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Statistical Properties of Cross-correlated Meteorological and OceanographicTime Series

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Abstract – **Most meteorological time series are long-range auto-correlated and cross-correlated. Due to the presence of highly periodic trends, these correlations are difficult to quantify and, very often, spurious correlations are reported. In order to eliminate the seasonal trends of meteorological data, we apply the detrending technique with varying order l of the polynomial and compute the DCCA cross-correlation coefficients. Furthermore, for the statistical properties of DCCA cross-correlation coefficients between those detrended meteorological time series, we performed the statistical test by introducing the cross-correlation coefficients computed from a thousand number of uncorrelated Gaussian time series as null hypothesis. In the 95% significance level, our analysis results show that the cross-correlation coefficients are very sensitive to the order of the polynomial. Also, the spurious correlations gradually decrease as the order of the polynomial increases. The utility of this new approach is illustrated from two pairs of selected meteorological time series, which are recorded at Cheol-Won and Seoul, and from a pair of oceanographic signals recorded at Hae-woon-dae near the south sea of Korean peninsula.**

Keywords – **DCCA Coefficient, DFA, Meteorological Time Series, Detrending Method.**

I. Introduction

Noisy signals in many real-world systems are recorded simultaneously and display long-range auto-correlations as well as cross-correlations. These correlations play a central role in a variety of disciplines such as physics, physiology, seismology, finance, meteorology, and so on [1-4]. To gain an insight into the correlation dynamics, the standard methods such as power spectral density and correlation analysis were developed and used widely. However, these standard methods assume stationarity and linearity in data and have limitations when applied to the real-world data, being commonly non-stationary and often exhibiting periodicity. Thus, detrending is essential to properly analyze the real-world time series because detrending prevents a time series being correlated if no correlation is present, and reveals a genuine correlation if correlations exist. In general, detrending is applied to data locally or globally. When applied locally, DFA (detrended fluctuation analysis) quantifies a single scaling parameter representing the long-range auto-correlation properties of a signal [5-7] while DCCA (detrended cross-correlation analysis) provides a single scaling parameter representing the long-range cross-correlation properties between two non-stationary signals [8,9]. The application of a global detrending does not guarantee that we can obtain a stationary signal because most complex signals are non-stationary. Also, the local linear detrending method is not suitable for obtaining a stationary signal from original noisy signals with highly nonlinear trends accompanied by periodic trends. In order to overcome the limitations mentioned above, different extensions of DFA have been proposed which locally subtract higher-order polynomials from the original signal together with the detrended moving average (DMA) [10,11] and multifractal DFA [12,13].

In the cross-correlation analysis, the DCCA method is very useful in detecting and quantifying power-law cross-correlations in many real-world non-stationary signals. However, the DCCA single scaling parameter is not suitable for quantifying the level of cross-correlations. Also, the well knownpearson correlation coefficient is not robust and often misleading if outliers are present, as in real-world data characterized by a high degree of non-stationarity. To overcome these limitations, a new cross-correlation coefficient based on DCCA was proposed by G. F. Zebende [14]. Now if we have two time series, $\{x_i\}$ and $\{x'_i\}$, we can compute the detrended variance function $F^2_{DFA}(n)$ and the detrended covariance function $F^2_{DCCA}(n)$ by applying DFA and DCCA methods to these data. This new coefficient is defined as the ratio between the detrended covariance function $F^2_{DCCA}(n)$ and the detrended variance function $F^2_{DFA}(n)$, i.e.,

$$\rho_{DCCA}(n) \equiv \frac{F^2_{DCCA}(n)}{\sqrt{F^2_{DFA\{x_i\}}(n) F^2_{DFA\{x'_i\}}(n)}} \quad (1)$$

Here, ρ_{DCCA} is a dimensionless coefficient that

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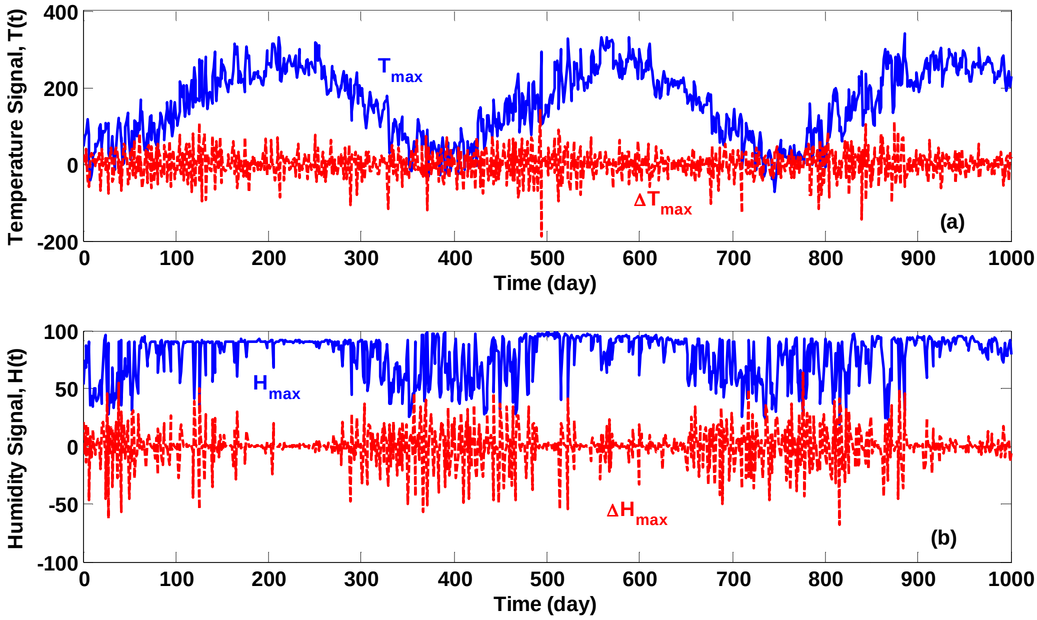

Fig. 1. The plots of temperature (a) and humidity (b)are presented. The maximum values are denoted by a blue solid line and the difference of successive maximum values is denoted by a red dotted line. A clear periodic trend is observed for the maximum values. The data set is recorded at Cheol-won.

ranges between $-1 \le \rho_{DCCA} \le 1$.

In our work, we extend the DCCA coefficient by applying the detrending technique with varying order of the polynomial. The detrending function with a higher order of the polynomial is very useful to eliminate periodic trends, which are present in most meteorological time series, without affecting the correlation property of the signal [15]. In the following, we perform the comparative study between the local linear detrending and the local nonlinear detrending. Also, we examine the utility of the local nonlinear detrending method by performing this study to both pairs of signals, namely the maximum values of air temperature and air relative humidity, and the successive difference of those maximum values. In addition, we include the oceanographic signals, which are differences of salinity and water temperature recorded every 30 minutes.

The organization of this paper is as follows. In the succeeding section, we give a brief description of data and discuss the methodology in section III. Our results are presented in section IV and, in the final section, we give a summary and concluding remarks.

II. Data

If The meteorological data under consideration are composed of a pair of air temperature and relative humidity over 2 stations in the South Korea. They are recorded hourly over 8 years and show a high degree of periodicity as in figure 1. In this work, we extract the daily maximum values of air temperature and relative humidity from the hourly recorded data covering the period from 01 January 2003 to 31 December 2010, and analyze the two datasets; one is the daily maximum values of air temperature $\{x_i\}$ and relative humidity $\{x'_i\}$, and the other is the successive differences of those maximum values given as $\{x_{i+1} - x_i\}$ and $\{x'_{i+1} - x'_i\}$, respectively. As for the oceanographic signals, we computed the difference of water temperature and salinity recorded every 30 minutes and applied the higher-order detrending method to them.

III. Methodology

In time series analysis there are some well-known methods to use. Among them, the most frequently cited method is the DFA to provide a relationship between $F_{DFA}(n)$ (root mean square fluctuation) and the scale n, exhibiting a power-law $F_{DFA}(n) \propto n^{\alpha}$ where α is the long-range auto-correlation scaling exponent. As a generalization of the DFA method, B. Podobnik et al. proposed the DCCA method to investigate power-law cross-correlations between different simultaneously recorded time series in the presence of non-stationarity [8]. Thus, for two time series of equal length N, we compute two integrated signals $y_k \equiv \sum_{i=1}^{k} x_i$ and $y'_k \equiv \sum_{i=1}^{k} x'_i$, where $k = 1, \ldots, N$. And then, we divide the entire time series into $N-n$ overlapping segments, each containing

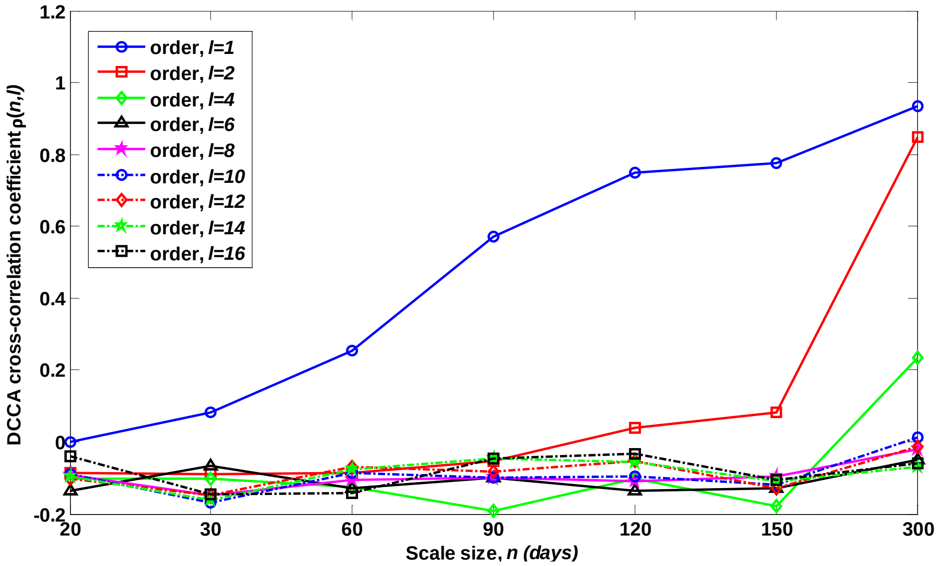

Fig. 2. The modified DCCA cross-correlation coefficients are presented for the maximum signals of air temperature and relative humidity. For the linear detrending (order $l=1$), $\rho_{DCCA}(n,1)$ exhibits a strongly positive cross-correlation but, as the fitting polynomial order increases, the DCCA coefficient decreases. This fact implies that a strong correlation is due to the periodic trend present in both temperature and humidity signals. Above the order $l=6$, there is no clear dependence on the fitting polynomial order. The data set is recorded at Cheol-won.

n values. For both time series, in each segment that starts at i and ends at $i+n-1$, we define the local trends, $\tilde{y}_{k,i}$ and $\tilde{y}'_{k,i}$, to be the ordinate of a linear least squares fit. Lastly, we define the detrended walk as the difference between the original walk and the local trend, that is, $y_k-\tilde{y}_{k,i}$ and $y'_k-\tilde{y}'_{k,i}$. Next, we calculate the covariance of the residuals in each segment defined as

$$f^2_{DCCA}(n,i)\equiv\frac{1}{n}\sum_{k=i}^{n+i-1}(y_k-\tilde{y}_{k,i})(y'_k-\tilde{y}'_{k,i}) \qquad (2)$$

where i denotes the segment index. Finally, we can obtain the detrended covariance function by summing over all overlapping $N-n$ segments of size n:

$$F^2_{DCCA}(n)\equiv\frac{1}{(N-n)}\sum_{i=1}^{N-n}f^2_{DCCA}(n,i) \qquad (3)$$

For a singular time series, $(y_k=y'_k)$, the detrended covariance $F^2_{DCCA}(n)$ reduces to the detrended variance $F^2_{DFA}(n)$ used in the DFA method. These variance and covariance functions are used to define the DCCA cross-correlation coefficient expressed in Equation (1).

For this new cross-correlation coefficient, B. Podobnik et al. derived that the Cauchy inequality, $-1\le\rho_{DCCA}\le1$, holds for a standard variance-covariance approach, and then for a detrending approach [16]. Also, to test whether the cross-correlations are genuine (statistically significant) or not, the authors carried out the statistical test and calculated critical points $\rho_C(N,n)$ for the 95% confidence level defined such that the integral between $-\rho_C(N,n)$ and $\rho_C(N,n)$ is equal to 0.95. Thus, the range of ρ_{DCCA}, within which the cross-correlations can be considered statistically significant, is determined. In this work, we do the same statistical test and determine the new critical points $\rho_C(N,n,l)$, which are also dependent of the order l of the polynomial in addition to the length of a time series N and the segment size n.

Lastly, the DFA and the DCCA methods are not suitable for complex time series with highly nonlinear trends accompanied by periodic trends. D. Horvatic et al. proposed a DCCA method with varying order of the polynomial fit [15]. This higher order fitting approach was shown not to affect correlation properties even when the polynomial order values are very large. The authors tested the validity of their approach by applying the higher-order

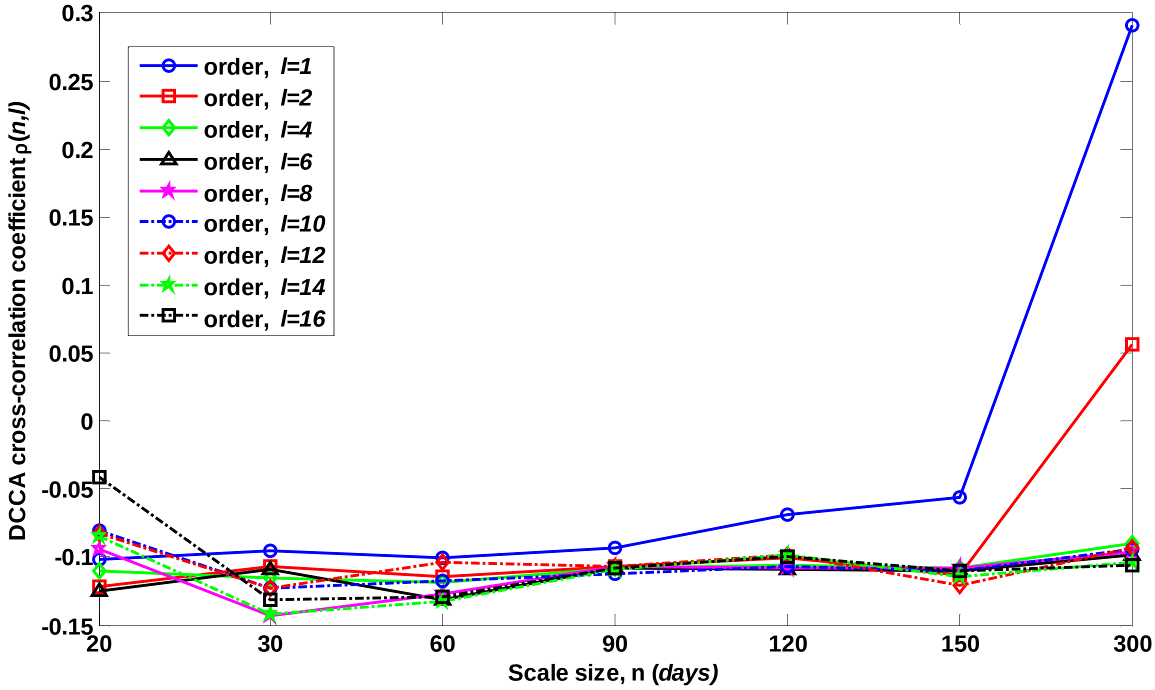

Fig. 3. The modified DCCA cross-correlation coefficients are presented for the successive differences of maximum air temperature and relative humidity. Since, the successive differencing eliminates the periodic trend present in the original signals, the dependence on the polynomial order is not clear. However, for the large scale size, a higher-order polynomial order is needed to well detrend the long-term periodic trend. The data set is recorded at Cheol-won.

polynomial fitting to two artificial time series generated from a periodic two-component fractionally autoregressive integrated moving average (ARFIMA) process [17,18],

$$z_i = \left[\sum_{n=1}^{\infty} a_n(\rho_1) z_{i-n}\right] + A_1 \sin\left(\frac{2\pi}{T_1} i\right) + \eta_i \quad (4)$$

$$z'_i = \left[\sum_{n=1}^{\infty} a_n(\rho_2) z'_{i-n}\right] + A_2 \sin\left(\frac{2\pi}{T_2} i\right) + \eta_i \quad (5)$$

Here, η_t is shared between z_i and z'_i in order to enable cross-correlations, $T_1(T_2)$ is the sinusoidal period, A_1 and A_2 are two sinusoidal amplitudes, and $a_n(\rho)$ is a statistical weight defined by $a_n(\rho) = \Gamma(n-\rho)/(\Gamma(-\rho)\Gamma(1+n))$, where Γ denotes the Gamma function and ρ is a parameter ranging from -0.5 to 0.5. According to the work of D. Horvatic et al. [15], the detrending method with varying order of the polynomial well eliminates the periodic trend and preserves the inherent cross-correlation between two signals. Importantly, the polynomial order l is increasing with the segment size n. In our work, we apply all possible orders of the polynomial to the segment with scale size n and examine the dependency of the DCCA cross-correlation coefficient $\rho(N,n,l)$ on the order l of the polynomial.

IV. Results

In the analysis, we investigate both the maximum values and the difference between successive maximum values by applying the varying order of the fitting polynomial. In order to examine the relationship between the scale size n of a segment and the polynomial order l, we set up the scale size set and the fitting order set as follows:

$$n = \{20, 30, 60, 90, 120, 150, 300\}$$
$$l = \{1, 2, 4, 6, 8, 10, 12, 14, 16\} \quad (6)$$

Here, all the components of the fitting order set l are applied to each component of the scale size set n. Now, we modify the equations (2) and (3) by replacing the linear local trend $\tilde{y}_{k,i}$ with the polynomial local trend $\tilde{y}_{k,i}(l)$. Then, the local detrended variance and covariance are given as follows:

$$f^2_{DCCA}(n,i;l) \equiv \frac{1}{n}\sum_{k=i}^{n+i-1}(y_k - \tilde{y}_{k,i}(l))(y'_k - \tilde{y}'_{k,i}(l)) \quad (7)$$

This local detrended covariance reduces to the detrended variance when two signals are same, $y_k = y'_k$.

$$F^2_{DCCA}(n,l) \equiv \frac{1}{(N-n)}\sum_{i=1}^{N-n} f^2_{DCCA}(n,i;l) \quad (8)$$

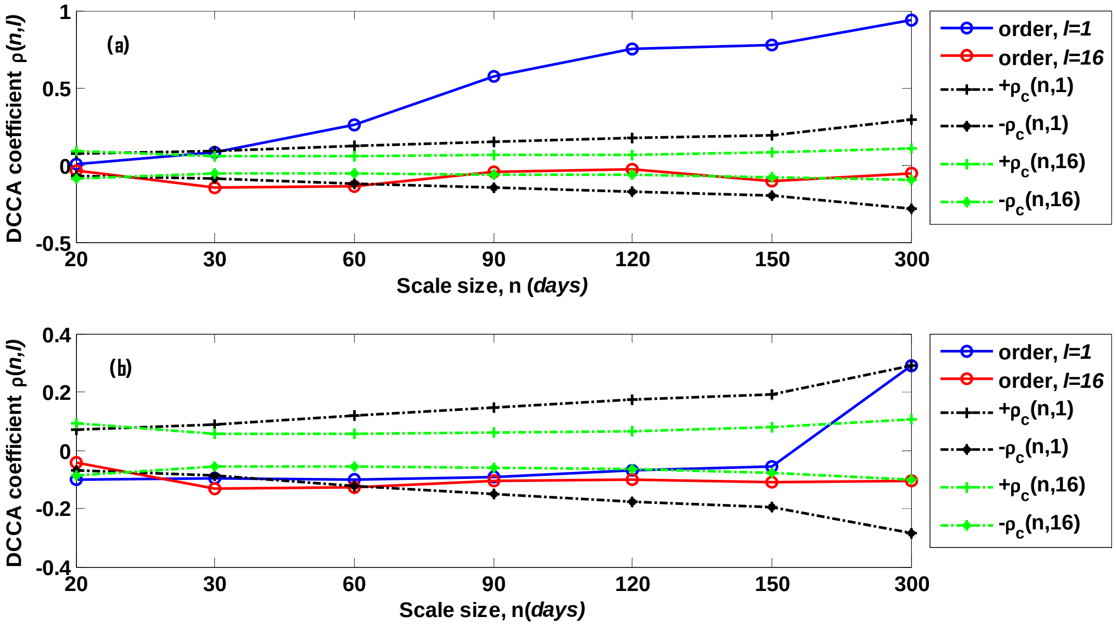

Fig. 4. The modified DCCA cross-correlation coefficients are presented with the critical points, for the successive differences of maximum air temperature and relative humidity. For maximum values (a), a strong positive cross-correlation seems to be only due to the periodic trend. At the higher order of polynomial fit, we find a negative or no cross-correlation over the scale range. However, for the difference signals (b), the cross-correlation under the linear least square fitting is statistically insignificant and, at the higher order of polynomial fit, the negative cross-correlation is statistically robust. The data set is recorded at Cheol-won.

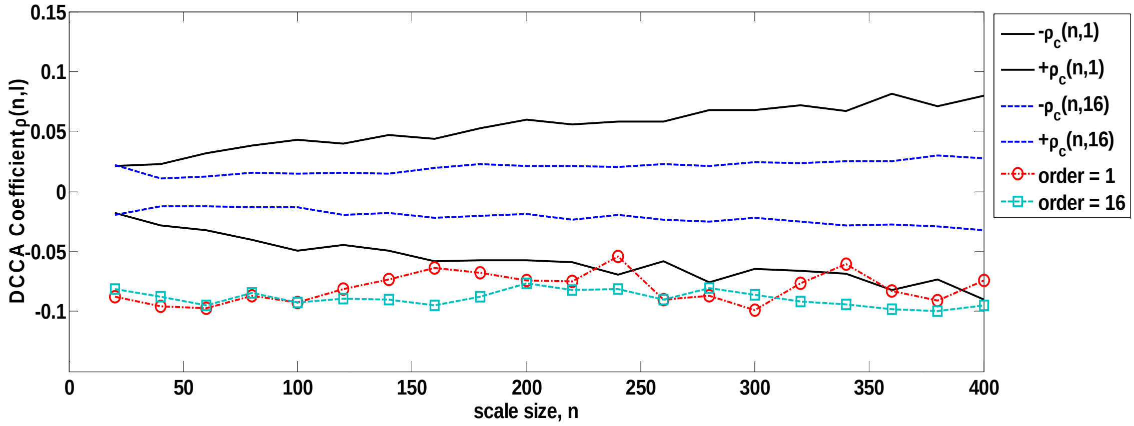

Fig. 5. The modified DCCA cross-correlation coefficients are presented with the critical points, for the successive differences salinity and water temperature. At the higher order of polynomial fit, we find a clear negative cross-correlation over the whole scale range. However, for the linear detrending fit, the cross-correlation is unclear in the long scale region.

By summing all local detrended covariance and variance, we obtain the detrended variance and covariance dependent on the fitting polynomial order l. Then, the modified DCCA cross-coefficient $\rho(N,n,l)$ is defined as

$$\rho_{DCCA}(N,n,l) \equiv \frac{F^2_{DCCA}(n,l)}{\sqrt{F^2_{DFA\{x_i\}}(n,l)\,F^2_{DFA\{x'_i\}}(n,l)}} \qquad (9)$$

Thus, we compute $\rho_{DCCA}(N,n,l)$ for the maximum signals and the difference signals. In fig. 2 and 3 we show the modified DCCA cross-correlation coefficients, which seems to be very sensitive to the fitting polynomial order.

The maximum value signals for air temperature and relative humidity have a strong periodic trend and may yield a spurious strong cross-correlation when the linear local detrending is applied. However, by applying a higher-order fitting polynomial to the original signal, we can obtain a genuine cross-correlation not contaminated by

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Table 1. The modified DCCA cross-correlation coefficient $\rho_C(N,n,l)$ are presented for the maximum values of air temperature and air relative humidity. The length of a signal $N=2922$ and the critical points are computed at 95% confidence level for a given couple of time series, each of which is Gaussian i.i.d. with zero mean and unit variance. The underlined numeric denotes being statistically significant under a given critical point.

Order L	Location	Segment size n						
		n=20	n=30	n=60	n=90	n=120	n=150	n=300
L=1	Seoul	0.104	0.185	0.149	0.444	0.486	0.580	0.830
	Cheol-won	0.000	0.083	0.254	0.571	0.751	0.777	0.936
$\rho_C(N,n,l=1)$	$-\rho_C$	-0.071	-0.086	-0.124	-0.150	-0.176	-0.195	-0.283
	$+\rho_C$	0.070	0.087	0.120	0.148	0.175	0.190	0.290
L=2	Seoul	0.132	0.095	0.085	0.054	-0.017	-0.023	0.715
	Cheol-won	-0.087	-0.090	-0.087	-0.052	0.041	0.082	0.848
$\rho_C(N,n,l=2)$	$-\rho_C$	-0.060	-0.069	-0.100	-0.133	-0.150	-0.163	-0.240
	$+\rho_C$	0.058	0.076	0.100	0.127	0.156	0.176	0.250
L=4	Seoul	0.084	0.103	0.082	0.027	-0.027	-0.013	0.159
	Cheol-won	-0.104	-0.103	-0.125	-0.193	-0.103	-0.178	0.236
$\rho_C(N,n,l=4)$	$-\rho_C$	-0.050	-0.062	-0.086	-0.101	-0.124	-0.135	-0.200
	$+\rho_C$	0.053	0.057	0.080	0.099	0.112	0.143	0.197
L=16	Seoul	0.070	0.003	0.085	0.160	0.177	0.133	0.055
	Cheol-won	-0.040	-0.147	-0.143	-0.047	-0.034	-0.106	-0.059
$\rho_C(N,n,l=16)$	$-\rho_C$	-0.090	-0.055	-0.055	-0.063	-0.065	-0.078	-0.101
	$+\rho_C$	0.091	0.058	0.057	0.060	0.066	0.080	0.103

Table 2. The modified DCCA cross-correlation coefficient $\rho_C(N,n,l)$ are presented for the successive difference signals of maximum daily temperature and relative humidity. The length of a signal $N=2922$ and the critical points are computed at 95% confidence level for a given couple of time series, each of which is Gaussian i.i.d. with zero mean and unit variance. The underlined numeric denotes being statistically significant under a given critical point.

Order L	Location	Segment size n						
		n=20	n=30	n=60	n=90	n=120	n=150	n=300
L=1	Seoul	0.093	0.088	0.087	0.090	0.080	0.066	0.242
	Cheol-won	-0.122	-0.095	-0.100	-0.093	-0.070	-0.056	0.291
$\rho_C(N,n,l=1)$	$-\rho_C$	-0.071	-0.086	-0.124	-0.150	-0.176	-0.195	-0.283
	$+\rho_C$	0.070	0.087	0.120	0.148	0.175	0.190	0.290
L=2	Seoul	0.075	0.095	0.085	0.081	0.080	0.072	0.107
	Cheol-won	-0.122	-0.107	-0.114	-0.107	-0.100	-0.114	0.056
$\rho_C(N,n,l=2)$	$-\rho_C$	-0.060	-0.069	-0.100	-0.133	-0.150	-0.163	-0.240
	$+\rho_C$	0.058	0.076	0.100	0.127	0.156	0.176	0.250
L=4	Seoul	0.070	0.073	0.089	0.085	0.078	0.077	0.092
	Cheol-won	-0.110	-0.115	-0.119	-0.108	-0.106	-0.108	-0.090
$\rho_C(N,n,l=4)$	$-\rho_C$	-0.050	-0.062	-0.086	-0.101	-0.124	-0.135	-0.200
	$+\rho_C$	0.053	0.057	0.080	0.099	0.112	0.143	0.197
L=16	Seoul	0.099	0.011	0.057	0.103	0.104	0.083	0.096
	Cheol-won	-0.041	-0.131	-0.130	-0.108	-0.100	-0.110	-0.106
$\rho_C(N,n,l=16)$	$-\rho_C$	-0.090	-0.055	-0.055	-0.063	-0.065	-0.078	-0.101
	$+\rho_C$	0.091	0.058	0.057	0.060	0.066	0.080	0.103

a strong periodic trend as shown in figure 2. And, in the region of small to middle scale size *n*, no sensitive dependence on the polynomial order is observed. For the successive difference signals of maximum temperature and relative humidity as shown in figure 3, the cross-correlation $\rho(N,n,l)$ looks different from figure 2. For this case, the periodic trend is weaker than in the original signals because the successive differencing eliminates a periodic trend. However, a long-period trend still survives and we need a higher-order polynomial fit. Also, for the difference signals, there is a negative cross-correlation in the whole range of scale size *n*. However, we need to determine whether this negative cross correlation is genuine or not, so we perform a statistical test.

As a practical problem in use of $\rho(N,n,l)$, for finite time series, due to the size effect, $\rho(N,n,l)$ is not zero but presumably some small nonzero value even if cross-correlations are not present. Thus, to test whether the cross-correlations are genuine or not, we use $\rho(N,n,l)$ as the first statistical test. First, we determine the null hypothesis. We begin by assuming that, under the null hypothesis, the time series are independent and identically distributed random variables (i.i.d) and calculate the range of $\rho(N,n,l)$ that can be obtained under the assumption that the time series are i.i.d. To this end, we obtain the PDF corresponding to the constraints $\{N,n,l\}$ by generating 1000 i.i.d. time series pairs taken from a Gaussian distribution, where for each time series pair we calculate the detrended variance $F^2_{DFA}(n,l)$ and the detrendedcovariance $F^2_{DCCA}(n,l)$, and then test it using Eq. (9). As expected, $P(\rho_{DCCA}(N,n,l))$ is almost symmetric and, with increasing n, the PDF converges to a Gaussian due to the central limit theorem. Next, for each PDF $P(\rho_{DCCA}(N,n,l))$ defined by $\{N,n,l\}$, we calculate the critical point $\rho_C(N,n,l)$ for the 95% confidence level. The critical values are given in Table 1 and 2. Figure 4 shows the DCCA cross-correlation $\rho_{DCCA}(N,n,l)$ of two pairs of signals, namely the maximum values and the successive difference time series, with respect to the critical points. Also, in figure 5, we show the DCCA cross-correlation coefficients for the oceanographic signals. In this analysis, as the statistical test, we apply the surrogate method by shuffling the series. At the higher order detrending, the cross-correlation pattern becomes clearer compared to the lower one.

This finding shows that the higher order of polynomial fitting and the statistical test are essential to investigate the cross-correlations between highly non-stationary time series with a periodic trend. Especially, the meteorological signals have a strong periodic trend and they are not well detrended simply by differencing the successive data values. Our all analysis results on 2 locations are presented in Table 1 and 2 with the critical points.

As shown in Table 1, the modified DCCA cross-correlation coefficient is very sensitive to the polynomial order and a highly periodic trend is well eliminated by increasing the order of a polynomial fit. Also, we can determine if a genuine cross-correlation is present between a pair of meteorological signals by performing a statistical test. We present the analysis results for the successive difference signals in Table 2.

The successive difference gives very similar results with those in the maximum values. For Seoul, the cross-correlation between air temperature and air humidity seems to be positive while, for Cheol-won, a negative cross-correlation is present. Also, for a large scale size *n*, the linear local detrending is not suitable for detecting a genuine cross-correlation due to a periodic trend. Since our data are recorded daily and have a yearly seasonality, the validity of our modified DCCA cross-correlation coefficient is conspicuous at the large scale size $n = 300$.

V. Conclusion

The higher-order polynomial fits are very useful in detecting the cross-correlation between highly non-stationary time series with a periodic trend, especially meteorological data such as air temperature and relative humidity. In this work, we presented a modified version of the DCCA cross-correlation coefficient based on the well-known DFA and DCCA methods. In order to prove the validity of our new approach, we applied the modified DCCA coefficient to meteorological data with yearly seasonality as a strong periodic trend. By increasing the order of polynomial fits, we found that the spurious cross-correlations appear via a linear local detrending at large scale size [19,20]. Also, we determined whether a DCCA cross-correlation coefficient value is statistically significant or not by performing a statistical test. The new cross-correlation analysis approach presented in this work will be very useful in investigating the cross-correlation between complex time series contaminated with a variety of periodic trends.

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