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Rayleigh-Taylor Instability through Porous Medium of Two Viscoelastic (Maxwellian) Superposed Fluids

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***Abstract* – An attempt has been made to investigate the instability of the plane interface between two Maxwellian viscoelastic superposed fluids in the presence of uniform rotation and variable magnetic field through porous medium. Following the linearized stability theory and normal mode analysis, the dispersion relation is obtained. The case of two uniform Maxwellian viscoelastic fluids separated by a horizontal boundary is studied. The stability analysis has been carried out, for mathematical simplicity, for highly viscous fluids of equal kinematic viscosities and equal Alfve'n velocities of the two fluids. It is found that for potentially stable configuration, the system is stable for disturbances of all wave numbers. However, the magnetic field succeeds in stabilizing certain wave-number range, which was unstable in the absence of magnetic field and rotation for the potentially unstable configuration. Sub cases of magnetic free and rotation free configurations are also considered, separately.**

***Keywords* – Maxwellian Viscoelastic Fluid, Porous Medium, Rayleigh-Taylor Instability, Uniform Rotation, Variable Magnetic Field.**

I. Introduction

When two fluids of different densities are superposed one over the other (or accelerated towards each other), the instability of the plane interface between the two fluids, when it occurs, is called Rayleigh-Taylor instability. Chandrasekhar [1] has given a detailed account of the instability of the plane interface between two incompressible and viscous fluids of different densities when the lighter fluid is accelerated into the heavier. A good account of hydrodynamic stability problems have been given by Drazin and Reid [2] and Joseph [3]. The influence of viscosity on the stability of the plane interface separating two electrically conducting, incompressible superposed fluids of uniform densities, when the whole system is immersed in a uniform horizontal magnetic field, has been studied by Bhatia [4]. He has carried out the stability analysis for two fluids of high viscosities and different uniform densities. Hide [5] has studied the case of a viscous conducting fluid in presence of a transverse magnetic field and found that magnetic field considerable stabilizes the configuration and it is possible to have oscillatory motion in presence of magnetic field even if the configuration is thoroughly unstable.

In recent years, the investigations of flow of fluids through porous media have become an important topic due to the recovery of crude oil from the pores of reservoir rocks. A great number of applications in geophysics may be found in the books by Phillips [6], Ingham and Pop [7], and Nield and Bejan [8]. The problem in porous medium is of importance in soil, ground water hydrology and in atmosphere. When the fluid slowly percolates through the pores of a macroscopically homogeneous and isotropic porous medium, the gross effect is represented by Darcy's law according to which the usual viscous term in the equations of fluid motion is replaced by the resistance term $-(\mu/k)v$, where μ is the viscosity of the fluid, k_1 is the permeability of the medium and v is the filter velocity of the fluid.

Many common materials such as paints, polymer's, plastics and more exotic one such as silicic magma, saturated soils and the Earth's lithosphere behaves as viscoelastic fluids. With the growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry, the investigations on such fluids are desirable. The stability of a horizontal layer of Maxwell's viscoelastic fluid heated from below has been investigated by Vest and Arpaci [9]. The nature of the instability and some factors may have different effects on viscoelastic fluids as compared to the Newtonian fluids. For example, Bhatia and Steiner [10] have considered the effect of a uniform rotation on the thermal instability of a Maxwell fluid and have found that rotation has a destabilizing effect in contrast to the stabilizing effect on Newtonian fluid. Experimental demonstration by Toms and Strawbridge [11] has revealed that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of the Oldroyd fluid. The thermal instability of an Oldroydian viscoelastic fluid has been considered in the presence of rotation (Eltayeb [12], Sharma [13]) and magnetic field (Sharma [14]). Sharma [15] has studied Oldroydian viscoelastic superposed conducting fluids in the presence of a uniform magnetic field. Generally, the magnetic field has a stabilizing effect on the stability, but there are a few exceptions. For example, Kent [16] has studied the effect of a horizontal magnetic field that varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable. Sharma and Kumar [17] have considered Rayleigh-Taylor instability of a Newtonian viscous fluid overlying an Oldroydian Viscoelastic fluid through porous medium. As in both Newtonian viscous-viscous fluids, the system is stable for potentially stable case and unstable for potentially unstable case. In another study, Sharma and Kumar [18] have considered Rayleigh-Taylor instability of electrically conducting Oldroydian viscoelastic fluid in the presence of a variable horizontal magnetic field through porous medium. Kumar [19] has considered the instability of the plane interface between two viscoelastic (Maxwellian) superposed conducting fluids in the presence of suspended particles and variable horizontal magnetic field in porous medium. Kumar and Singh [20] studied the superposed Maxwellian viscoelastic fluids through porous media in hydromagnetics. The double-diffusive convection in an Oldroydian viscoelastic fluid is mathematically investigated under the simultaneous effects of magnetic field and suspended particles through porous medium by Kumar and Mohan [21].

Keeping in mind the importance of viscoelastic fluids in modern technology and industries and owing to the importance of variable magnetic field, rotation and porous medium in chemical engineering and geophysics, the stability of the plane interface separating two incompressible superposed rotating Maxwellian viscoelastic fluids in porous medium in presence of a variable magnetic field has been considered in the present paper.

II. Formulation of the Problem and Perturbation Equations

Here we consider a static state in which an incompressible Maxwellian viscoelastic fluid is arranged in horizontal strata in porous medium and the pressure p and the density ρ are functions of the vertical co-ordinate z only. The system is acted on by a variable horizontal magnetic field $\vec{H}$ ($H_o(z)$,0,0), a uniform rotation $\vec{\Omega}$ (0,0,Ω) and a gravity force $\vec{g}$ (0,0,-g). The character of the equilibrium of this initial static state is determined, as usual, by supposing that the system is slightly disturbed and then by following its further evolution.

Let Γ_{ij}, τ_{ij}, e_{ij}, μ, λ, p, δ_{ij}, v_i, x_i and $\frac{d}{dt}$ denote respectively the total stress tensor, the shear stress tensor, the rate-of-strain tensor, the viscosity, the stress relaxation time, the isotropic pressure, the Kroneckor delta, the velocity vector, the position vector and the convective derivative. Then the Maxwellian fluid is described by the constitutive relations

$$\Gamma_{ij} = -p\delta_{ij} + \tau_{ij},$$

$$\left(1+\lambda\frac{d}{dt}\right)\tau_{ij} = 2\mu e_{ij},$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{1}$$

Let ρ, p and $\vec{v}$ (u, v, w) denote respectively the density, pressure and velocity of the fluid; μ_e is the magnetic permeability, ε is the medium porosity, k_1 is the medium permeability and $\bar{x} = (x, y, z)$. Then the hydromagnetic equations relevant to the problem are

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\varepsilon}(\vec{v}.\nabla)\vec{v}\right] = \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\left(p - \frac{\rho}{2}\left|\vec{\Omega}\times\bar{x}\right|^2\right) + \rho\vec{g} + \frac{2\rho}{\varepsilon}\left(\vec{v}\times\vec{\Omega}\right) + \frac{\mu_e}{4\pi}\left(\nabla\times\vec{H}\right)\times\vec{H}\right] - \frac{\mu}{k_1}\vec{v}, \tag{2}$$

$$\nabla\cdot\vec{v} = 0, \tag{3}$$

$$\varepsilon\frac{\partial \vec{H}}{\partial t} = \nabla\times\left(\vec{v}\times\vec{H}\right), \tag{4}$$

$$\nabla.\vec{H} = 0. \tag{5}$$

Since the density of a fluid particle moving with the fluid remains unchanged, we have

$$\varepsilon\frac{\partial\rho}{\partial t} + (\vec{v}.\nabla)\rho = 0. \tag{6}$$

Let $\vec{v}(u, v, w)$, δρ, δp and $\vec{h}$ (h_x, h_y, h_z) denote respectively the perturbations in fluid velocity (0,0,0), fluid density ρ, fluid pressure p and magnetic field $\vec{H}$ ($H_0(z)$,0,0). Then the linearized hydromagnetic perturbation equations of rotating Maxwellian viscoelastic fluid in porous medium are

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial\vec{v}}{\partial t} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\delta p + \vec{g}\delta\rho + \frac{2\rho}{\varepsilon}\left(\vec{v}\times\vec{\Omega}\right) + \frac{\mu_e}{4\pi}\left\{\left(\nabla\times\vec{h}\right)\times\vec{H} + \left(\nabla\times\vec{H}\right)\times\vec{h}\right\}\right] - \frac{\mu}{k_1}\vec{v}, \tag{7}$$

$$\nabla\cdot\vec{v} = 0, \tag{8}$$

$$\in\frac{\partial\vec{h}}{\partial t} = \left(\vec{H}\cdot\nabla\right)\vec{v} - (\vec{v}\cdot\nabla)\vec{H}, \tag{9}$$

$$\nabla\cdot\vec{h} = 0, \tag{10}$$

$$\in\frac{\partial}{\partial t}\delta\rho = -w(D\rho), \tag{11}$$

where $D = \frac{d}{dz}$.

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y and t is given by

$$\exp(ik_x x + ik_y y + nt), \tag{12}$$

where k_x, k_y are horizontal wave numbers, $k^2 = k_x^2 + k_y^2$, and n is a complex constant.

For perturbations of the form (12), equations (7)-(11) give

$$\frac{\rho}{\varepsilon}(1+\lambda n)nu = -(1+\lambda n)ik_x\delta p + (1+\lambda n)\frac{\mu_e}{4\pi}h_z(DH) + (1+\lambda n)\frac{2\rho\Omega}{\varepsilon}v - \frac{\mu}{k_1}u, \tag{13}$$

$$\frac{\rho}{\varepsilon}(1+\lambda n)nv = -(1+\lambda n)ik_y\delta p + (1+\lambda n)\frac{\mu_e H}{4\pi}\left(ik_x h_y - ik_y h_x\right) - (1+\lambda n)\frac{2\rho\Omega}{\varepsilon}u - \frac{\mu}{k_1}v, \tag{14}$$

$$\frac{\rho}{\varepsilon}(1+\lambda n)nw = -(1+\lambda n)[D\delta p + g\delta\rho] + (1+\lambda n)\frac{\mu_e H}{4\pi}\left[\left(ik_x h_z - Dh_x\right) - h_x\frac{(DH)}{H}\right] - \frac{\mu}{k_1}w, \tag{15}$$

$$ik_x u + ik_y v + Dw = 0, \tag{16}$$

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$$ik_x h_x + ik_y h_y + Dh_z = 0, \quad (17)$$

$$\in nh_x = ik_x Hu - w(DH), \quad (18)$$

$$\in nh_y = ik_x Hv, \quad (19)$$

$$\in nh_z = ik_x Hw, \quad (20)$$

$$\in n\delta\rho = -wD\rho. \quad (21)$$

Multiplying equations (13) and (14) by $-ik_x$, $-ik_y$, respectively, adding and using equations (16), (18) – (20), we obtain

$$\frac{\rho}{\in} n(1+\lambda n)Dw = -k^2(1+\lambda n)\delta p - \frac{\mu}{k_1}Dw - \frac{2\rho\Omega}{\in}(1+\lambda n)\zeta + (1+\lambda n)k_x k_y \frac{\mu_e H^2}{4\pi n \in}\zeta + \frac{(1+\lambda n)\mu_e H k^2}{4\pi n \in} w(DH), \quad (22)$$

where $\zeta = ik_x v - ik_y u$, the z-component of vorticity, is given by

$$\zeta = \frac{2(1+\lambda n)\Omega Dw}{(1+\lambda n)n + \frac{\nu \in}{k_1} + (1+\lambda n)\frac{k_x^2 V_A^2}{n}}, \quad (23)$$

which is obtained by multiplying equation (13) by $-ik_y$, equation (14) by $+ik_x$, adding and using equations (16), (18) – (20),

where $V_A^2 = \frac{\mu_e H^2}{4\pi\rho}$ is the square of the Alfv′en velocity and $\nu\left(=\frac{\mu}{\rho}\right)$ stands for kinematic viscosity.

Eliminating δp between equations (15) and (22) and using equations (23), (16)-(21) and the relation

$$ik^2 u = -\left(k_x Dw + k_y \zeta\right)$$

$$= -\left[k_x + \frac{2(1+\lambda n)k_y \Omega n}{(1+\lambda n)n^2 + \frac{n\nu \in}{k_1} + (1+\lambda n)k_x^2 V_A^2}\right]Dw, \quad (24)$$

we get after simplification, the dispersion relation as

$$\left[(1+\lambda n)n + \frac{\nu \in}{k_1}\right]\left[D(\rho Dw) - k^2\rho w\right] + \frac{gk^2(1+\lambda n)}{n}(D\rho)w + 4(1+\lambda n)^2\Omega^2 n \left[D\left\{\frac{\rho Dw}{(1+\lambda n)n^2 + \frac{n\nu \in}{k_1} + (1+\lambda n)k_x^2 V_A^2}\right\}\right] + \frac{(1+\lambda n)\mu_e k_x^2}{4\pi n}\left[D\left(H^2 Dw\right) - k^2 H^2 w\right] = 0. \quad (25)$$

III. Two Uniform Maxwellian Viscoelastic Fluids Separated by a Horizontal Boundary

Here we consider the case when two rotating superposed uniform Maxwellian viscoelastic fluids of densities ρ_1 and ρ_2, uniform viscosities μ_1 and μ_2 and magnetic fields H_1 and H_2 separated by a horizontal boundary z = 0 in porous medium. The subscripts 1 and 2 distinguish the lower and the upper fluids, respectively. Then, in each region of constant ρ, μ and $\vec{H}$, equation (25) reduces to

$$\left(D^2 - \kappa^2\right)w = 0, \quad (26)$$

where

$$\kappa = \frac{k}{\sqrt{1 + \frac{4(1+\lambda n)^2\Omega^2}{\left[(1+\lambda n)n + \frac{\nu\varepsilon}{k_1} + (1+\lambda n)\frac{k_x^2 V_A^2}{n}\right]^2}}},$$

and in case of highly viscous fluid

$$\kappa = \frac{k}{\left[1 + \frac{2(1+\lambda n)^2\Omega^2}{\left[(1+\lambda n)n + \frac{\nu\varepsilon}{k_1} + (1+\lambda n)\frac{k_x^2 V_A^2}{n}\right]^2}\right]}. \quad (27)$$

The general solution of equation (26) is

$$w = Ae^{+\kappa z} + Be^{-\kappa z}, \quad (28)$$

where A and B are arbitrary constants.

The boundary conditions to be satisfied here are:

(i) The velocity w should vanish when $z \to +\infty$ (for the upper fluid) and $z \to -\infty$ (for the lower fluid).

(ii) w (z) is continuous at z = 0.

(iii) The pressure should be continuous across the interface.

Applying the boundary conditions (i) and (ii), we have

$$w_1 = Ae^{+\kappa z}, \quad (z<0), \quad (29)$$

$$w_2 = Ae^{-\kappa z}, \quad (z>0), \quad (30)$$

the same constant A being chosen to ensure the continuity of w at z = 0.

Now by assuming the kinematic viscosities of both fluids to be equal i.e. $\nu_1 = \nu_2 = \nu$ (Chandrasekhar [1], p.443) and the Alfv′en velocities of the two fluids to be equal i.e.

$V_A^2 = \frac{\mu_e H_1^2}{4\pi\rho_1} = \frac{\mu_e H_2^2}{4\pi\rho_2}$, as these simplifying assumptions do not obscure any of the essential features of the problem, the continuity of pressure implies that

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$$\left[(1+\lambda n)n+\frac{\nu\varepsilon}{k_1}\right]\Delta_0(\rho Dw)+$$
$$\frac{4(1+\lambda n)^2\Omega^2}{(1+\lambda n)n+\frac{\nu\varepsilon}{k_1}+(1+\lambda n)\frac{k_x^2V_A^2}{n}}\Delta_0(\rho Dw)$$
$$+\frac{(1+\lambda n)k_x^2V_A^2}{n}\Delta_0(\rho Dw)=-\frac{gk^2(1+\lambda n)}{n}\Delta_0(\rho)w_0, \quad (31)$$

where $\Delta_0(f)$ denotes a jump which a quantity 'f' experiences at the interface z = 0, and w_0 is the common value of w at z = 0.

IV. Discussion

Applying the condition (31) to the solutions (29) and (30) and using relation (27), we get

$$A_9n^9 + A_8n^8 + A_7n^7 + \ldots\ldots + A_2n^2 + A_1n + A_0 = 0, \quad (32)$$

where

$$A_9 = \lambda^2 \quad ,$$
$$A_0 = -k_x^2V_A^2\left[gk(\alpha_2-\alpha_1)-k_x^2V_A^2\right], \quad (33)$$

where $\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1+\rho_2}$, and the coefficients A_1 - A_8 , being quite lengthy and not needed in the discussion of stability, have not been written here.

For the **potentially stable arrangement** $(\alpha_2 < \alpha_1)$, all the coefficients of equation (32) are either real and negative, or there are complex roots (which occurs in pairs) with negative real parts and the rest negative real roots. The system is therefore stable in each case.

For the **potentially unstable arrangement** $(\alpha_2 > \alpha_1)$, if

$$k_x^2V_A^2 > gk(\alpha_2-\alpha_1), \quad (34)$$

equation (32) does not admit of any change of sign and so has no positive root. The system is therefore stable.

But if

$$k_x^2V_A^2 < gk(\alpha_2-\alpha_1), \quad (35)$$

the constant term in equation (32) is negative. Equation (32) therefore allows one change of sign and so has one positive root. The occurrence of positive root implies that the system is unstable.

Thus for the unstable case, the system is stable or unstable according as $k_x^2V_A^2$ is greater than or less than $gk(\alpha_2-\alpha_1)$. In the absence of rotation and magnetic field, it can be checked, that the system is unstable for all wave numbers for the potentially unstable case. But the magnetic field has got a stabilizing effect and completely stabilizes the wave number band $k > k^*$, where

$$k^* = \frac{g(\rho_2-\rho_1)}{(\rho_2+\rho_1)V_A^2}\sec^2\theta,$$

and θ is the inclination of the wave number $\vec{k}$ to the direction of magnetic field.

Discussion of Some Important Sub-Cases

(i) Case of Magnetic Free Configuration

$(\vec{H}=0)$**:** For the case of magnetic free configuration, equation (26) reduces to

$$\left(D^2-\kappa_1^2\right)w=0, \quad (36)$$

where

$$\kappa_1 = \frac{k}{\left[1+\frac{2(1+\lambda n)^2\Omega^2}{\left[(1+\lambda n)n+\frac{\nu\varepsilon}{k_1}\right]^2}\right]} \quad . \quad (37)$$

The solutions appropriate for the two regions can be written as

$$w_1 = Ae^{+\kappa_1 z} \quad , \quad (z<0) \quad , \quad (38)$$
$$w_2 = Ae^{-\kappa_1 z} \quad , \quad (z>0) \quad . \quad (39)$$

The condition (31), in the limit of vanishing magnetic field, reduces to

$$\left[(1+\lambda n)n+\frac{\nu\varepsilon}{k_1}\right]\Delta_0(\rho Dw)+\frac{4(1+\lambda n)^2\Omega^2}{(1+\lambda n)n+\frac{\nu\varepsilon}{k_1}}\Delta_0(\rho Dw)$$
$$=-\frac{gk^2(1+\lambda n)}{n}\Delta_0(\rho)w_0 \quad . \quad (40)$$

Applying the condition (40) to the solutions (38), (39) and using relation (37), we obtain

$$A_9n^9 + A_8n^8 + A_7n^7 + \ldots\ldots\ldots\ldots + A_2n^2 + A_1n + A_0 = 0, \quad (41)$$

where

$$A_9 = \lambda^2 \quad ,$$
$$A_0 = -\frac{\nu\,\varepsilon\,gk(\alpha_2-\alpha_1)}{k_1}\left[\frac{\nu^2\varepsilon^2}{k_1^2}+2\Omega^2\right] \quad , \quad (42)$$

and the coefficients A_1 - A_8 , being quite lengthy and not needed in the discussion of stability, have not been written here.

Now, for the **potentially stable arrangement** $(\alpha_2 < \alpha_1)$, all the coefficients of equation (41) are either real and negative, or there are complex roots (which occurs in pairs) with negative real parts and the rest negative real roots. The system is therefore stable in each case.

For the **potentially unstable arrangement** $(\alpha_2 > \alpha_1)$, the constant term in equation (41) is negative and so there is atleast one change of sign in equation (41). Equation (41), therefore, allows atleast one positive root of n meaning thereby instability of the system.

The system is, therefore, stable for potentially stable arrangement and unstable for potentially unstable arrangement for the magnetic free case.

(ii) Case of Rotation Free Configuration: For rotation free configuration, equation (26) reduces to

$$\left(D^2-k^2\right)w=0, \quad (43)$$

where

$$k=\kappa \tag{44}$$

The solutions appropriate for the two regions can be written as

$$w_1 = Ae^{+kz} \quad , \quad (z<0) \quad , \tag{45}$$

$$w_2 = Ae^{-kz} \quad , \quad (z>0) \quad . \tag{46}$$

The condition (31), for the rotation free case, reduces to

$$\left[(1+\lambda n)n+\frac{\nu\varepsilon}{k_1}\right]\Delta_0(\rho Dw)+\frac{(1+\lambda n)k_x^2V_A^2}{n}\Delta_0(\rho Dw)$$
$$=-\frac{gk^2(1+\lambda n)}{n}\Delta_0(\rho)w_0 \quad . \tag{47}$$

Applying the condition (47) to the solutions (45) and (46), we obtain

$$A_3n^3 + A_2n^2 + A_1n + A_0 = 0, \tag{48}$$

where

$$A_3=\lambda\,,$$

$$A_2=1\,,$$

$$A_1=\left[\frac{\nu\varepsilon}{k_1}+\left\{k_x^2V_A^2-gk(\alpha_2-\alpha_1)\right\}\right]\,,$$

$$A_0=\left[k_x^2V_A^2-gk(\alpha_2-\alpha_1)\right] \tag{49}$$

Now for the **potentially stable arrangement** $(\alpha_2 < \alpha_1)$, equation (48) does not involve any change of sign and so does not allow any positive root. The system is therefore stable.

For the **potentially unstable arrangement** $(\alpha_2 > \alpha_1)$, it is clear from equation (48) that the system is stable or unstable according as

$$k_x^2V_A^2 > or < gk(\alpha_2-\alpha_1)\,. \tag{50}$$

Thus, for rotation free configuration, the system is stable for potentially stable arrangement but for the potentially unstable arrangement, the magnetic field stabilizes a certain wave number range (which was unstable in the hydrodynamic case).

V. Conclusion

The effect of uniform rotation and variable horizontal magnetic field on the Rayleigh-Taylor instability of two superposed Maxwellian viscoelastic fluids in porous medium is considered in the present paper. This investigation is motivated due to the importance of viscoelastic fluids in chemical engineering, modern technology and industry; and importance of porous media in the hydrology of soil, ground water and in atmosphere.

The main conclusions from the analysis of this paper are as follows:

(a) For the case of combined effect of uniform rotation and variable horizontal magnetic field the following observations are made:

(i) For the potentially stable configuration, the system is found to be stable for disturbances of all wave numbers.

(ii) For potentially unstable configuration, the system is found to be stable or unstable according as $k_x^2V_A^2$ is greater than or less than $gk(\alpha_2-\alpha_1)$.

(iii) In the absence of rotation and magnetic field, the system is unstable for all wave numbers for the potentially unstable configuration.

(b) For the case of magnetic free configuration, the system is found to be stable for potentially stable arrangement and unstable for potentially unstable arrangement.

(c) For rotation free configuration, the system is found to be stable for potentially stable arrangement but for the potentially unstable arrangement, the magnetic field stabilizes a certain wave number range (which was unstable in the hydrodynamic case).

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