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Common Fixed Point Theorems of Integral Type in Menger Probabilistic Metric Spaces

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Abstract **– In this paper, we prove common fixed point theorems of integral type in Menger probabilistic metric spaces satisfying Common limit in range property. Our results generalize several previously known results of Menger spaces as well as metric spaces.**

Keywords **– Menger Space, Common Limit in Range Property, Weakly Compatible Mappings, T-norm.**

I. Introduction

In 1942 Menger [19] initiated the study of probabilistic metric space (often abbreviated as PM space) and by now the theory of probabilistic metric spaces has already made a considerable progress in several directions (see [26]). The idea of Menger was to use distribution functions (instead of nonnegative real numbers) as values of a probabilistic metric. This probabilistic metric space can cover even those situations where in one can not exactly ascertain a distance between two points, but can only know the possibility of a possible value for the distance (between a pair of points).

The theory of fixed points in probabilistic metric spaces is a part of probabilistic analysis and continues to be an active area of mathematical research. Thus far, several authors studied fixed point and common fixed point theorems in PM spaces which include [5, 7, 8, 10, 15, 16, 17, 20, 21, 22, 24, 27, 25, 29, 30] besides many more. In 2002, Branciari [3] obtained a fixed point result for a mapping satisfying an integral analogue of Banach contraction principle. The authors of the papers [2, 4, 6, 21, 11, 23] proved a host of fixed point theorems involving relatively more general integral type contractive conditions. In an interesting note, Suzuki [28] showed that Meir-Keeler contractions of integral type are still Meir-Keeler contractions.

In 1986, Jungck [13] introduced the notion of compatible mappings and utilized the same to improve commutativity conditions in common fixed point theorems. This concept has been frequently employed to prove existence theorems on common fixed points. However, the study of common fixed points of non-compatible mappings was initiated by Pant [26]. Aamri and Moutawakil [1] and Liu et al. [31] respectively defined the property (E.A) and the common property (E.A) and proved interesting common fixed point theorems in metric spaces. Kubiaczyk and Sharma [14] adopted the property (E. A) in probabilistic metric spaces and used it to prove results on common fixed points. Although E. A property is generalization of the concept of noncompatible mappings, yet it requires either completeness of the whole space or any of the range spaces or continuity of mappings. But on the contrary, the new notion of the common limit range property recently given by Sintunavarat and Kumam [32] does not require such conditions. The importance of the common limit range property ensures that one does not require the closeness of range subspaces.

The aim of this paper is to prove some common fixed point theorems for mappings satisfying integral type contractive condition in Menger probabilistic metric space using common limit in range property.

II. Preliminaries

Definition 2.1. [7] A Mapping $F : R \to R^+$ is called distribution function if it is non-decreasing, left continuous with $\inf\{F(t) : t \in R\} = 0$ and $\sup\{F(t) : t \in R\} = 1$.

Let L be the set of all distribution functions whereas H be the set of specific distribution functions (also known as Heaviside function) defined by

$$H(x) = \begin{cases} 0, & \text{if } x \le 0 \\ 1, & \text{if } x > 0 \end{cases}$$

Definition 2.2. [19] Let X be a nonempty set. An ordered pair (X, F) is called a PM space if F is a mapping from X * X into L satisfying the following conditions:

(i) $F_{p,q}(x) = H(x)$ if and only if $p = q$,

(ii) $F_{p,q}(x) = F_{q,p}(x)$,

(iii) $F_{p,q}(x) = 1$ and $F_{q,r}(y) = 1$, then $F_{p,r}(x + y) = 1$,

for all p, q, r ∈ X and x, y ≥ 0.

Every metric space (X, d) can always be realized as a PM space by considering $F : X * X \to L$ defined by $F_{p,q}(x) = H(x\text{-}d(p, q))$ for p, q ∈ X. So PM spaces offer a wider framework (than that of the metric spaces) and are general enough to cover even wider statistical situations.

Definition 2.3. [7] A mapping $\Delta : [0,1] * [0,1] \to [0,1]$ is called a t-norm if

(i) $\Delta(a, 1) = a, \Delta(0, 0) = 0$,

(ii) $\Delta(a, b) = \Delta(b, a)$,

(iii) $\Delta(c, d) \ge \Delta(a, b)$ for $c \ge a, d \ge b$,

(iv) $\Delta(\Delta(a, b)c) = \Delta(a, \Delta(b, c))$ for all $a, b, c \in [0, 1]$.

Example 2.4. The following are the four basic t-norms:

(i) The minimum t-norm: TM (a, b) = min {a, b}.

(ii) The product t-norm: TP (a, b) = a, b.

(iii) The Lukasiewiez t –norm: TL(a, b) = max{a+b-1, 0}.

(iv) The weakest t-norm, the drastic product:

$$TD(a, b) = \begin{cases} \min\{a, b\}, & \text{if } \max\{a, b\} = 1 \\ 0, & \text{otherwise} \end{cases}$$

In respect of above mention t-norms, we have the following ordering TD < TL < TP < TM.

Definition 2.5. [19] A Menger probabilistic metric space (X, F, Δ) is a tripiet where (X, F) is a probabilistic metric space and Δ is a t-norm satisfying the following condition:

$$F_{p,r}(x + y) \ge \Delta(F_{p,q}(x)\, F_{q,r}(y)).$$

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Definition 2.6. [22] A sequence {pn} in a Menger space (X, F, Δ) is said to be convergent to a point p in X if $\in > 0$ and $\lambda > 0$, there is a integer M (∈, λ) such that $F_{pn,p} > 1 - \lambda$ for all n≥ M (∈, λ).

Lemma 2.7. [7, 9] Let (X, F, Δ) be a Menger space with a continuous t-norm Δ with {xn}, {yn} ⊂ X such that {xn}converges to x and {yn} Converges to y. If $F_{x,y}(.)$is continuous at the point t_0, then $\lim_{n\to\infty} F_{x_n,y_n}(t_0) = F_{x,y}(t_0)$.

Definition 2.8. Let (A, S) be a pair of maps from a Menger probabilistic metric space (X, F,Δ) into itself. Then the pair of maps (A, S) is said to be weakly commuting if $F_{ASx,SAx}(t) \geq F_{Ax,Sx}(t)$, for each x ∈ X,t > 0.

Definition 2.9. [27] A pair (A, S) of self mapping of a Menger probabilistic metric Space (X, F, Δ) is said to be compatible if $F_{ASp_n,SAp_n}(x) \to 1$ for all x > 0, whenever {pn} is a sequence in X such that {pn} is the sequence in X such that Apn, Spn → t, for some t in X as n→1. Clearly, a weakly commuting pair is compatible but every compatible pair need not be weakly commuting.

Definition 2.10. [18] A pair (A, S) of a self mapping of a Menger probabilistic metric space (X, F, Δ) is said to be non-compatible if and only if there exists at least one sequence {xn} in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t \in X$ For some t ∈ X

Implies that $\lim_{n\to\infty} F_{ASx_n,SAx_n}(t_0)$ (for some t0>0) is either less than 1 or non-existent.

Definition 2.11. [14] A pair (A, S) of a self mapping of a Menger probabilistic metric space (X, F, Δ) is said to satisfy the property (E. A) if there exists a sequence {xn} in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t \in X$$

Clearly, a pair of compatible mappings as well as noncompatible mappings satisfies the property (E.A).

Inspired by Liu et al. [31], Imdad et al. [21] defined the following:

Definition 2.12. Two pair (A, S) and (B, T) of self mappings of a Menger probabilistic metric space (X, F, Δ) are said to satisfy the common property (E. A), if there exist two sequences {xn}, {yn} in X and some t in X such that

$$\lim_{n\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = \to \lim_{n\to\infty} By_n = t.$$

Definition 2.13. [32] A pair (A, S) of self mappings of a Menger probabilistic metric space (X, F, Δ) is said to satisfy common limit in range of S property (in short CLRs property) if there exists a sequence {xn}, such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = Su$ for Some u ∈ X.

Definition 2.14. [12] A pair (A, S) of self mappings of a nonempty set X is said to be weakly compatible if the pair commutes on the set of their coincidence points i.e. Ap = Sp (for some p ∈ X) implies ASp = SAp.

III. Main Result

The following lemma is useful for the proof of succeeding theorems

Lemma 3.1. Let (X, F, Δ) be a Menger space. If there exists some k ∈ (0, 1) such that for all p, q ∈ X and all x > 0,

$$\int_0^{F_{p,q}(kx)} \phi(t)dt \geq \int_0^{F_{p,q}(x)} \phi(t)dt,$$

where ϕ : [0, ∞)⟶ [0, ∞) is a summable non-negative Lebesgue integrable function such that

$\int_c^1 \phi(t)dt \geq 0$ for each t ∈ [0, 1], then p = q.

Remark 3.2. By setting $\phi(t) = 1$ (for each t ≥ 0) in Lemma 3.1, we have

$$\int_0^{F_{p,q}(kx)} \phi(t)dt = F_{p,q}(kx) \geq F_{p,q}(x) = \int_0^{F_{p,q}(x)} \phi(t)dt,$$

which shows that Lemma 3.1 is a generalization of Lemma 2 (contained in [8]).

Theorem 3.3. Let A, B, S and T be four self mappings of a Menger space (X, F, Δ) Satisfying the following conditions:

(3.1) (A, S) satisfies CLR_S property or (B, T) satisfies CLR_T property,

(3.2) A(X) ⊆ T(X) (or B(X) ⊆ S(X)),

(3.3) (A, S) and (B, T) are weak compatible,

(3.4) one of the range of the mapping A, B, S, or T is a closed subspace of X,

(3.5) for any p, q ∈ X and for all x > 0,

$$\int_0^{F_{Ap,Bq}(kx)} \phi(t)dt \geq \int_0^{\min\left\{F_{Sp,Tq}(x), F_{Sp,Ap}(x), F_{Tq,Bq}(x), F_{Sp,Bq}(x), F_{Tq,Ap}(x), \frac{F_{Sp,Tq}(x)F_{Tp,Bq}(x)}{F_{Sp,Bq}(x)}, \frac{F_{Sp,Tq}(x)F_{Sp,Ap}(x)}{F_{Tq,Ap}(x)}\right\}} \phi(t)dt,$$

where ϕ : [0, ∞)⟶[0, ∞) is a non-negative summable Lebesgue integrable function such that

$\int_0^1 \phi(t)dt > 0$ for each t ∈ [0,1), and 0 < k < 1.

Then A, B, S and T have a unique common fixed point in X.

Proof. Suppose that the pair (A, S) satisfies the CLR_S property, then exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, \text{where } z \in S(X),$$

then there exists a point u in X such that z = Su.

Since A(X) ⊆ T(X), then there exists a sequence $\{y_n\}$ in X such that $Ax_n = Ty_n$. Hence $\lim_{n\to\infty} Ty_n = z$. Thus, we have

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = z.$$

Now we are required to show that $\lim_{n\to\infty} By_n = z$. Putting p = x_n and q = y_n in (3.5), we get

$$\int_0^{F_{Ax_n,By_n}(kx)} \phi(t)dt \geq \int_0^{\min\left\{F_{Sx_n,Ty_n}(x), F_{Sx_n,Ax_n}(x), F_{Ty_n,By_n}(x), F_{Sx_n,By_n}(x), F_{Ty_n,Ax_n}(x), \frac{F_{Sx_n,Ty_n}(x)F_{Ty_n,By_n}(x)}{F_{Sx_n,By_n}(x)}, \frac{F_{Sx_n,Ty_n}(x)F_{Sx_n,Ax_n}(x)}{F_{Ty_n,Ax_n}(x)}\right\}} \phi(t)dt$$

Let $\lim_{n\to\infty} By_n = l \neq z$, then taking limit as n ⟶∞, we have

$$\int_0^{F_{z,l}(kx)} \phi(t)dt \geq \int_0^{\min\left\{F_{z,z}(x), F_{z,z}(x), F_{z,l}(x), F_{z,l}(x), F_{z,z}(x), \frac{F_{z,z}(x)F_{z,l}(x)}{F_{z,l}(x)}, \frac{F_{z,z,}(x)F_{z,z}(x)}{F_{z,z}(x)}\right\}} \phi(t)dt,$$

or

$$\int_0^{F_{z,l}(kx)} \phi(t)dt \ge \int_0^{F_{z,l}(x)} \phi(t)dt,$$

by Lemma 3.1, we have z = l, thus $\lim_{n\to\infty} By_n = z$.

Hence
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} By_n = z = Su$ for some u ∈ X.

Now we claim that Au = Su. For this purpose putting p = u and q = y_n in (3.5), we have

$$\int_0^{F_{Au,By_n}(kx)} \phi(t)dt \ge \int_0^{\min\{F_{Su,Ty_n}(x),F_{Su,Au}(x),F_{Ty_n,By_n}(x),F_{Su,By_n}(x),F_{Ty_n,Au}(x),\frac{F_{Su,Ty_n}(x)F_{Ty_n,By_n}(x)}{F_{Su,By_n}(x)},\frac{F_{Su,Ty_n}(x)F_{Su,Au}(x)}{F_{Ty_n,Au}(x)}\}} \phi(t)dt,$$

taking limit as n ⟶ ∞, we get

$$\int_0^{F_{Au,Su}(kx)} \phi(t)dt \ge \int_0^{\min\{F_{Su,Su}(x),F_{Su,Au}(x),F_{Su,Su}(x),F_{Su,Su}(x),F_{Su,Au}(x),\frac{F_{Su,Su}(x)F_{Su,Su}(x)}{F_{Su,Su}(x)},\frac{F_{Su,Su}(x)F_{Su,Au}(x)}{F_{Su,Au}(x)}\}} \phi(t)dt \text{ ,or}$$

$$\int_0^{F_{Au,su}(kx)} \phi(t)dt \ge \int_0^{F_{Au,su}(x)} \phi(t)dt.$$

By Lemma 3.1, we have
Au = Su, hence Au = Su = z.

Since A(X) ⊆ T(X), there exists a point v in X, Such that Au = Tv = z.

Next, we claim that Tv = Bv, putting p = u and q = v in (3.5), we have

$$\int_0^{F_{Au,Bv}(kx)} \phi(t)dt \ge \int_0^{\min\{F_{Su,Tv}(x),F_{Su,Au}(x),F_{Tv,Bv}(x),F_{Su,Bv}(x),F_{Tv,Au}(x),\frac{F_{Su,Tv}(x)F_{Tv,Bv}(x)}{F_{Su,Bv}(x)},\frac{F_{Su,Tv}(x)F_{Su,Au}(x)}{F_{Tv,Au}(x)}\}} \phi(t)dt$$

or

$$\int_0^{F_{Au,Bv}(kx)} \phi(t)dt \ge \int_0^{F_{Au,Bv}(x)} \phi(t)dt$$

By Lemma 3.1, we have Au = Bv. Hence Au=Tv=Bv=z. Thus we have Au = Su =Tv = Bv =z.

Since the pairs (A, S) and (B, T) are weakly compatible also Au = Su and Bv = Tv, we have Az=ASu=SAu=Sz and Bz= BTv= TBv = Tz.

Now, we prove that z is common fixed point of A, B, S and T. Putting p =z and q=v in (3.5), we have

$$\int_0^{F_{Az,Bv}(kx)} \phi(t)dt \ge \int_0^{\min\{F_{Sz,Tv}(x),F_{Sz,Au}(x),F_{Tv,Bv}(x),F_{Sz,Bv}(x),F_{Tv,Az}(x),\frac{F_{Sz,Tv}(x)F_{Tv,Bv}(x)}{F_{Sz,Bv}(x)},\frac{F_{Su,Tv}(x)F_{Sz,Az}(x)}{F_{Tv,Az}(x)}\}} \phi(t)dt$$

or

$$\int_0^{F_{Az,z}(kx)} \phi(t)dt \ge \int_0^{\min\{F_{Az,z}(x),F_{Az,Az}(x),F_{z,z}(x),F_{Az,z}(x),F_{z,Az}(x),\frac{F_{Az,z}(x)F_{z,z}(x)}{F_{z,z}(x)},\frac{F_{Az,z}(x)F_{Az,Az}(x)}{F_{z,Az}(x)}\}} \phi(t)dt$$

or

$$\int_0^{F_{Az,z}(kx)} \phi(t)dt \ge \int_0^{F_{Az,z}(kx)} \phi(t)dt$$

By Lemma 3.1, we have Az = z, hence Az = Sz = z, which shows that z is common fixed point of mappings A and S.

Also the pair (B, T) is weakly compatible and Bv = Tv, hence
Bz = BTv =TBv=Tz

Now we show that z is common fixed point of mappings B and T. In order to accomplish this, using (3.5) with p = u and q = z, we have

$$\int_0^{F_{Au,Bz}(kx)} \phi(t)dt \ge \int_0^{\min\{F_{Su,Tz}(x),F_{Su,Au}(x),F_{Tz,Bz}(x),F_{Su,Bz}(x),F_{Tz,Au}(x),\frac{F_{Su,Tz}(x)F_{Tz,Bz}(x)}{F_{Su,Bz}(x)},\frac{F_{Su,Tz}(x)F_{Su,Au}(x)}{F_{Tz,Au}(x)}\}} \phi(t)dt$$

or

$$\int_0^{F_{z,Bz}(kx)} \phi(t)dt \ge \int_0^{\min\{F_{z,Tz}(x),F_{z,z}(x),F_{z,z}(x),F_{Bz,Bz}(x),F_{Bz,z}(x),\frac{F_{z,Bz}(x)F_{Bz,Bz}(x)}{F_{z,Bz}(x)},\frac{F_{z,Bz}(x)F_{z,z}(x)}{F_{Bz,z}(x)}\}} \phi(t)dt$$

or

$$\int_0^{F_{z,Bz}(kx)} \phi(t)dt \ge \int_0^{F_{z,Bz}(kx)} \phi(t)dt$$

Using Lemma 3.1, we have Bz = z, hence Bz = Tz = z, which shows that z is common fixed point of mappings Band T. Hence z is a common fixed point both the mappings A, B, S and T.

Corollary 3.6. Let A and S be self mappings on a Menger space (X, F, Δ). Suppose that
(i) The pair (A, S) satisfies CLR_S property,
(ii) for all p, q ∈ X and for all x > 0,

$$\int_0^{F_{Ap,Aq}(kx)} \phi(t)dt \ge \int_0^{\min\{F_{Sp,Sq}(x),F_{Sp,Ap}(x),F_{Sq,Aq}(x),F_{Sp,Aq}(x),F_{Sq,Ap}(x)\}} \phi(t)dt,$$

where ϕ : [0, ∞)⟶[0, ∞) is a non-negative summable Lebesgue integrable function such that
$\int_0^1 \phi(t)dt > 0$ for each t ∈ [0,1), where 0 < k < 1.
(iii) S(X) is a closed subset of X.

Then A and S have a coincidence point. Moreover, if the pair (A, S) is weakly compatible, then A and S have a unique common fixed point.

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