International Journal of Applied Science and Mathematics
Volume 3, Issue 3, ISSN (Online): 2394-2894

On Certain Generalized $\mathcal{B}$R–Recurrent Finsler Space

Fahmi Yaseen Abdo Qasem
Dept. of Math., Faculty of Education-Aden, Univ. of Aden, Khormakssar, Aden, Yemen Email ID : fahmiyaseen 1@ gmail. com

Ala'a Abdalnasser Awad Abdallah
Dept. of Math., Faculty of Education-Aden, Univ. of Aden, Khormakssar, Aden, Yemen Email ID : ala733.ala00@Gmail.com

Abstract **– In 1970 G.P. Pokhariyal and R.S. Mishra [7] have introduced a w-curvature tensor and this tensor is defined in local coordinates as [3]**

$$W_{pjkh} = R_{pjkh} + \frac{1}{n-1}(g_{pk}R_{jk} - g_{jk}R_{ph}).$$

For a Riemannian space V_4, the projective curvature tensor P^i_{jkh} is defined as [3]

$$P^i_{jkh} = R^i_{jkh} - \frac{1}{3}(\delta^i_h R_{jk} - \delta^i_k R_{jh}).$$

The conformal curvature tensor C_{ijkh} (Weyl's conformal curvature tensor), for a V_4 is defined through the equation [3]

$$C_{ijkh} = R_{ijkh} - \frac{1}{2}(g_{ik}R_{jh} + g_{jh}R_{ik} - g_{ih}R_{jk} - g_{jk}R_{ih}) - \frac{R}{6}(g_{ih}R_{jk} - g_{ik}R_{jh}).$$

The aim of this paper is to develop the above curvature tensors by study their properties in generalized $\mathcal{B}R$–recurrent space which characterized by the condition

$$\mathcal{B}_m R^i_{jkh} = \lambda_m R^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}),$$

$R^i_{jkh} \neq 0$, where $\mathcal{B}_m$ is covariant derivative operator with respect to x^m in sense of Berwald, λ_m and μ_m are the recurrence vectors.

Keywords **– W–Curvature Tensor Space, Projective Curvature Tensor P^i_{jkh}, Conformal Curvature Tensor C_{ijkh} (Weyl's Conformal Curvature Tensor), Generalized $\mathcal{B}R$ – Recurrent Space.**

I. Introduction

H.S. Ruse [9] considered a three dimensional Riemannian space having the recurrent of curvature tensor and he called such space as *Riemannian space of recurrent curvature* and extended to n–dimensional Riemannian space by A.G. Walker [1], Y.C. Wong [13], Y.C. Wong and K. Yano [14] and others. This idea was extended to Finsler space by A. Moor [2] for the first time. Due to different connections of Finsler space, the recurrent of Cartan's third curvature tensor R^i_{jkh} have been discussed by R. Verma [12].

P.N. Pandey, S. Saxena and A. Goswami [11] introduced a generalized H–recurrent Finsler space, F.Y.A. Qasem and A.M.A. Al-qashbari ([4],[6]) introduced a generalized H^h - recurrent space and studied some types of this space, also they defined a generalized R^h-recurrent space and obtained some identities satisfied in such space [5].

Let F_n be an n-dimensional Finsler space equipped with the metric function F(x,y) satisfying the request conditions[8].

The vector y_i is defined by

(1.1) $y_i = g_{ij}(x,y)y^j$.

The two sets of quantities g_{ij} and its associative g^{ij}, which are components of a metric tensor connected by

(1.2) $g_{ij}g^{ik} = \delta^k_j = \begin{cases} 1 & if \;\; j = k\,, \\ 0 & if \;\; j \neq k\,. \end{cases}$

In view of (1.1) and (1.2), we have

(1.3) a) $\delta^i_k y^k = y^i$, b) $\delta^i_k y_i = y_k$ and c) $\delta^i_j g_{ir} = g_{jr}$.

The tensor C_{ijk} is defined by

$$C_{ijk} = \frac{1}{2}\dot{\partial}_k g_{ij}$$

which is positively homogeneous of degree -1 in y^i and symmetric in all its indices and called *(h)hv-torsion tensor* [10]. According to Euler's theorem on homogeneous functions, this tensor satisfies the following:

(1.4) $C_{ijk}y^i = C_{jki}y^i = C_{kij}y^i = 0$.

Berwald covariant derivative $\mathcal{B}_k T^i_j$ of an arbitrary tensor field T^i_j with respect to x^k is given by

$$\mathcal{B}_k T^i_j \coloneqq \partial_k T^i_j - (\dot{\partial}_r T^i_j)\, G^r_k + T^r_j G^i_{rk} - T^i_r G^r_{jk}.$$

In most of existing literature, Berwald covariant derivative $\mathcal{B}_k T^i_j$ appears as $T^i_{j(k)}$. Berwald covariant derivative of the vector y^i vanish identically, i.e.

(1.5) $\mathcal{B}_k y^i = 0$.

But, in general, Berwald covariant derivative of the metric tensor g_{ij} does not vanish and given by

(1.6) $\mathcal{B}_k g_{ij} = -2\, y^h\, \mathcal{B}_h C_{ijk}$.

A Finsler space whose connection parameter $G^i_{jkh}$is independent of y^i is called an *affinely connected space* (*Berwald space*).

In particular, $\mathcal{B}_k g_{ij}$ vanishes for affinely connected space, i.e.

(1.7) $\mathcal{B}_k g_{ij} = 0$.

The hv-curvature tensor (*Cartan's second curvature tensor*)is defined by [8].

The tensor P^i_{jkh} is positively homogeneous of degree zero in y^i and satisfies

(1.8) $P^i_{jkh}\, y^j = P^i_{kh}$.

The associate curvature tensor P_{rjkh} is given by

(1.9) $P_{rjkh} = P^i_{jkh}\, g_{ir}$.

P-Ricci tensor P_{jk} and the curvature vector P_k (of Cartan's second curvature tensor) are given by

(1.10) $P_{jk} = P^i_{jki}$

and

(1.11) $P_k = P^i_{ki}$.

The h- curvature tensor (*Cartan's third curvature tensor*) is defined by

$$R^i_{jkh} = \partial_h \Gamma^{*i}_{jk} + (\partial_l \Gamma^{*i}_{jk})\, G^l_h + C^i_{jm}(\partial_k G^m_h - G^m_{kl} G^l_h) + \Gamma^{*i}_{mk}\Gamma^{*m}_{jh} - k/h^*.$$

This tensor satisfies the following relations

(1.12) $R^i_{jkh}g^{jk} = R^i_h$

and

(1.13) $R^i_{jki} = R_{jk}$.

The associate curvature tensor R_{rjkh} satisfies

(1.14) $R_{pjkh}g^{pi} = R^i_{jkh}$.

The deviation tensor R^i_k, R-Ricci tensor R_{jk} and the curvature vector R_k are related by

(1.15) $g^{ij}R_{jk} = R^i_k$

and

(1.16) $R_{jk}y^j = R_k$.

L. Berwald deduces a tensor

$$W^i_h = H^i_h - H\delta^i_h - \{(\dot{\partial}_r H_h - \dot{\partial}_h H)y^i\}/(n+1).$$

---

* k/h means the subtraction from the former terms by interchanging the indices k and h.

This tensor is called *projective deviation tensor*. The tensor W^i_h is positively homogeneous of degree two in y^i, we defined the following projective tensors

$$W^i_{kh} = \frac{1}{3}(\dot{\partial}_k W^i_h - \dot{\partial}_h W^i_k)$$

and

$$W^i_{jkh} \coloneqq \dot{\partial}_j W^i_{kh} = \tfrac{1}{3}\dot{\partial}_j(\dot{\partial}_k W^i_h - \dot{\partial}_h W^i_k).$$

The tensor W^i_{jkh} is known as *projective curvature tensor* (*generalized Weyl's projective curvature tensor*) and the tensor W^i_{kh} is known as *projective torsion tensor* (*Weyl's projective torsion tensor*). They are skew- symmetric in their lower indices k and h.

These tensors are satisfies the following:

(1.17) $W^i_{jkh}y^j = W^i_{kh}$

and

(1.18) $W^i_{kh}y^k = W^i_h$.

The associate curvature tensor W_{pjkh} satisfies

(1.19) $W_{pjkh}\, g^{pi} = W^i_{jkh}$.

II. A Generalized $\mathcal{BR}$-Recurrent Space

Let us consider a Finsler space F_n whose Cartan's third curvature tensor R^i_{jkh} satisfies

(2.1) $\mathcal{B}_m R^i_{jkh} = \lambda_m R^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh})$, $R^i_{jkh} \neq 0$,

where λ_m and μ_m are non-zero covariant vectors field. The space and the tensor satisfying the condition (2.1) will be celled a *generalized* $\mathcal{BR}$*-recurrent space* and a *generalized* $\mathcal{B}$ *-recurrent tensor*, respectively and denoted them briefly by G($\mathcal{BR}$) $- RF_n$ and G$\mathcal{B} - R$, respectively.

Let us consider a G($\mathcal{BR}$) $- RF_n$.

Contracting the indices i and h in the condition (2.1), using (1.13), (1.5) and (1.3c), we get

(2.2) $\mathcal{B}_m R_{jk} = \lambda_m R_{jk}$.

Transvecting the condition (2.2) by y^k and using (1.16), we get

(2.3) $\mathcal{B}_m R_k = \lambda_m R_k$.

Thus, we conclude

Theorem 2.1. In G ($\mathcal{B}$R) $-$ RF$_n$, R-Ricci tensor R_{jk} and the curvature vector R_k behave as recurrent.

Transvecting the condition (2.1) by g^{jk}, using (1.12) and (1.2), we get

(2.4) $\mathcal{B}_m R^i_h = \lambda_m R^i_h$

if and only if

$$\mathcal{B}_m g^{jk} = 0 ,$$

since $R^i_{jkh} \neq 0$.

Transvecting the condition (2.2) by g^{jk}, we get

(2.5) $\mathcal{B}_m R = \lambda_m R$

If and only if

$$\mathcal{B}_m g^{jk} = 0,$$

since $R_{jk} \neq 0$.

Thus, we conclude

Theorem 2.2. In G ($\mathcal{B}$R) $-$ RF$_n$, the deviation tensor R^i_h and the curvature scalar R behave as recurrent if and only if $\mathcal{B}_m g^{jk} = 0$.

In view of (1.7), we may rewrite theorem 2.2 as follows :

Theorem 2.3. The G($\mathcal{B}$R) $-$ RF$_n$ is affinely connected space provided the deviation tensor R^i_h or the curvature scalar R behaves as recurrent.

In 1970 G.P. Pokhariyal and R.S. Misra [7] have introduced a W- curvature tensor and or W_2-curvature tensor and studied its properties,

This tensor is defined in local coordinates as [3]

(2.6) $W_{pjkh} = R_{pjkh} + \frac{1}{n-1}(g_{pk}R_{jh} - g_{jk}R_{ph})$.

Transvecting (2.6) by g^{pi}, using (1.19), (1.14), (1.2) and (1.15), we get

(2.7) $W^i_{jkh} = R^i_{jkh} + \frac{1}{n-1}(\delta^i_k R_{jh} - g_{jk}R^i_h)$.

Let us consider a G($\mathcal{B}R$) $- RF_n$.

Taking the covariant derivative of (2.7) with respect to x^m in sense of Berwald,we get

(2.8) $\mathcal{B}_m W^i_{jkh} = \mathcal{B}_m R^i_{jkh} + \frac{1}{n-1}(\delta^i_k \mathcal{B}_m R_{jh} - g_{jk}\mathcal{B}_m R^i_h - R^i_h \mathcal{B}_m g_{jk})$.

Using the conditions (2.1), (2.2) and (2.4) in (2.8), we get

(2.9) $\mathcal{B}_m W^i_{jkh} = \lambda_m R^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}) + \frac{\lambda_m}{n-1}(\delta^i_k R_{jh} - g_{jk}R^i_h) + \frac{2}{n-1}R^i_h y^h \mathcal{B}_h C_{jkm}$.

Using (2.7) in (2.9), we get

(2.10) $\mathcal{B}_m W^i_{jkh} = \lambda_m W^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}) + \frac{2}{n-1}R^i_h y^h \mathcal{B}_h C_{jkm}$.

This shows that

$$\mathcal{B}_m W^i_{jkh} = \lambda_m W^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh})$$

if and only if

(2.11) $y^h \mathcal{B}_h C_{jkm} = 0,$

since $R^i_h \neq 0$.

Thus, we conclude

Theorem 2.4. In $G(\mathcal{B}R)-RF_n$, Weyl's projective curvature tensor is generalized recurrent if and only if (2.11) holds [provided $\mathcal{B}_m g^{jk}=0$].

In view of (1.6) and (1.7), we may rewrite theorem 2.4 as follows :

Theorem 2.5. The $G(\mathcal{B}R)-RF_n$, is affinely connected space provided Weyl's projective curvature tensor W^i_{jkh} is generalized recurrent.

Transvecting (2.10) by y^j, using (1.17), (1.5), (1.3a), (1.1) and (1.4), we get

(2.12) $$\mathcal{B}_m W^i_{kh}=\lambda_m W^i_{kh}+\mu_m\left(y^i g_{kh}-\delta^i_k y_h\right).$$

Thus, we conclude

Theorem 2.6. In $G(\mathcal{B}R)-RF_n$, Berwald covariant derivative of the first order for Weyl's projective torsion tensor W^i_{kh} is given by the condition(2.12) [provided$\mathcal{B}_m g^{jk}=0$]. Transvecting the condition (2.12) by y^k, using (1.1),(1.3a), (1.5) and (1.18), we get

$$\mathcal{B}_m W^i_h=\lambda_m W^i_h.$$

Thus, we conclude

Theorem 2.7. In $G(\mathcal{B}R)-RF_n$, Weyl's projective deviation tensor W^i_h behaves as recurrent [provided $\mathcal{B}_m g^{jk}=0$].

For a Riemannian space V_4, the projective curvature tensor P^i_{jkh} (Cartan's second curvature tensor) is defined as [3]

(2.13) $$P^i_{jkh}=R^i_{jkh}-\frac{1}{3}\left(\delta^i_h R_{jk}-\delta^i_k R_{jh}\right).$$

Let us consider a $G(\mathcal{B}R)-RF_n$.

Taking the covariant derivative of (2.13) with respect to x^m in sense of Berwald,we get

(2.14) $$\mathcal{B}_m P^i_{jkh}=\mathcal{B}_m R^i_{jkh}-\frac{1}{3}\left(\delta^i_h\mathcal{B}_m R_{jk}-\delta^i_k\mathcal{B}_m R_{jh}\right).$$

Using the conditions (2.1) and (2.2) in (2.14), we get

(2.15) $$\mathcal{B}_m P^i_{jkh}=\lambda_m\{R^i_{jkh}-\frac{1}{3}\left(\delta^i_h R_{jk}-\delta^i_k R_{jh}\right)\}+\mu_m\left(\delta^i_j g_{kh}-\delta^i_k g_{jh}\right).$$

Using (2.13) in (2.15), we get

(2.16) $$\mathcal{B}_m P^i_{jkh}=\lambda_m P^i_{jkh}+\mu_m\left(\delta^i_j g_{kh}-\delta^i_k g_{jh}\right).$$

Thus, we conclude

Theorem 2.8. In $G(\mathcal{B}R)-RF_n$ (for n=4), Cartan's second curvature tensor P^i_{jkh} is generalized recurrent.

Transvecting the condition (2.16) by y^j, using (1.1), (1.5), (1.8) and (1.3a), we get

(2.17) $$\mathcal{B}_m P^i_{kh}=\lambda_m P^i_{kh}+\mu_m\left(y^i g_{kh}-\delta^i_k y_h\right).$$

Thus, we conclude

Theorem 2.9. In $G(\mathcal{B}R)-RF_n$ (for n=4), Berwald covariant derivative of the first order for the (v)hv-torsion tensor P^i_{kh} is given by the condition (2.17).

Transvecting the condition (2.16) by g_{ir}, using (1.6), (1.14) and (1.3c), we get

$$\mathcal{B}_m P_{rjkh}=\lambda_m P_{rjkh}+\mu_m\left(g_{jr}g_{kh}-g_{kr}g_{jh}\right)-2\,P^i_{jkh}y^h\mathcal{B}_h C_{irm}.$$

This shows that

$$\mathcal{B}_m P_{rjkh}=\lambda_m P_{rjkh}+\mu_m\left(g_{jr}g_{kh}-g_{kr}g_{jh}\right)$$

if and only if

(2.18) $$y^h\mathcal{B}_h C_{irm}=0,$$

since $P^i_{jkh}\neq 0$.

Thus, we conclude

Theorem 2.10. In $G(\mathcal{B}R)-RF_n$(for n=4), the associate curvature tensor P_{rjkh} is generalized recurrent if and only if (2.18) holds.

In view of (1.6) and (1.7), we may rewrite theorem 2.10 as follows :

Theorem 2.11. The $G(\mathcal{B}R)-RF_n$, is affinely connected space provided the associate curvature tensor P_{rjkh} is generalized recurrent.

Contracting the indices i and h in the condition (2.16), using (1.3c) and (1.10), we get

(2.19) $$\mathcal{B}_m P_{jk}=\lambda_m P_{jk}.$$

Thus, we conclude

Theorem 2.12. In $G(\mathcal{B}R)-RF_n$ (for n=4), R-Ricci tensor R_{jk} behaves as recurrent. Contracting the indices i and h in the condition (2.17), using (1.11), (1.1) and (1.3b), we get

(2.20) $$\mathcal{B}_m P_k=\lambda_m P_k.$$

Thus, we conclude

Theorem 2.13. In $G(\mathcal{B}R)-RF_n$ (for n=4), the curvature vector P_k behaves as recurrent.

The conformal curvature tensor C_{ijkh} (Weyl's conformal curvature tensor), for a V_4 is defined through the equation [3]

$$C_{ijkh}=R_{ijkh}-\frac{1}{2}\left(g_{ik}R_{jh}+g_{jh}R_{ik}-g_{ih}R_{jk}-g_{jk}R_{ih}\right)-\frac{R}{6}\left(g_{ih}g_{jk}-g_{ik}g_{jh}\right)$$

so that [3]

(2.21) $$C^i_{jkh}=R^i_{jkh}-\frac{1}{2}\left(\delta^i_k R_{jh}+g_{jh}R^i_k-\delta^i_h R_{jk}-g_{jk}R^i_h\right)-\frac{R}{6}\left(\delta^i_h g_{jk}-\delta^i_k g_{jh}\right).$$

Let us consider a $G(\mathcal{B}R)-RF_n$.

Taking the covariant derivative of (2.21) with respect to x^m in sense of Berwald, we get

(2.22) $$\mathcal{B}_m C^i_{jkh}=\mathcal{B}_m R^i_{jkh}-\frac{1}{2}\{\delta^i_k\left(\mathcal{B}_m R_{jh}\right)+\left(\mathcal{B}_m g_{jh}\right)R^i_k+g_{jk}\left(\mathcal{B}_m R^i_k\right)-\delta^i_h\left(\mathcal{B}_m R_{jk}\right)-\left(\mathcal{B}_m g_{jk}\right)R^i_h-g_{jk}\left(\mathcal{B}_m R^i_h\right)\}-\frac{1}{6}\mathcal{B}_m R\left(\delta^i_h g_{jk}-\delta^i_k g_{jh}\right)-\frac{R}{6}\{\delta^i_h\left(\mathcal{B}_m g_{jk}\right)-\delta^i_k\left(\mathcal{B}_m g_{jh}\right)\}.$$

Using the conditions (2.1), (2.2), (2.3) and (2.4) in (2.22), we get

(2.23) $$\mathcal{B}_m C^i_{jkh}=\lambda_m R^i_{jkh}+\mu_m\left(\delta^i_j g_{kh}-\delta^i_k g_{jh}\right)-\frac{1}{2}\{\delta^i_k\lambda_m R_{jh}+g_{jh}\lambda_m R^i_k-\delta^i_h\lambda_m R_{jk}-g_{jk}\lambda_m R^i_h\}-\frac{1}{2}\{\left(\mathcal{B}_m g_{jh}\right)R^i_k-\left(\mathcal{B}_m g_{jk}\right)R^i_h\}-\frac{\lambda_m}{6}R\left(\delta^i_h g_{jk}-\delta^i_k g_{jh}\right)-\frac{R}{6}\{\delta^i_h\left(\mathcal{B}_m g_{jk}\right)-\delta^i_k\left(\mathcal{B}_m g_{jh}\right)\}.$$

Using (2.21) in (2.23), we get

(2.24) $$\mathcal{B}_m C^i_{jkh} = \lambda_m C^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}) - \tfrac{1}{2}\{(\mathcal{B}_m g_{jh})R^i_k - (\mathcal{B}_m g_{jk})R^i_h\} - \tfrac{R}{6}\{\delta^i_h(\mathcal{B}_m g_{jk}) - \delta^i_k(\mathcal{B}_m g_{jh})\}.$$

This shows that

$$\mathcal{B}_m C^i_{jkh} = \lambda_m C^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh})$$

if and only if

(2.25) $$-\tfrac{1}{2}\{(\mathcal{B}_m g_{jh})R^i_k - (\mathcal{B}_m g_{jk})R^i_h\} - \tfrac{R}{6}\{\delta^i_h(\mathcal{B}_m g_{jk}) - \delta^i_k(\mathcal{B}_m g_{jh})\} = 0.$$

Thus, we conclude

Theorem 2.14. In $G(\mathcal{B}R) - RF_n$ (for n=4), Weyl's conformal curvature tensor C^i_{jkh} is generalized recurrent if and only if (2.25) holds [provided (2.5) holds].

Transvecting (2.24) by y^j, using (1.1), (1.5), (1.6), (1.3a) and Putting $(C^i_{jkh}y^j = C^i_{kh})$, we get

(2.26) $$\mathcal{B}_m C^i_{kh} = \lambda_m C^i_{kh} + \mu_m(y^i g_{kh} - \delta^i_k y_h).$$

Thus, we conclude

Theorem 2.15. In $G(\mathcal{B}R) - RF_n$ (for n=4), Berwald covariant derivative of the first order for the (v)hv-torsion tensor C^i_{kh} is given by the condition (2.26).

III. Conclusion

(3.1) The space whose defined by the condition (2.1) is called generalized $\mathcal{B}R$-recurrent Finsler space.

(3.2) In generalized $\mathcal{B}R$-recurrent Finsler space, R-Ricci tensor R_{jk} and the curvature vector R_k behave as recurrent.

(3.3) In generalized $\mathcal{B}R$-recurrent Finsler space, the deviation tensor R^i_h and the curvature scalar R behave as recurrent if and only if $\mathcal{B}_m g^{jk} = 0$.

(3.4) The generalized $\mathcal{B}R$-recurrent Finsler space is affinely connected space provided the deviation tensor R^i_h or the curvature scalar R behaves as recurrent.

(3.5) Thegeneralized $\mathcal{B}R$-recurrent Finsler space is affinely connected space provided Weyl's projective curvature tensor W^i_{jkh} is generalized recurrent.

(3.6) In generalized $\mathcal{B}R$-recurrent Finsler space, Weyl's projective deviation tensor W^i_h behaves as recurrent provided $\mathcal{B}_m g^{jk} = 0$.

(3.7) In generalized $\mathcal{B}R$-recurrent Finsler space (for n=4), Cartan's second curvature tensor P^i_{jkh} is generalized recurrent.

(3.8) In generalized $\mathcal{B}R$-recurrent Finsler space (for n=4), the associate curvature tensor P_{rjkh} is generalized recurrent if and only if $y^h\mathcal{B}_h C_{irm} = 0$ holds.

(3.9) The generalized $\mathcal{B}R$-recurrent Finsler space provided the associate curvature tensor P_{rjkh} is generalized recurrent.

(3.10) In generalized $\mathcal{B}R$-recurrent Finsler space (for n=4), the curvature vector P_k behaves as recurrent.

References

[1] **A.G. Walker**: On Ruse's space of recurrent curvature, proc. Lond.Math.Soc.,52, (1950),36 – 64.

[2] **A. Moor**: Untersuchungenüber Finslerränme von rekurrenterkrümmung,Tensor N.S,13, (1963),1 -18.

[3] **A. Zafar** and **A. Musavvir**: On some properties of W –curvature tensor, Palestine Journal of Mathematics, Vol.3(1), (2014),61 – 69.

[4] **F.Y.A. Qasem** and **A.M.A. Al–qashbari**: On study generalized H^h- recurrent Finsler space, Journal of Yemen Engineer university of Aden, (Aden), (Yemen), Vol.14, November, (2015), (Accepted).

[5] **F.Y.A. Qasem** and **A.M.A. Al–qashbari**: Certain identities in generalized R^h - recurrent Finsler space, International Journal of Innovation in Science Mathematics, Volume 4, Issues 2, (2016), 66 – 69.

[6] **F.Y.A. Qasem** and **A.M.A. Al -qashbari**: Some types of generalized H^h-recurrent in Finsler space, International Journal of Mathematics and its Application, Volume 4,Issues 1– C, (2016),1 – 10.

[7] **G.P. Pokhariyal** and **R.S. Mishra**: Curvature tensor and their relativistic significance, Yokohama Math. Journal, 18, (1970),105 – 108.

[8] **H. Rund.**: *The differential geometry of Finsler spaces*, Springer-Verlag, Berlin Göttingen, (1959); 2nd Edit. (in Russian), Nauka, (Moscow), (1981).

[9] **H.S. Ruse**: Three dimensional spaces of recurrent curvature. Proc. Lond. Math. Soc.,50, (1949),438 – 446.

[10] **M. Matsumoto**: On C-reducible Finsler space, Tensor N.S., 24, (1972), 29-37.

[11] **P.N. Pandey**, **S. Saxena** and **A. Goswani**: On a generalized H-recurrent space, Journal of International Academy of Physical Science, Vol.15,(2011),201 – 211.

[12] **R. Verma**: *Some transformations in Finsler space*, Ph.D. Thesis, University of Allahabad, (Allahabad), (India), (1991).

[13] **Y.C. Wong**: Linear connections with zero torsion and recurrent curvature, Trans. Amer. Math. Soc.,102, (1962),471 – 506.

[14] **Y.C. Wong** and **K. Yano**: Projectively flat spaces with recurrent curvature, Comment Math.Helv.,35, (1961),223 – 232.