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On R^h-Generalized Trirecurrent Curvature Tensor

Fahmi Yaseen Abdo Qasem
Dept. of Maths., Faculty of Edu.-Aden,
Univ. of Aden, Khormaksar, Aden, Yemen
fahmi.yaseen@yahoo.com

Khaled Mohammed A. Alhamadi
Dept. of Maths., Faculty of Edu.-Aden,
Univ. of Aden, Khormaksar, Aden, Yemen
abusuliman88@yahoo.com

Ali Ali Ali Muhib
Dept. of Maths. , Faculty of Edu.-Thamar,
Univ. of Thamar, Thamar, Yemen
moheb20052000@yahoo.com

Abstract **– In this paper we introduced a R^h-generalized trirecurrent Finsler space and R^h-special generalized trirecurrent Finsler space. The aim of this paper to study the above spaces and discussed different Finsler spaces with generalized and special generalized trirecurrent of Cartan's fourth curvature tensor K^i_{jkh} whereas generalized and special generalized trirecurrent of associative curvature tensor R_{ijkh} are investigated, various theorems concerning Ricci generalized trirecurrent and Ricci special generalized trirecurrent space have been obtained, different theorems regarding the necessary and sufficient condition for Berwald curvature tensor H^i_{jkh} and Cartan's fourth curvature tensor K^i_{jkh} to be generalized trirecurrent and special generalized trirecurrent, certain identities have been obtained.**

Keywords **– R^h-generalized trirecurrent space, R^h-special generalized trirecurrent space, Ricci-generalized trirecurrent space, Ricci-special generalized trirecurrent space R^h-generalized trirecurrent affinely connected space and R^h-special generalized trirecurrent affinely connected space.**

I. Introduction

The concept of recurrent curvature of n-dimensional Riemannian space was extended to a Finsler space by A. Moór ([15], [16]) for the first time. Finsler spaces with different types of recurrent curvature tensors have been discussed by R.N. Sen [30], R.S. Mishra and H.D. Pande [9], R.B. Misra ([10], [11], [12]), R.B. Misra and F.M. Meher [13], H.D. Pande and B. Singh [19], P.N. Pandey ([20], [21], [22]), P.N. Pandey and R.B. Misra [25], R.S.D. Dubey and A.K. Srivastava [4], P.N. Pandey and V.J. Dwivedi [24], V.J. Dwivedi [5], R. Verma [31], P.N. Pandey and S. Dikshit [23], S. Dikshit [3] and others. A. Moór [14] studied a Finsler space of recurrent torsion. M. Matsumoto ([6], [7]) introduced a Finsler space in which torsion tensor is recurrent and called it C^h-*recurrent space*. Though several contributions have been made by above authors to spaces of recurrent curvature. P.N. Pandey and R. Verma [26], R. Verma [31] and C.K. Mishra and G. Lodhi [8] discussed C^h-birecurrent space. We have little informations about the space in which curvature tensor R^i_{jkh} is h-recurrent. H.D. Pande and K.K. Gupta ([17], [19]) introduced a Finsler space in which Cartan's curvature tensor R^i_{jkh} is recurrent of first, second and third order with non-symmetric connection and called them R-⊕*recurrent space of first*, *second* and *third order*. R. Verma [31] introduced a Finsler space in which Cartan's third curvature tensor R^i_{jkh} is recurrent and called it R^h-*recurrent space*. S. Dikshit [3] discussed a Finsler space in which Cartan's curvature tensor R^i_{jkh} is birecurrent and called it R^h-*birecurrent space*. F.Y.A. Qasem [27] discussed a Finsler spaces in which Cartan's third curvature tensor R^i_{jkh} generalized (special generalized) birecurrent of first and second kind and called them R^h-*generalized* (R^h-*special generalized*) *birecurrent space of first* and *second kind*. F.Y.A. Qasem and A.A.A. Muhib [28] discussed a Finsler space in which Cartan's third curvature tensor R^i_{jkh} is trirecurrent and called it R^h-*trirecurrent space*.

In the present paper we introduce R^h-generalized trirecurrent and R^h-special generalized trirecurrent Finsler spaces, we discuss different Finsler spaces with generalized and special generalized trirecurrent of Cartan's curvature tensor R^i_{jkh}, where as generalized and special generalized trirecurrent of associative curvature tensor R_{ijkh} are investigated.

II. Preliminaries

Let F_n be an n-dimensional Finsler space equipped with the metric function F satisfies the requisite conditions [29]. The components of the corresponding metric tensor be denoted by g_{ij} . This is positively homogeneous of degree zero in y^i. Due to it homogeneity in y^i, we have

(1.1) a) $C_{ijk}\, y^i = C_{kij}\, y^i = C_{jki}\, y^i = 0$

and

b) $C^h_{ik}\, y^i \coloneqq g^{hj} C_{ijk}$,

where g^{ij} is the associate metric tensor of the metric tensor and the (v) hv-torsion tensor C_{ijk} [8]. These are positively homogeneous of degree -1 in y^i and they are symmetric in the lower indices j and k.

É. Cartan deduced ([1], [2]) and [29] the h-covariant derivative of an arbitrary vector field X^i with respect to x^k which given by

(1.2) $X^i_{|k} \coloneqq \partial_k X^i - (\dot\partial_r X^i)\, G^r_k + X^r \Gamma^{*i}_{rk}$,

where $\partial_j \equiv \frac{\partial}{\partial x^j}$, $\dot\partial_r \equiv \frac{\partial}{\partial y^r}$, $G^i_j \coloneqq \dot\partial_j G^i$, $G^i_j = \Gamma^{*i}_{sj}\, y^s$ and the connection of Cartan

$\Gamma^{*i}_{sj} \coloneqq \Gamma^i_{sj} - C^i_{ms}\, \Gamma^m_{rj}\, y^r$.

The associative metric tensor g^{ij} and the vector y^i vanish under h-covariant differentiation, i.e.

(1.3) a) $g^{ij}_{|k} = 0$ and b) $y^i_{|k} = 0$.

The process of h-covariant differentiation with respect to x^k, defined above commute with partial differentiation with respect to y^j according to

(1.4) a) $\dot\partial_j (X^i_{|k}) - (\dot\partial_j X^i)_{|k}$
$= X^r (\dot\partial_j\, \Gamma^{*i}_{rk}) - (\dot\partial_r\, X^i) P^r_{jk}$,

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where

b) $P^r_{jk} := (\dot\partial_j \Gamma^{*i}_{rk})\, y^h = \Gamma^{*r}_{jkh}\, y^h$.

Cartan's fourth curvature tensor K^i_{jkh} is defined by

(1.5) $K^i_{rkh} := \partial_h \Gamma^{*i}_{kr} + (\dot\partial_l \Gamma^{*i}_{rh})\, G^l_k + \Gamma^{*i}_{mh}\, \Gamma^{*m}_{kr} - \mathrm{h/k}^*$.

This tensor is skew symmetric in its last two lower indices h and k and positively homogeneous of degree zero in y^i.

The tensor K^i_{jkh} satisfies the relation

(1.6) $K^i_{jkh}\, y^j = H^i_{kh}$.

Berwald curvature tensor H^i_{jkh} and the h(v)-torsion tensor H^i_{kh} are defined by

(1.7) a) $H^i_{jkh} := \partial_h G^i_{jk} + G^s_{jk} G^i_{sh} + G^i_{sjh} G^s_k - h/k$

and

b) $H^i_{kh} := \partial_h G^i_k + G^r_k G^i_{rh} - h/k$

or

c) $H^i_{kh} := \frac{1}{3}(\dot\partial_k H^i_h - \dot\partial_h H^i_k)$,

where

(1.8) $H^i_h := 2\partial_h G^i - \partial_r G^i_h\, y^r + 2G^i_{hr} G^i - G^i_r G^i_h$

is the deviation tensor which is positively homogeneous of degree two in y^i.

Berwald curvature tensor H^i_{jkh} and the h(v)-torsion tensor H^i_{kh} are skew-symmetric in their lower indices k and h. These tensor are positively homogeneous of degree zero and one in y^i, respectively. These tensors are connected by

(1.9) a) $H^i_{rkh}\, y^r = H^i_{kh}$

and

b) $H^i_{rkh} := \dot\partial_k H^i_{hk}$.

In view of Euler's theorem on homogeneous functions we have the following:

(1.10) $H^i_{jk}\, y^j = H^i_k = -H^i_{kj}\, y^j$.

The contraction of the indices i and h in (1.7c) and (1.8) yield the following:

(1.11) a) $H_r = H^i_{ri}$

and

b) $H = \frac{1}{n-1} H^i_i$,

where H is a scalar called *curvature scalar*.

The tensor $H_{ij.k}$ defined by

(1.12) $H_{ij.k} := g_{ik}\, H^i_{jh}$.

Cartan's third curvature tensor R^i_{jkh} is defined by

(1.13) $R^i_{jkh} := \partial_h \Gamma^{*i}_{jk} + (\dot\partial_l \Gamma^{*i}_{jh})\, G^l_k + G^i_{jm}(\partial_h G^m_h - G^m_{hl} G^l_k) + \Gamma^{*i}_{mh}\, \Gamma^{*m}_{jk} - h/k$

which is skew-symmetric in its last two lower indices and satisfies the relations

(1.14) a) $R^i_{jkh} = K^i_{jkh} + C^i_{jr}\, H^r_{kh}$

and

b) $R^i_{jkh}\, y^i = H^i_{kh} = K^i_{jkh}\, y^i$.

$* -h/k$ means the subtraction from the former term by interchanging the indices h and k.

Cartan's second curvature tensor P^i_{jkh} is defined by

(1.15) a) $P^i_{jkh} := \dot\partial_h \Gamma^{*i}_{jk} + C^i_{jm}\, P^m_{kh} - C^i_{jh|k}$

or equivalent by

b) $P^i_{jkh} = \dot\partial_h \Gamma^{*i}_{jk} + C^i_{jr} C^r_{kh|s}\, y^s - C^i_{jh|k}$

or

c) $P^i_{jkh} = C^i_{kh|j} - g^{ir} C_{jkh|r} + C^r_{jk} P^i_{rh} - P^r_{jh} C^i_{rk}$.

The tensor P^i_{jkh} satisfies the relation

(1.16) $P^i_{jkh}\, y^j = \Gamma^{*i}_{jkh}\, y^j = P^i_{kh} = C^i_{kh|r} y^r$.

A Finsler space whose connection parameter G^i_{jk} is independent of y^i is called *an affinely connected space* (*Berwald space*). Thus, an affinely connected space characterized by any one of the equivalent conditions

(1.17) a) $G^i_{jkh} = 0$ and b) $C_{ijk|h} = 0$.

The connection parameters Γ^{*i}_{kh} of Cartan and G^i_{kh} of Berwald coincide in affinely connected space and they are independent of the directional argument [29], i.e.

(1.18) a) $G^i_{jkh} = \dot\partial_j\, G^i_{kh} = 0$

and b) $\dot\partial_j \Gamma^{*i}_{hk} = 0$.

Cartan's connection paramete Γ^{*i}_{jk} coincides with Berwald's connection parameter G^i_{jk} for a Landsberg space, which is characterized by the condition

(1.19) $y_r\, G^r_{jkh} = -2\, C_{jkh|l}\, y^l = -2\, P_{jkh} = 0$.

The associate curvature tensor P_{rkh} of the v(hv)-torsion tensor P^i_{kh} is given by

(1.20) $g_{ir}\, P^i_{kh} = P_{krh}$.

III. A R^h- Generalized Trirecurrent Finsler Space

Let us consider a Finsler space whose Cartan's third curvature tensor R^i_{jkh} satisfies the condition

(2.1) $R^i_{jkh|m|l|n} = a_{nlm} R^i_{jkh} + \lambda_l R^i_{jkh|m|n}$, $R^i_{jkh} \neq 0$,

where λ_l and a_{nlm} are non-zero covariant vector field and covariant tensor field of third order, respectively. The space and the tensor satisfy the condition (2.1) will be called R^h*-generalized trirecurrent space* and *h-generalized trirecurrent tensor*, respectively. We shall denote such space and tensor briefly by R^h-GTR-F_n and h-GTR .

Let us consider a Finsler space for which Cartan's third curvature tensor R^i_{jkh} satisfies the condition

(2.2) $R^i_{jkh|m|l|n} = \lambda_l\, R^i_{jkh|m|n}$, $R^i_{jkh} \neq 0$,

where λ_l is non-zero covariant vector field. The space and the tensor satisfy the condition (2.2) will be called R^h-*special generalized trirecurrent space* and *h-special generalized trirecurrent tensor*, respectively. We shall denote such space and tensor briefly by R^h-$SGTR$-F_n and h-$SGTR$, respectively.

Let us consider an R^h-GTR-F_n and R^h-$SGTR$-F_n characterized by the conditions (2.1) and (2.2), respectively. Since the metric tensor g_{ip} of a Finsler space is a covariant constant, it may proved that the conditions (2.1) and (2.2) are equivalent to the conditions

(2.3) $R_{jpkh|m|l|n} = a_{nlm}\, R_{jpkh}$

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$+\lambda_l \, R_{jpkh|m|n}$

and

(2.4) $\quad R_{jpkh|m|l|n} = \lambda_l \, R_{jpkh|m|n}$,

respectively. Conversely, the transvection of the conditions (2.3) and (2.4) by the associate metric tensor g^{ij} of the metric tensor g_{ij} yield the condition (2.1) and (2.2), respectively.

Therefore R^h - GTR - F_n and R^h - $SGTR$ - F_n may characterized by the conditions (2.3) and (2.4), respectively.

Thus, we conclude

Theorem 2.1. *An* R^h-GTR-F_n , may characterized by the condition (2.3).

Theorem 2.2. *An* R^h-$SGTR$-F_n , may characterized by the condition (2.4).

The contraction of the indices i and h in the conditions (2.1) and (2.2) leads to

(2.5) $\quad R_{jk|m|l|n} = a_{nlm} \, R_{jk} + \lambda_l R_{jk|m|n}$

and

(2.6) $\quad R_{jk|m|l|n} = \lambda_l \, R_{jk|m|n}$,

respectively, which show that Ricci tensor R_{jk} of R^h-GTR-F_n and R^h-$SGTR$-F_n are generalized trirecurrent and special generalized trirecurrent, respectively. A space whose Ricci tensor is generalized trirecurrent and special generalized trirecurrent with respect to Cartan's connection parameter called as *Ricci generalized trirecurrent space* and *Ricci special generalized trirecurrent space*, respectively. Thus, we see that every R^h-GTR-F_n and R^h-$SGTR$-F_n are Ricci generalized trirecurrent space and Ricci special generalized trirecurrent space, respectively.

Thus, we conclude

Theorem 2.3. *An* R^h-GTR-F_n , is Ricci-GTR-F_n.

Theorem 2.4. *An* R^h-$SGTR$-F_n is Ricci-$SGTR$-F_n .

Now, transvecting the conditions (2.1) and (2.2) by y^i and using (1.14b), we get

(2.7) $\quad H^i_{kh|m|l|n} = a_{nlm} \, H^i_{kh} + \lambda_l \, H^i_{kh|m|n}$

and

(2.8) $\quad H^i_{kh|m|l|n} = \lambda_l \, H^i_{kh|m|n}$,

respectively.

Transvecting the conditions (2.7) and (2.8) by g_{ij} and using (1.12), we get

(2.9) $\quad H_{kj.h|m|l|n} = a_{nlm} \, H_{kj.h}$
$\qquad\qquad +\lambda_l \, H_{kj.h|m|n}$

and

(2.10) $\quad H_{kj.h|m|l|n} = \lambda_l H_{kj.h|m|n}$,

respectively.

Transvecting the conditions (2.7) and (2.8) by y^k and using (1.10), we get

(2.11) $\quad H^i_{h|m|l|n} = a_{nlm} \, H^i_h + \lambda_l \, H^i_{h|m|n}$

and

(2.12) $\quad H^i_{h|m|l|n} = \lambda_l \, H^i_{h|m|n}$,

respectively.

In view of (1.11a) and (1.11b), the contraction of the indices i and h in the conditions (2.7), (2.8), (2.11) and (2.12) gives

(2.13) $\quad H_{k|m|l|n} = a_{nlm} \, H_k + \lambda_l \, H_{k|m|n}$,

(2.14) $\quad H_{k|m|l|n} = \lambda_l \, H_{k|m|n}$,

(2.15) $\quad H_{|m|l|n} = a_{nlm} \, H + \lambda_l \, H_{|m|n}$

and

(2.16) $\quad H_{|m|l|n} = \lambda_l \, H_{|m|n}$, respectively.

Thus, we may conclude

Theorem 2.5. *In* R^h-GTR-F_n *, the* $h(v)$*-torsion tensor* H^i_{kh} *, the tensor* $H_{kj.h}$ *, the deviation tensor* H^i_h *,the curvature vector* H_k *and the curvature scalar* H *are* h-GTR.

Theorem 2.6. *In* R^h-$SGTR$-F_n *, the* $h(v)$*-torsion tensor* H^i_{kh} *, the tensor* $H_{kj.h}$ *, the deviation tensor* H^i_h *,the curvature vector* H_k *and the curvature scalar* H *are* h-$SGTR$.

Cartan ([1], [2]) deduced the identity

(2.17) $\quad R_{ijhk} + R_{ikjh} + R_{ihkj}$
$\qquad +(C_{ijs}K^s_{rhk} + C_{iks}K^s_{rjh} + C_{ihs}K^s_{rkj})y^r = 0$

which in view of (1.6), the identity (2.17) reduces to

(2.18) $\quad R_{ijhk} + R_{ikjh} + R_{ihkj} + (C_{ijs}H^s_{hk} + C_{iks}H^s_{jh} + C_{ihs}H^s_{kj}) = 0$.

Taking the h-covariant derivative for (2.18) twice with respect to x^m and x^l ,successively, we get

(2.19) $\quad R_{ijhk|m|l} + R_{ikjh|m|l} + R_{ihkj|m|l}$
$+(C_{ijs}H^s_{hk} + C_{iks}H^s_{jh} + C_{ihs}H^s_{kj})_{|m|l} = 0$.

Again, taking the h-covariant derivative for (2.19) with respect to x^n , using the condition (2.3) and (2.4), we get

(2.20) $\quad a_{nlm}\left(R_{ijhk} + R_{ikjh} + R_{ihkj}\right) +$
$\lambda_l(R_{ijhk|m|n} + R_{ikjh|m|n} + R_{ihkj|m|n}) + (C_{ijs}H^s_{hk} + C_{iks}H^s_{jh} + C_{ihs}H^s_{kj})_{|m|l|n} = 0$

and

(2.21) $\lambda_l\left(R_{ijhk|m|n} + R_{ikjh|m|n} + R_{ihkj|m|n}\right)$
$+(C_{ijs}H^s_{hk} + C_{iks}H^s_{jh} + C_{ihs}H^s_{kj})_{|m|l|n} = 0.$

In view of (2.18) and (2.19), the equations (2.20) and (2.21) can be written as

(2.22) $(C_{ijs}H^s_{hk} + C_{iks}H^s_{jh} + C_{ihs}H^s_{kj})_{|m|l|n}$
$= a_{nlm}(C_{ijs}H^s_{hk} + C_{iks}H^s_{jh} + C_{ihs}H^s_{kj})$
$+ \lambda_l(C_{ijs}H^s_{hk} + C_{iks}H^s_{jh} + C_{ihs}H^s_{kj})_{|m|n}$

and

(2.23) $(C_{ijs}H^s_{hk} + C_{iks}H^s_{jh} + C_{ihs}H^s_{kj})_{|m|l|n}$
$= \lambda_l(C_{ijs}H^s_{hk} + C_{iks}H^s_{jh} + C_{ihs}H^s_{kj})_{|m|n}$.

Transvecting (2.22) and (2.23) by y^j , using (1.1a), (1.10) and (1.3b), we get

(2.24) $\quad (C_{iks}H^s_h - C_{ihs}H^s_k)_{|m|l|n}$
$\qquad = a_{nlm}(C_{iks}H^s_h - C_{ihs}H^s_k)$
$\qquad +\lambda_l(C_{iks}H^s_h - C_{ihs}H^s_k)_{|m|n}$

and

(2.25) $\quad (C_{iks}H^s_h - C_{ihs}H^s_k)_{|m|l|n}$
$\qquad = \lambda_\ell(C_{iks}H^s_h - C_{ihs}H^s_k)_{|m|n}$.

Transvecting (2.24) and (2.25) by g^{pi} , using the symmetric property of the (h)hv-torsion tensor C_{ijk} in all lower indices, (1.1b) and (1.3a), we get

(2.26) $\quad (C^p_{ks}H^s_h - C^p_{hs}H^s_k)_{|m|l|n}$
$\qquad = a_{nlm}\left(C^p_{ks}H^s_h - C^p_{hs}H^s_k\right)$
$\qquad +\lambda_\ell(C^p_{ks}H^s_h - C^p_{hs}H^s_k)_{|m|n}$

and

(2.27) $\quad (C^p_{ks}H^s_h - C^p_{hs}H^s_k)_{|m|l|n}$
$\qquad = \lambda_\ell(C^p_{ks}H^s_h - C^p_{hs}H^s_k)_{|m|n}$.

Thus, we conclude

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Theorem 2.7. *In* R^h-GTR-F_n *, the tensors* $(C_{ijs}H^s_{hk} + C_{iks}H^s_{jh} + C_{ihs}H^s_{kj})$, $(C_{iks}H^s_h - C_{ihs}H^s_k)$ *and* $(C^p_{ks}H^s_h - C^p_{hs}H^s_k)$ *are h-GTR.*

Theorem 2.8. *In* R^h-$SGTR$-F_n *, the tensors* $(C_{ijs}H^s_{hk} + C_{iks}H^s_{jh} + C_{ihs}H^s_{kj})$, $(C_{iks}H^s_h - C_{ihs}H^s_k)$ *and* $(C^p_{ks}H^s_h - C^p_{hs}H^s_k)$ *are h-SGTR.*

IV. Necessary and Sufficient Condition

We shall find the necessary and sufficient condition for some tensors to be h-GTR and h-$SGTR$ in R^h-GTR-F_n and R^h-$SGTR$-F_n , respectively.

Let us consider an R^h-GTR-F_n which is characterized by the condition (2.1) and an R^h-$SGTR$-F_n characterized by the condition (2.2).

Firstly, we shall find the necessary and sufficient condition for Berwald curvature tensor H^i_{jkh} to be h-GTR and h-$SGTR$ in R^h-GTR-F_n and R^h-$SGTR$-F_n , respectively.

Differentiating (2.3)and (2.4) partially with respect to y^i and using (1.9b),we get

(3.1) $$\dot\partial_j\big(H^i_{kh|m|l|n}\big) = \big(\dot\partial_j a_{nlm}\big)H^i_{kh} + a_{nlm}H^i_{jkh} + \big(\dot\partial_j\lambda_l\big)H^i_{kh|m|n} + \lambda_l\dot\partial_j\big(H^i_{kh|m|n}\big)$$

and

(3.2) $$\dot\partial_j\big(H^i_{kh|m|\ell|n}\big) = \big(\dot\partial_j\lambda_l\big)\, H^i_{kh|m|n} + \lambda_l\,\dot\partial_j\big(H^i_{kh|m|n}\big)\,,$$

respectively.

Using the commutation formula exhibited by (1.4a) for $\big(H^i_{kh|m|l}\big)$ in (3.1) and (3.2), we get

(3.3) $$H^i_{jkh|m|l|n} + \big[\{H^r_{kh}\dot\partial_j\Gamma^{*i}_{rm} - H^i_{rh}\dot\partial_j\Gamma^{*i}_{km} - H^i_{kr}\dot\partial_j\Gamma^{*r}_{hm} - H^i_{rkh}P^r_{jm}\}_{|l} + H^r_{kh|m}\dot\partial_j\Gamma^{*i}_{rl} - H^i_{rh|m}\dot\partial_j\Gamma^{*r}_{kl} - H^i_{kr|m}\dot\partial_j\Gamma^{*r}_{hl} - H^i_{kh|r}\dot\partial_j\Gamma^{*r}_{ml} - H^i_{ks}\dot\partial_r\Gamma^{*s}_{hm} - H^i_{skh}P^s_{rm}\}P^r_{jl}\big]_{|n} + H^r_{kh|m|l}\dot\partial_j\Gamma^{*i}_{rn} - H^i_{rh|m|l}\dot\partial_j\Gamma^{*i}_{kn} - H^i_{kr|m|l}\dot\partial_j\Gamma^{*r}_{hn} - H^i_{kh|r|l}\,\dot\partial_j\Gamma^{*r}_{mn} - H^i_{kh|m|r}\,\dot\partial_j\Gamma^{*r}_{ln} - \big[\{H^i_{rkh|m} + H^s_{kh}\,\dot\partial_r\Gamma^{*i}_{sm} - H^i_{sh}\dot\partial_r\Gamma^{*s}_{km} - H^i_{ks}\,\dot\partial_r\Gamma^{*s}_{hm} - H^i_{skh}\,P^s_{rm}\}_{|l} + H^s_{kh|m}\,\dot\partial_r\Gamma^{*i}_{sl} - H^i_{sh|m}\,\dot\partial_r\Gamma^{*s}_{kl} - H^i_{ks|m}\dot\partial_r\Gamma^{*s}_{hl} - H^i_{kh|s}\dot\partial_r\Gamma^{*s}_{ml} - \{H^i_{skh|m} + H^t_{kh}\dot\partial_s\Gamma^{*i}_{tm} - H^i_{th}\dot\partial_s\Gamma^{*t}_{km} - H^i_{kt}\dot\partial_s\Gamma^{*t}_{hm} - H^i_{tkh}P^t_{sm}\}P^s_{rl}\big]P^r_{jn} = b_{hlm}H^i_{jkh} + \lambda_\ell H^i_{jkh|m|n}\big(\dot\partial_j b_{hlm}\big)H^i_{kh} + \big(\dot\partial_j\lambda_l\big)H^i_{kh|m|n} + \lambda_l\big[\{H^r_{kh}\dot\partial_j\Gamma^{*i}_{rm} - H^i_{rh}\,\dot\partial_j\Gamma^{*r}_{km} - H^i_{kr}\,\dot\partial_j\Gamma^{*r}_{hm} - H^i_{rkh}\,P^r_{jm}\}_{|n} + H^r_{kh|m}\,\dot\partial_j\Gamma^{*i}_{rn} - H^i_{rh|m}\dot\partial_j\Gamma^{*r}_{kn} - H^i_{kr|m}\dot\partial_j\Gamma^{*r}_{hn} - H^i_{kh|r}\dot\partial_j\Gamma^{*r}_{mn} - \{H^i_{rkh|m} + H^s_{kh}\dot\partial_r\Gamma^{*i}_{sm} - H^i_{sh}\,\dot\partial_r\Gamma^{*s}_{km} - H^i_{ks}\dot\partial_r\Gamma^{*s}_{hm} - H^i_{skh}P^s_{rm}\}P^r_{jn}\big]$$

and

(3.4) $$H^i_{jkh|m|l|n} + \big[\{H^r_{kh}\dot\partial_j\Gamma^{*i}_{rm} - H^i_{rh}\dot\partial_j\Gamma^{*r}_{km} - H^i_{kr}\dot\partial_j\Gamma^{*r}_{hm} - H^i_{rkh}P^r_{jm}\}_{|l} + H^i_{kh|m}\dot\partial_j\Gamma^{*i}_{rl} - H^i_{rh|m}\dot\partial_j\Gamma^{*r}_{kl} - H^i_{kr|m}\dot\partial_j\Gamma^{*r}_{hl} - H^i_{kh|r}\dot\partial_j\Gamma^{*r}_{ml} - H^i_{ks}\,\dot\partial_r\Gamma^{*s}_{hm} - H^i_{skh}P^s_{rm}\}\,P^r_{jl}\big]_{|n} + H^r_{kh|m|l}\dot\partial_j\Gamma^{*i}_{rn} - H^i_{rh|m|l}\dot\partial_j\Gamma^{*i}_{kn} - H^i_{kr|m|l}\dot\partial_j\Gamma^{*r}_{hn} - H^i_{kh|r|l}\,\dot\partial_j\Gamma^{*r}_{mn} - H^i_{kh|m|r}\,\dot\partial_j\Gamma^{*r}_{ln} - \big[\{H^i_{rkh|m} + H^s_{kh}\,\dot\partial_r\Gamma^{*i}_{sm} - H^i_{sh}\,\dot\partial_r\Gamma^{*s}_{km} - H^i_{ks}\dot\partial_r\Gamma^{*s}_{hm} - H^i_{skh}P^s_{rm}\}_{|l} + H^s_{kh|m}\,\dot\partial_r\Gamma^{*i}_{sl} - H^i_{sh|m}\,\dot\partial_r\Gamma^{*s}_{kl} - H^i_{ks|m}\,\dot\partial_r\Gamma^{*s}_{hl} - H^i_{kh|s}\dot\partial_r\Gamma^{*s}_{ml} - \{H^i_{skh|m} + H^t_{kh}\dot\partial_s\Gamma^{*i}_{tm} - H^i_{th}\dot\partial_s\Gamma^{*t}_{km} - H^i_{kt}\dot\partial_s\Gamma^{*t}_{hm} - H^i_{tkh}P^t_{sm}\}P^s_{rl}\big]P^r_{jn} = \lambda_l H^i_{jkh|m|n} + \big(\dot\partial_j\lambda_l\big)H^i_{kh|m|n} + \lambda_l\big[\{H^r_{kh}\dot\partial_j\Gamma^{*i}_{rm} - H^i_{rh}\,\dot\partial_j\Gamma^{*r}_{km} - H^i_{kr}\,\dot\partial_j\Gamma^{*r}_{hm} - H^i_{rkh}P^r_{jm}\}_{|n} + H^r_{kh|m}\,\dot\partial_j\Gamma^{*i}_{rn} - H^i_{rh|m}\,\dot\partial_j\Gamma^{*r}_{kn} - H^i_{kr|m}\,\dot\partial_j\Gamma^{*r}_{hn} - H^i_{kh|r}\,\dot\partial_j\Gamma^{*r}_{mn} - \{H^i_{rkh|m} + H^s_{kh}\dot\partial_r\Gamma^{*i}_{sm} - H^i_{sh}\dot\partial_r\Gamma^{*s}_{km} - H^i_{ks}\dot\partial_r\Gamma^{*s}_{hm} - H^i_{skh}P^s_{rm}\}P^r_{jn}\big]\,,$$

respectively.

This shows that

(3.5) $$H^i_{jkh|m|l|n} = a_{nlm}\,H^i_{jkh} + \lambda_l\,H^i_{jkh|m|n}$$

and

(3.6) $$H^i_{jkh|m|l|n} = \lambda_l\,H^i_{jkh|m|n}$$

if and only if

(3.7) $$\big[\{H^r_{kh}\dot\partial_j\Gamma^{*i}_{rm} - H^i_{rh}\dot\partial_j\Gamma^{*i}_{km} - H^i_{kr}\dot\partial_j\Gamma^{*i}_{hm} - H^i_{rkh}P^r_{jm}\}_{|l} + H^r_{kh|m}\dot\partial_j\Gamma^{*i}_{rl} - H^i_{rh|m}\dot\partial_j\Gamma^{*r}_{kl} - H^i_{kr|m}\dot\partial_j\Gamma^{*r}_{hl} - H^i_{kh|r}\dot\partial_j\Gamma^{*r}_{ml} - \{H^i_{rkh|m} + H^s_{kh}\dot\partial_r\Gamma^{*i}_{sm} - H^i_{sh}\dot\partial_r\Gamma^{*s}_{km} - H^i_{ks}\dot\partial_r\Gamma^{*s}_{hm} - H^i_{skh}\,P^s_{rm}\}\,P^r_{jl}\big]_{|n} + H^r_{kh|m|l}\,\dot\partial_j\,\Gamma^{*i}_{rn} - H^i_{rh|m|l}\,\dot\partial_j\Gamma^{*r}_{kn} - H^i_{kr|m|l}\,\dot\partial_j\,\Gamma^{*r}_{hn} - H^i_{kh|r|l}\,\dot\partial_j\Gamma^{*r}_{mn} - H^i_{kh|m|r}\,\dot\partial_j\Gamma^{*r}_{ln} - \big[\{H^i_{rkh|m} + H^s_{kh}\,\dot\partial_r\Gamma^{*i}_{sm} - H^i_{sh}\,\dot\partial_r\Gamma^{*s}_{km} - H^i_{ks}\dot\partial_r\Gamma^{*s}_{hm} - H^i_{skh}P^s_{rm}\}_{|l} + H^s_{kh|m}\dot\partial_r\Gamma^{*i}_{sl} - H^i_{sh|m}\dot\partial_r\Gamma^{*s}_{kl} - H^i_{ks|m}\dot\partial_r\Gamma^{*s}_{hl} - H^i_{kh|s}\,\dot\partial_r\Gamma^{*s}_{ml} - \{H^i_{skh|m} + H^t_{ks}\,\dot\partial_s\Gamma^{*i}_{tm} - H^i_{th}\dot\partial_s\Gamma^{*t}_{km} - H^i_{kt}\,\dot\partial_s\,\Gamma^{*t}_{hm} - H^i_{tkh}P^t_{sm}\}\,P^s_{rl}\big]P^r_{jn} = \big(\dot\partial_j b_{hlm}\big)H^i_{kh} + \big(\dot\partial_j\lambda_l\big)\,H^i_{kh|m|n} + \lambda_l\big[\,\{H^r_{kh}\dot\partial_j\Gamma^{*i}_{rm}\, y - H^i_{rh}\dot\partial_j\Gamma^{*r}_{km} - H^i_{kr}\,\dot\partial_j\Gamma^{*r}_{hm} - H^i_{rkh}P^r_{jm}\}_{|n} + H^r_{kh|m}\dot\partial_j\Gamma^{*i}_{rn} - H^i_{rh|m}\dot\partial_j\Gamma^{*r}_{kn} - H^i_{kr|m}\,\dot\partial_j\,\Gamma^{*r}_{hn} - H^i_{kh|r}\,\dot\partial_j\Gamma^{*r}_{mn} - \{H^i_{rkh|m} + H^s_{kh}\,\dot\partial_r\,\Gamma^{*i}_{sm} - H^i_{sh}\,\dot\partial_r\Gamma^{*s}_{km} - H^i_{ks}\,\dot\partial_r\Gamma^{*s}_{hm} - H^i_{skh}P^s_{rm}\}P^r_{jn}\big]$$

and

(3.8) $$\big[\{H^r_{kh}\dot\partial_j\Gamma^{*i}_{rm} - H^i_{rh}\dot\partial_j\Gamma^{*i}_{km} - H^i_{kr}\dot\partial_j\Gamma^{*i}_{hm} - H^i_{rkh}P^r_{jm}\}_{|l} + H^r_{kh|m}\dot\partial_j\Gamma^{*i}_{rl} - H^i_{rh|m}\dot\partial_j\,\Gamma^{*r}_{kl} - H^i_{kr|m}\dot\partial_j\Gamma^{*r}_{hl} - H^i_{kh|r}\dot\partial_j\Gamma^{*r}_{ml} - \{\,H^i_{rkh|m} + H^s_{kh}\dot\partial_r\Gamma^{*i}_{sm} - H^i_{sh}\dot\partial_r\Gamma^{*s}_{km} - H^i_{ks}\dot\partial_r\Gamma^{*s}_{hm}$$

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$$-H^i_{skh}P^s_{rm}\}\, P^r_{jl}\big]_{|n} + H^r_{kh|m|\ell}\dot\partial_j \Gamma^{*i}_{rn}$$
$$- H^i_{rh|m|\ell}\, \dot\partial_j \Gamma^{*i}_{kn} - H^i_{kr|m|\ell}\dot\partial_j \Gamma^{*r}_{hn}$$
$$-H^i_{kh|r|\ell}\dot\partial_j \Gamma^{*r}_{mn} - H^i_{kh|m|r}\, \dot\partial_j\, \Gamma^{*r}_{ln}$$
$$-[\{H^i_{rkh|m} + H^s_{kh}\dot\partial_r \Gamma^{*i}_{sm} - H^i_{sh}\, \dot\partial_r \Gamma^{*s}_{km}$$
$$- H^i_{ks}\dot\partial_r \Gamma^{*s}_{hm} - H^i_{skh}P^s_{rm}\}_{|l} + H^s_{kh|m}\, \dot\partial_r \Gamma^{*i}_{sl}$$
$$-H^i_{sh|m}\, \dot\partial_r \Gamma^{*s}_{kl} - H^i_{ks|m}\, \dot\partial_r \Gamma^{*s}_{hl} - H^i_{kh|s}\dot\partial_r \Gamma^{*s}_{ml}$$
$$-\{H^i_{skh|m} + H^t_{kh}\dot\partial_s \Gamma^{*i}_{tm} - H^i_{th}\dot\partial_s \Gamma^{*t}_{km}$$
$$-H^i_{kt}\dot\partial_s \Gamma^{*t}_{hm} - H^i_{tkh}\, P^t_{sm}\}P^s_{rl}]P^r_{jn}$$
$$= (\dot\partial_j \lambda_l)H^i_{kh|m|n} + \lambda_\ell[\{H^r_{kh}\dot\partial_j \Gamma^{*i}_{rm}$$
$$-H^i_{rh}\dot\partial_j \Gamma^{*r}_{km} - H^i_{kr}\dot\partial_j \Gamma^{*r}_{hm} - H^i_{rkh}P^r_{jm}\}_{|n}$$
$$+ H^r_{kh|m}\, \dot\partial_j \Gamma^{*i}_{rn} - H^i_{rh|m}\dot\partial_j \Gamma^{*r}_{kn} - H^i_{kr|m}\dot\partial_j \Gamma^{*r}_{hn}$$
$$-H^i_{kh|r}\dot\partial_j \Gamma^{*r}_{mn} - \{H^i_{rkh|m} + H^s_{kh}\, \dot\partial_r \Gamma^{*i}_{sm}$$
$$-H^i_{sh}\dot\partial_r \Gamma^{*s}_{km} - H^i_{ks}\dot\partial_r \Gamma^{*s}_{hm} - H^i_{skh}\, P^s_{rm}\}P^r_{jn}],$$

respectively.

Thus, we conclude

Theorem 3.1. *In R^h-GTR-F_n , Berwald curvature tensor H^i_{kh} , is h-GTR if and only if (3.7) holds good.*

Theorem 3.2. *In R^h-$SGTR$-F_n , h(v)-torsion tensor H^i_{kh} , is h-GTR if and only if (3.8) holds good.*

Since Cartan's third curvature tensor R^i_{jkh} and Cartan's fourth curvature tensor K^i_{jkh} are connected by the formula

(3.9) $\quad R^i_{jkh} = K^i_{jkh} + C^i_{js}\, K^s_{rkh}\, y^r.$

Taking the h-covariant derivative for (3.9) three times with respect to x^m, x^l and x^n , successively , using (1.3a) and (2.1), we get

(3.10) $$a_{nlm}\, R^i_{jkh} + \lambda_l\, R^i_{jkh|m|n} = K^i_{jkh|m|l|n}$$
$$+ C^i_{js|m|l|n}\, K^s_{rkh}\, y^r + C^i_{js|m|n}\, K^s_{rkh|l}\, y^r$$
$$+C^i_{js|m}K^s_{rkh|l|n}\, y^r + C^i_{js|l|n}K^s_{rkh|m}y^r$$
$$+C^i_{js|l}K^s_{rkh|m|n}\, y^r + C^i_{js|n}K^s_{rkh|m|l}\, y^r$$
$$+ C^i_{js}\, K^s_{rkh|m|l|n}\, y^r.$$

Using (1.3a), (1.6) and the condition (2.3) in (3.10), we get

(3.11) $$a_{nlm}\, R^i_{jkh} + \lambda_l\, R^i_{jkh|m|n} = K^i_{jkh|m|l|n}$$
$$+ C^i_{js|m|l|n}\, H^s_{kh} + C^i_{js|m|l}\, H^s_{kh|n}$$
$$+ C^i_{js|m|n}\, H^s_{kh|l} + C^i_{js|m}\, H^s_{kh|l|n}$$
$$+C^i_{js|l|n}H^s_{kh|m} + C^i_{js|l}H^s_{kh|m|n}$$
$$+C^i_{js|n}H^s_{kh|m|l} + C^i_{js}\big(a_{n\ell m}H^s_{kh} + \lambda_l H^s_{kh|m|n}\big).$$

In view of (3.9), the condition (3.11) can be written as

(3.12) $$a_{nlm}\, \big(R^i_{jkh} - C^i_{js}\, H^s_{kh}\big)$$
$$+\lambda_l\big(R^i_{jkh} - C^i_{js}H^s_{kh}\big) = K^i_{jkh|m|l|n}$$
$$+ C^i_{js|m|l|n}\, H^s_{kh} + C^i_{js|m|l}H^s_{kh|n}$$
$$+ C^i_{js|m|n}H^s_{kh|l} + C^i_{js|m}H^s_{kh|l|n}$$
$$+ C^i_{js|l|n}H^s_{kh|m} + C^i_{js|l}H^s_{kh|m|n}$$
$$+ C^i_{js|n}H^s_{kh|m|l} - \lambda_l(C^i_{js|m|n}H^s_{kh}$$
$$+ C^i_{js|m}H^s_{kh|n} + C^i_{js|n}H^s_{kh|m}) \quad .$$

Using (1.14 a) in (3.12), we get

(3.13) $$a_{nlm}K^i_{jkh} + \lambda_l\, K^i_{jkh|m|n} = K^i_{jkh|m|l|n}$$
$$+ C^i_{js|m|l|n}\, H^s_{kh} + C^i_{js|m|l}\, H^s_{kh|n}$$
$$+ C^i_{js|m|n}H^s_{kh|l} + C^i_{js|m}H^s_{kh|l|n}$$
$$+ C^i_{js|l|n}H^s_{kh|m} + C^i_{js|l}H^s_{kh|m|n}$$
$$+ C^i_{js|n}H^s_{kh|m|l} - \lambda_l(C^i_{js|m|n}H^s_{kh}$$
$$+ C^i_{js|m}H^s_{kh|n} + C^i_{js|n}H^s_{kh|m}) \quad .$$

This shows that

(3.14) $\quad K^i_{jkh|m|l|n} = b_{nlm}K^i_{jkh} + K^i_{jkh|m|l|n}$

if and only if

(3.15) $$(C^i_{js|m|l|n} - \lambda_l C^i_{js|m|n})\, H^s_{kh}$$
$$+ (C^i_{js|m|l} - \lambda_l C^i_{js|m})\, H^s_{kh|n}$$
$$+ (C^i_{js|l|n} - \lambda_l C^i_{js|n})\, H^s_{kh|m}$$
$$+ C^i_{js|m|n}\, H^s_{kh|l} + C^i_{js|m}\, H^s_{kh|l|n}$$
$$+C^i_{js|l}H^s_{kh|m|n} + C^i_{js|n}H^s_{kh|m|l} = 0\, .$$

Thus, we may conclude the above discussion as follows:

Theorem 3.3. *In R^h-GTR-F_n , Cartan's fourth curvature tensor K^i_{jkh} is h-GTR if and only if (3.15) holds good.*

Similarly, we can prove that

(3.16) $\quad K^i_{jkh|m|l|n} = \lambda_l\, K^i_{jkh|m|n}$

if and only if

(3.17) $$(C^i_{js|m|l|n} - \lambda_l C^i_{js|m|n})\, H^s_{kh}$$
$$+(C^i_{js|m|l} - \lambda_l C^i_{js|m})\, H^s_{kh|n}$$
$$+(C^i_{js|l|n} - \lambda_l C^i_{js|n})\, H^s_{kh|m}$$
$$+ C^i_{js|m|n}\, H^s_{kh|l} + C^i_{js|m}\, H^s_{kh|l|n}$$
$$+C^i_{js|l}H^s_{kh|m|n} + C^i_{js|n}H^s_{kh|m|l} = 0\ .$$

Thus, we conclude

Theorem 3.4. *In R^h- $SGTR$- F_n , Cartan's fourth curvature tensor K^i_{jkh} is h-$SGTR$ if and only if (3.17) holds good.*

V. Certain Identities

Bianchi identities for the tensor R^i_{jkh} are given by[31]

(4.1) $$R^i_{jkh|m} + R^i_{jmk|h} + R^i_{jhm|k}$$
$$+y^s(R^r_{shm}P^i_{jkr} + R^r_{skh}P^i_{jmr} + R^r_{smk}P^i_{jhr}) = 0,$$

where

(4.2) $\quad P^i_{jkh} = \dot\partial_h \Gamma^{*i}_{jk} - C^i_{jh|k} + C^i_{jr}C^r_{kh|s}\, y^s.$

Taking the h-covariant derivative for (4.1) with respect to x^l and using (1.3b), we get

(4.3) $$R^i_{jkh|m|l} + R^i_{jmk|h|l} + R^i_{jhm|k|l} +$$
$$y^s(R^r_{shm}P^i_{jkr} + R^r_{skh}P^i_{jmr} + R^r_{smk}P^i_{jhr})_{|l} = 0\, .$$

Again, taking the h-covariant derivative for (4.3) with respect to x^n, using (2.1), (2.2) and (1.14b), we get

(4.4) $$a_{nlm}R^i_{jkh} + a_{nlh}R^i_{jmk} + a_{nlk}R^i_{jhm}$$
$$+\lambda_l(R^i_{jkh|m|n} + R^i_{jmk|h|n})$$
$$+(H^r_{hm}P^i_{jkr} + H^r_{kh}P^i_{jmr} + H^r_{mk}P^i_{jhr})_{|l|n} = 0$$

and

(4.5) $$\lambda_l\big(R^i_{jkh|m|n} + R^i_{jmk|h|n} + R^i_{jhm|k|n}\big)$$
$$+(H^r_{hm}P^i_{jkr} + H^r_{kh}P^i_{jmr} + H^r_{mk}P^i_{jhr})_{|l|n} = 0\, .$$

Transvecting (4.4) and (4.5) by y^j , using (1.3b), (1.14b) and (1.16), we get

(4.6) $$a_{nlm}H^i_{kh} + a_{nlh}H^i_{mk} + a_{nlk}H^i_{hm}$$
$$+\lambda_l(H^i_{kh|m|n} + H^i_{mk|h|n} + H^i_{hm|h|n})$$
$$+ (H^r_{hm}P^i_{kr} + H^r_{kh}P^i_{mr} + H^r_{mk}P^i_{hr})_{|l|n} = 0$$

and

(4.7) $$\lambda_l\big(H^i_{kh|m|n} + H^i_{mk|h|n} + H^i_{hm|k|n}\big)$$
$$+(H^r_{hm}P^i_{kr} + H^r_{kh}P^i_{mr} + H^r_{mk}P^i_{hr})_{|l|n} = 0\, .$$

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Thus, we conclude

Theorem 4.1. *In R^h- GTR- F_n, the identity* (4.6) *holds.*

Theorem 4.2. *In R^h- $SGTR$- F_n, the identity* (4.7) *holds.*

Collecting terms in (4.6), we see that this may be written as

$$(4.8)\quad \left(a_{nlm}H^i_{kh} + \lambda_l H^i_{kh|m|n}\right) + \left(a_{nlh}H^i_{mk} + \lambda_l H^i_{mk|h|n}\right) + \left(a_{nlk}H^i_{hm} + \lambda_l H^i_{mk|h|n}\right) + (H^r_{hm}P^i_{kr} + H^r_{kh}P^i_{mr} + H^r_{mk}P^i_{hr})_{|l|n} = 0.$$

In view of the conditions (2.7) and (2.8), the identities (4.8) and (4.7), give same identity which can be written as

$$(4.9)\quad H^i_{kh|m|l|n} + H^i_{mk|h|l|n} + H^i_{hm|k|l|n} + (H^r_{hm}P^i_{kr} + H^r_{kh}P^i_{mr} + H^r_{mk}P^i_{hr})_{|l|n} = 0\ .$$

Theorem 4.3. *In R^h-GTR-F_n and R^h-$SGTR$-F_n , the identity (4.9) holds.*

If the R^h-GTR-F_n and R^h-$SGTR$-F_n are affinely connected spaces, then we get the same identity

$$(4.10)\ H^i_{kh|m|l|n} + H^i_{mk|h|l|n} + H^i_{hm|k|l|n} = 0$$

in both spaces.

Thus, we conclude

Theorem 4.4. *If the R^h-GTR-F_n and R^h-$SGTR$-F_n are affinely connected spaces, then we have the identity (4.10).*

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