Technology,
Zibo 255049, China

Hongkui Li
School of Science, Shandong University of Technology,
Zibo 255049, China

Tao Li
Innovation and entrepreneurship College,
Shandong University of Technology,
Zibo 255049, China

***Abstract* – The concept of compact space was first put forward in 1921, the articles referenced in the concept of compactness is given to posterity, origin of the concept of column tightly associated with the compactness characterizations of metric space, belongs to the popular concept in the development history of topology. The article mainly introduces the concepts and several relative compactness.**

***Keywords* – Compact Space, Locally Compact Space, Strong Locally Compact, Sequence, Subset of Tight.**

I. Preface

With the progress of the society topology is applied to more and more widely, but also permeated various fields of engineering, theory of topological system is limited by the logic and topology, produced by a blend of both point convenience and logic of the system. Literature [1] this paper discusses the locale compactness and Hausdorff separability of convergence, the literature [2] [3] topology system is introduced, the topological system compactness Tychonoff product theorems are proved. Many scholars also proposed the concept of a variety of compactness, such as: XuanLiXin is set in a good compactness in article [4] introduced the concept of a few good compactness: shui-li Chen in [5] proposed a Fuzzy L almost good compactness concept: article [6] system was introduced and the study of the characteristics of the approximate good compactness and topological properties: article [7] in 1978 defines the compactness, article [8] using relevant far to redefine the concept of the family and depict the compactness; article [9] using relevant far clan, we define the topology of approximate compactness. In this paper, using the correlation almost related far and far domain family domain group introduced the concept of an almost compactness, discussed it with super good compactness, approximate compactness and almost compactness.

II. Some Concepts

Theorem 1 according to the topological space is tight, refers to the X each open cover shall be limited coverage. Direct product topological Spaces on product topology is a compact space. [10]Is a locally compact topological space space x ∈ x means for each point, U ∈ mu (x), makes U is tight.

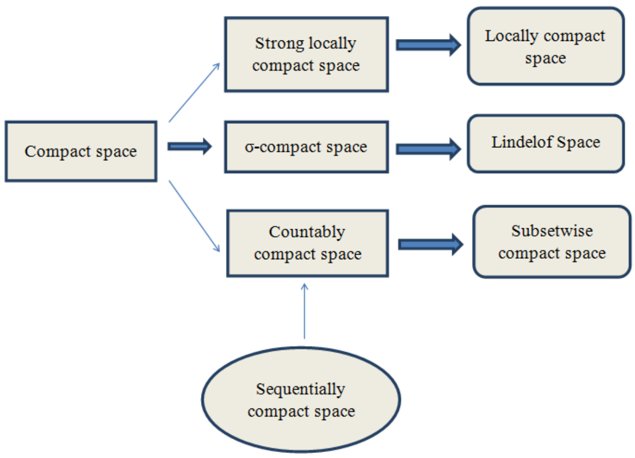
Theorem 2 topological space called a sequence of convergence child refers to a bit of X column contains columns; Refers to any of the X of X as a subset of tight infinite subset and accumulation point; According to X is a countable compact or countably compact refers to X arbitrary countable open cover shall be limited to cover.

Theorem 3 topological space sigma refers to X Bryant abet [11]into up to countably compact set and set.

Theorem 4 strong locally compact refers to every point in an open set of X, and the closure of open set is tight.

Theorem 5 topological space is called the K space refers to the subset if X and each closed set for A closed set, is A closed set.

III. About the Space of the Compactness of Implication Relationships



IV. Some Special Case

Example 1 there are locally compact and not strong locally compact topological space. N is a natural number set, order

$$A_n=\{n\}\times N\times\{1\}, n=1,2\ldots$$

$B_1=\emptyset$, $B_n=\{(n,1,2),(n,2,2),\ldots,(n,n-1,2)\}$, n=2,3…. To make

$$C_n=A_n\cup B_n,\quad n=1,2,\ldots.$$

$$X=\cup_{n=1}^{\infty}C_n.$$

Take the topology of X for all such set C, makes for a n, C belongs to C_n, and C_n \ C is limited, so the X is strong locally compact, and X is T_1space. make

Y=（N×{1}）∪（N×{2}），And take the topological tau on Y are as follows:

τ={C⊆Y|（N×{1}）\C is Finite set}∪{∅}。

Topological space Y is T_1 space, because Y each non-empty open set of closure is the whole space, it is not tight, so Y is not strong locally compact. However, Y is clearly locally compact.

Example 2 There is a certain Lindelof space, it is not a sigma is tight.

Proof the authenticated sigma tight must be Lindelof space. But Lindelof doesn't have to be a sigma tight space. X, for example, is a set of uncountable, life of open sets X itself, empty set ∅, as well as the complement to all at most countable subset. For every open set of complementary set X is at most countable, therefore, X is Lindelof space. And because only a limited set of X is tight, Therefore, X is unlikely to be sigma tigh.

Example 3 There is a subset tightly and tightly uncountable topological space.

Set X is into the set of all natural Numbers, for every natural number n, life u_n= {2n - 1, 2n}, is $\{u_n\}$ is clearly a topology of X, resulting in a topology of X

(1) The topological space X is a subset of tight. In fact, for every natural number n, 2n is single point set{2n-1}a focal point, and 2n - 1 is a single point set{2n}a focal point, and each the loophole of X set at least one point, namely X is a subset of tight.

(2) Topological space X is not tight

In fact, $\{u_n\}$ is a countable open covering of X, but it has no limited coverage.

Example 4 R^2 in two locally compact subspace, it is not a locally compact.

Let A={（x,y）|(x,y)∈R^2，x>0}，B ={（0,0）}. A and B are two dimensional European space of locally compact subspace of R^2. Due to point (0, 0) in A∪B not tight in the neighborhood, so A∪B not locally compact.

V. Corollary

Set K is a compact subset in Hausdorff space X, p∈X-K, there is X mutually disjoint open set U, V, K belong to U and p ∈ V.

Subspace is set Y X, Y is tight, if and only if any are covered in X Y open set contains a cover Y finite sub groups.

F: X→Y is a continuous map, X is a compact space, then f（x）is tight.

VI. Conclusion

The compactness conditions for promotion to the general topological space, there are a lot of compactness of the corresponding concept. So the compactness of the pivotal position in the topology. So well, the compactness of topological space topology has a great influence in learning.

Reference

[1] Liang Jihua. The convergence on the locale structure [J]. Journal of mathematics, 1995, 20 (3) : 294 ~ 300.

[2] Chen Yixiang. Stable Domain theory and Stone representation theorem [D]. Sichuan university, 1995.

[3] Chen Yixiang. Topology system category and son topology system [J]. Journal of shaanxi normal university (natural science edition), 1994, 22 (4) : 19 ~ 24.

[4] Wang G J.A new fuzzy compactness defined by fuzzy sets. The Math [J]. Math. Anal. Appl. 1983 hanjie xuebao/transactions: 1-23.

[5] Chen S L.L - fuzzy Almost nice compactness [J]. Proc. 6 th. IFSA World united V0l. (II). 1995:467-470.

[6] Shui-li Chen. L - good approximate compactness in Fuzzy topological space [J]. Journal of mathematics miscellaneous: base, 1996 (1) : 68-71.

[7] Lowen state Richard armitage comparison of company's compactness notions in fuzzy topo logical Spaces [J]. Journal Math. Anal. Appl., (1978:446-454.

[8] Meng Peiyuan, tempo can. Related to the family of far field does not clear subset of compactness [J]. Journal of mathematics journal, 1996 (4) : 534-538.

[9] Taupo. Topological space of the approximate compactness [J]. Fuzzy systems and mathematics, 2005, 12 (2) : 71-76.

[10] Zhao D, Zhao D. The N -compactness in L -fuzzy topological spaces[J]. Journal of Mathematical Analysis & Applications, 1987, 128(1):64-79.

[11] Chen Y. On compactness of induced I (L)-fuzzy topological spaces[J]. Fuzzy Sets & Systems, 1997, 88(88):373-378.

Author's Profile

Corresponding Author:
Hongkui Li

Acknowledgment

This work is supported by Doctoral Fund of Shandong University of Technology of China (4041/415061), Comprehensive budget management system design (9101/215043), Shandong Research project of teaching reform in undergraduate colleges and universities of China (2015Z071).