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T-Rough Fuzzy Subalgebras of Boolean Algebras

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***Abstract* – The rough set theory was introduced by Pawlak in 1982. It was proposed for presentation equivalence relations. But the concept of fuzzy set was introduced by Zadeh in 1965. In this paper, the concepts of the rough sets, T-rough sets, Boolean algebras, T-rough fuzzy sets, fuzzy subalgebras and set-valued homomorphism of Boolean algebras will be given. A necessary and sufficient condition for a fuzzy subset of Boolean algebra to be a T-rough fuzzy subalgebra is stated, and images and inverse images of fuzzy subalgebra under Boolean algebra homomorphism are studied. The purpose of this paper is to introduce and discuss the concept of T-rough fuzzy subalgebras of Boolean Algebras that those have been proved in several papers.**

***Keywords* – T-rough set, Boolean Algebra, T-rough Fuzzy set, Fuzzy Subalgebra.**

I. Introduction

The concept of rough set was originally proposed by Pawlak in [13, 14], as a formal tool for modeling and processing incomplete information in information systems. Rough set theory is a mathematical framework for dealing with uncertainty and to some extent overlapping fuzzy set theory. The rough set theory approach is based on indiscernibility relations and approximations. The theory of rough set is an extension of set theory. Dubois and Prade in [5], began to investigate the problem of fuzzyification of a rough set. Rough fuzzy sets are a notion introduced as a further extension of the idea of rough sets. A fuzzy set is a class of objects with a continuum of grades of membership such a set is characterized by a membership(characteristic)function which as signs to each object a grade of membership ranging between zero and one.

The concept of fuzzy sets was initiated by Zadeh [18]. In 1971, Rosenfeld [17], introduced fuzzy sets in the realm of group theory and formulated the concept of a fuzzy subgroup of a group. Since then many researchers are engaged in extending the concepts of abstract algebra to the broader framework of the fuzzy setting. In 1982, Liu [9] defined and studied fuzzy subrings and fuzzy ideals of a ring. Reader in [1, 6, 10], will get some definitions and basic results about fuzzy algebras that their properties were carefully studied to a certain extent. Fuzzy set theory has been shown to be a useful tool to describe situation in which the data are imprecise or vague. Fuzzy sets handle such situations by attributing a degree to which a certain object belongs to a set. In fuzzy sets theory, there is no means to incorporate the hesitation or uncertainty in the membership degrees.

In this paper, the concepts of the rough sets, T-rough sets, Boolean algebras, T-rough fuzzy sets, T-rough fuzzy subalgebras and set-valued homomorphism of Boolean algebras will be given. A necessary and sufficient condition for a fuzzy subset of Boolean algebra to be a T-rough fuzzy subalgebra is stated, and images and inverse images of fuzzy subalgebra under Boolean algebra homomorphism are studied. The purpose of this paper is to introduce and discuss the concept of T-rough fuzzy subalgebras of Boolean Algebras that those have been proved in [15, 16].

II. Preliminaries

The following definitions and preliminaries are required in the sequel of our work and hence presented in brief. Some of them were in [13, 14]. Suppose that U is a non-empty set. A partition or classification of U is a family θ of non-empty subsets of U such that each element of U is contained in exactly one element of θ. Recall that an equivalence relation θ on a set U is a reflexive, symmetric, and transitive binary relation on U. Eech partition θ induces an equivalence relation θ on U by setting.

$x\theta y \Leftrightarrow x$ and y are in the same class of θ

Conversely, each equivalence relation θ on U induces a partition θ of U whose classes have the form

$$[x]_\theta = \{y \in U \,|\, x\theta y\}.$$

Definition 2.1

A pair (U,θ) where $U \neq \phi$ and θ is an equivalence relation on U is called an approximation space.

Definition 2.2

For an approximation space (U,θ) by a rough approximation in (U,θ) we mean a mapping $Apr : P(U) \to P(U) \times P(U)$ defined by for every $X \in P(U)$, $Apr(X) = (\underline{Apr}(X), \overline{Apr}(X))$, where

$$\underline{Apr}(X) = \left\{x \in X \,\middle|\, [x]_\theta \subseteq X\right\},$$
$$\overline{Apr}(X) = \left\{x \in X \,\middle|\, [x]_\theta \cap \right\} \tag{1}$$

$\underline{Apr}(X)$ is called a lower rough approximation of X in (U,θ) whereas $\overline{Apr}(X)$ is called a upper rough approximation of X in (U,θ).

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Definition 2.3

Given an approximation space (U,θ) a pair (A,B) in $P(U)\times P(U)$ is called a rough set in (U,θ) if $(A,B)=(\underline{Apr}(X),\overline{Apr}(X))$ for some $X\in P(U)$.

Definition 2.4

Let (U,θ) be an approximation space. A subset fuzzy is a mapping μ from U to $[0,1]$. If $x\in U$. We define

$$\underline{Apr}(\mu)(x)=\bigwedge_{a\in[x]_\theta}\mu(a)\quad,$$
$$\overline{Apr}(\mu)(x)=\bigvee_{a\in[x]_\theta}\mu(a) \tag{2}$$

They are called, respectively, the lower and the upper approximation of the fuzzy subset μ. $Apr(\mu)=(\underline{Apr}(\mu),\overline{Apr}(\mu))$ is called a rough fuzzy set respect to θ if $\underline{Apr}(\mu)\neq\overline{Apr}(\mu)$.

Definition 2.5. [17]

Let μ be a fuzzy subset in a set U. For $t\in[0,1]$, set $\mu_t=\{x\in U\mid\mu(x)\geq t\}$ is called a level subset of μ.

Definition 2.6. [12]

The algebra system $\{R;+,\cdot,0,1\}$ two binary algebra operations is called Boolean algebra if at least there are two different elements in R and the following axioms are correct:

(1) For any $a\in R$, $a+a=a$, $a.a=a$;

(2) For any $a,b\in R$, $a+b=b+a$, $a.b=b.a$;

(3) For any $a,b,c\in R$,
$(a+b)+c=a+(b+c)$, $(ab)c=a(bc)$;

(4) For any $a,b,c\in R$,
$a(b+c)=ab+ac$, $a+bc=(a+b)(a+c)$;

(5) There is elements $0,1\in \mathrm{R}$, for any $a\in R$, $a+0=a$, $a1=a$;

(6) For any $a\in R$, there is $\bar{a}\in R$ to satisfy $a+\bar{a}=1$, $a\bar{a}=0$.

Remark 2.7

Boolean algebra $\{R;+,\cdot,0,1\}$ is also noted by $\{R;+,\cdot,-,0,1\}$.

Definition 2.8. [12]

Let $\{R;+,\cdot,-,0,1\}$ be Boolean algebra, for any $a,b\in R$, if $a+b=b$, then a is not larger than b and using $a\leq b$ to stand for.

Remark 2.9

From now on, R and R' stands for Boolean algebras $\{R;+,\cdot,-,0,1\}$ and $\{R';+,\cdot,-,0',1'\}$, respectively.

Definition 2.10. [15]

A fuzzy subset μ of a Boolean algebras R is said to be a fuzzy subalgebra of R if for all $x,y\in R$,

$$\begin{aligned}&(1)\ \mu(x+y)\geq\mu(x)\wedge\mu(y);\\&(2)\ \mu(xy)\geq\mu(x)\wedge\mu(y);\\&(3)\ \mu(\bar{x})\geq\mu(x).\end{aligned}\tag{3}$$

III. Set- Valued Homomorphism Which Induced By Boolean Algebras

Definition 3.1. [2]

Let X and Y be two non-empty sets and $B\subseteq Y$. Let $T:X\to P^*(Y)$ be a set-valued mapping where $P^*(Y)$ denotes the set of all non-empty subsets of *Y*. The lower inverse and upper inverse of B under T are defined by

$$\begin{aligned}&T^{+}(B)=\{x\in X\mid T(x)\subseteq B\}\ ;\\&T^{-1}(B)=\{x\in X\mid T(x)\cap\end{aligned}\tag{4}$$

Definition 3.2. [2]

Let X and Y be two non-empty sets and $B\subseteq Y$. Let $T:X\to P^*(Y)$ be a set-valued mapping where $P^*(Y)$ denotes the set of all non-empty subsets of *Y*. $(T^{+}(B),T^{-1}(B))$ is called *T*-rough set of *R*.

Example 3.3

Let (U,θ) be an approximation space and $T:U\to P^*(U)$ be a set-valued mapping where $T(x)=[x]_\theta$, then for any $B\subseteq U$, $T^{+}(B)=\underline{Apr}(B)$ and $T^{-1}(B)=\overline{Apr}(B)$.

Definition 3.4. [7]

Let R and R' be two Boolean algebras and $T:R\to P^*(R')$ be a set-valued mapping. T is called a set-valued homomorphism if for all $x_1,x_2\in R$,

$$\begin{aligned}&(1)\ T(x_1+x_2)=T(x_1)+T(x_2);\\&(2)\ T(x_1x_2)=T(x_1)T(x_2);\\&(3)\ T(0)=\{0\};\\&(4)\ T(1)=\{1\}.\end{aligned}\tag{5}$$

Remark 3.5

Example 3.3 is a set-valued homomorphism. So a Boolean algebra homomorphism is a special case of a set-valued homomorphism. Let θ be a complete congruence relation, i.e., $[x]_\theta[y]_\theta=[xy]_\theta$ for all $x,y\in R$. Define

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$T:R \to P^*(R)$ by $T(x)=[x]_\theta$ for all $x \in R$, then T is a set-valued homomorphism. Further, by Example 3.3, the rough sets are T-rough sets.

Definition 3.6

Let R and R' be two Boolean algebras and $T:R \to P^*(R')$ be a set-valued homomorphism. Let μ be a fuzzy subset of R'. For every $x \in R$, we define

$$T^+(\mu)(x) = \bigwedge_{a \in T(x)} \mu(a) \ ;$$
$$T^{-1}(\mu)(x) = \bigvee_{a \in T(x)} \mu(a). \tag{6}$$

$T^+(\mu)$ and $T^{-1}(\mu)$ are called, respectively, the T-rough lower and the T- rough upper fuzzy subsets of R.

Proposition 3.7

Let R and R' be two Boolean algebras and $T:R \to P^*(R')$ be a set-valued homomorphism. Let μ be a fuzzy subset of a Boolean algebra R', if for all $x, y \in R$, the following is correct:

(1) $\mu(x+y) \geq \mu(x) \wedge \mu(y)$;

(2) $\mu(\bar{x}) \geq \mu(x)$.

Then $T^{-1}(\mu)$ is a fuzzy subalgebra of R.

Proof.

For all $x, y \in R$, we have

$$\begin{aligned} T^{-1}(\mu)(x+y) &= \bigvee_{z \in T(x+y)} \mu(z) \\ &\geq \bigvee_{a \in T(x), b \in T(y)} \mu(a+b) \\ &\geq \bigvee_{a \in T(x), b \in T(y)} (\mu(a) \wedge \mu(b)) \\ &= (\bigvee_{a \in T(x)} \mu(a)) \wedge (\bigvee_{b \in T(y)} \mu(b)) \\ &= T^{-1}(\mu)(x) \wedge T^{-1}(\mu)(y). \end{aligned}$$

Hence $T^{-1}(\mu)(x+y) \geq T^{-1}(\mu)(x) \wedge T^{-1}(\mu)(y)$. And

$$\begin{aligned} T^{-1}(\mu)(xy) &= T^{-1}(\mu)(\overline{\bar{x}+\bar{y}}) \\ &= \bigvee_{\bar{z} \in T(\bar{x}+\bar{y})} \mu(\bar{z}) \\ &\geq \bigvee_{\bar{a} \in T(\bar{x}), \bar{b} \in T(\bar{y})} \mu(\bar{a}+\bar{b})) \\ &= \bigvee_{\bar{a} \in T(\bar{x}), \bar{b} \in T(\bar{y})} (\mu(\bar{a})) \wedge \mu(\bar{b})) \\ &= (\bigvee_{\bar{a} \in T(\bar{x})} \mu(\bar{a})) \wedge (\bigvee_{\bar{b} \in T(\bar{y})} \mu(\bar{b})) \\ &\geq (\bigvee_{a \in T(x)} \mu(a)) \wedge (\bigvee_{b \in T(y)} \mu(b)) \\ &= T^{-1}(\mu)(x) \wedge T^{-1}(\mu)(y). \end{aligned}$$

Hence $T^{-1}(\mu)(xy) \geq T^{-1}(\mu)(x) \wedge T^{-1}(\mu)(y)$. Also

$$\begin{aligned} T^{-1}(\mu)(\bar{x}) &= \bigvee_{\bar{a} \in T(\bar{x})} \mu(\bar{a}) \geq \bigvee_{a \in T(x)} \mu(a) \\ &= T^{-1}(\mu)(x) \end{aligned}$$

It implies that $T^{-1}(\mu)$ is a fuzzy subalgebra of R.

Proposition 3.8

Let R and R' be two Boolean algebras and $T:R \to P^*(R')$ be a set-valued homomorphism. Let μ be a fuzzy subset of a Boolean algebra R', if for all $x, y \in R$, the following is correct:

(1) $\mu(x+y) \geq \mu(x) \wedge \mu(y)$;

(2) $\mu(\bar{x}) \geq \mu(x)$.

Then $T^+(\mu)$ is a fuzzy subalgebra of R.

Proof.

The proof is similar to the proof of proposition 3.7.

Corollary 3.9

Let R and R' be two Boolean algebras and $T:R \to P^*(R')$ be a set-valued homomorphism. Let μ be a fuzzy subset of a Boolean algebra R', if for all $x, y \in R$, the following is correct:

(1) $\mu(x+y) \geq \mu(x) \wedge \mu(y)$;

(2) $\mu(\bar{x}) \geq \mu(x)$.

Then $(T^+(\mu), T^{-1}(\mu))$ is a T-rough fuzzy subalgebra of R.

Proposition 3.10

Let R and R' be two Boolean algebras and $T:R \to P^*(R')$ be a set-valued homomorphism. Let μ be a fuzzy subset of a Boolean algebra R', if for all $x, y \in R$, the following is correct:

(1) $\mu(xy) \geq \mu(x) \wedge \mu(y)$;

(2) $\mu(\bar{x}) \geq \mu(x)$.

Then $(T^+(\mu), T^{-1}(\mu))$ is a T-rough fuzzy subalgebra of R.

Proof

For all $x, y \in R$, we have

$$\begin{aligned} T^+(\mu)(x+y) &= T^+(\mu)(\overline{\bar{x}\bar{y}}) = \bigwedge_{\bar{z} \in T(\overline{xy})} \mu(\bar{z}) \\ &\geq \bigwedge_{\bar{a} \in T(\bar{x}), \bar{b} \in T(\bar{y})} \mu(\overline{a b}) \\ &\geq \bigwedge_{\bar{a} \in T(\bar{x}), \bar{b} \in T(\bar{y})} (\mu(\bar{a})) \wedge \mu(\bar{b})) \\ &= (\bigwedge_{\bar{a} \in T(\bar{x})} \mu(\bar{a})) \wedge (\bigwedge_{\bar{b} \in T(\bar{y})} \mu(\bar{b})) \\ &\geq (\bigwedge_{a \in T(x)} \mu(a)) \wedge (\bigwedge_{b \in T(y)} \mu(b)) \\ &= T^+(\mu)(x) \wedge T^+(\mu)(y). \end{aligned}$$

Thus $T^{+}(\mu)(x+y) \geq T^{+}(\mu)(x) \wedge T^{+}(\mu)(y)$. And

$$
\begin{aligned}
T^{+}(\mu)(xy) &= \bigwedge_{z \in T(xy)} \mu(z) \\
&\geq \bigwedge_{a \in T(x), b \in T(y)} \mu(ab) \\
&\geq \bigwedge_{a \in T(x), b \in T(y)} (\mu(a) \wedge \mu(b)) \\
&= (\bigwedge_{a \in T(x)} \mu(a)) \wedge (\bigwedge_{b \in T(y)} \mu(b)) \\
&= T^{+}(\mu)(x) \wedge T^{+}(\mu)(y).
\end{aligned}
$$

Therefore $T^{+}(\mu)(xy) \geq T^{+}(\mu)(x) \wedge T^{+}(\mu)(y)$. Also

$$
\begin{aligned}
T^{+}(\mu)(\bar{x}) &= \bigwedge_{\bar{a} \in T(\bar{x})} \mu(\bar{a}) \geq \bigwedge_{a \in T(x)} \mu(a) \\
&= T^{+}(\mu)(x).
\end{aligned}
$$

Then $T^{+}(\mu)$ is a fuzzy subalgebra of R. The same method, $T^{-1}(\mu)$ is a fuzzy subalgebra of R.

Therefore $(T^{+}(\mu), T^{-1}(\mu))$ is a T-rough fuzzy subalgebra of R.

Theorem 3.11

Let μ be a fuzzy subset of a Boolean algebra R, then μ is a fuzzy subalgebra of R if and only if μ_t is a subalgebra of R for all $t \in [0,1]$.

Proof.

Let μ is a fuzzy subalgebra of R, $t \in [0,1]$ and $x, y \in \mu_t$, then $\mu(x+y) \geq \mu(x) \wedge \mu(y) \geq t$, $\mu(xy) \geq \mu(x) \wedge \mu(y) \geq t$, And $\mu(\bar{x}) \geq \mu(x) \geq t$. So $x+y, xy \in \mu_t$ and $\bar{x} \in \mu_t$. Therefore μ_t is a subalgebra of R.

Conversely, let μ_t is a subalgebra of R for all $t \in [0,1]$. If there exist $x, y \in R$, such that $\mu(x+y) < z = \mu(x) \wedge \mu(y)$.

Then $x, y \in \mu_z$, $z \in [0,1]$ and $\mu(x+y) < z$. Since μ_z is a subalgebra of R, so $x+y \in \mu_z$ and $\mu(x+y) \geq z$. This is a contradiction with $\mu(x+y) < z$. Therefore for all $x, y \in R$, $\mu(x+y) \geq \mu(x) \wedge \mu(y)$.

The same method, for all $x, y \in R$, we have $\mu(xy) \geq \mu(x) \wedge \mu(y)$ and $\mu(\bar{x}) \geq \mu(x)$. Hence μ is a fuzzy subalgebra of R.

Lemma 3.12

Let R and R' be two Boolean algebras and $T: R \to P^{*}(R')$ be a set-valued homomorphism. If μ is a fuzzy subalgebra of R', Then for all $t \in [0,1]$,

(1) $(T^{+}(\mu))_t = T^{+}(\mu_t)$;

(2) $(T^{-1}(\mu))_t = T^{-1}(\mu_t)$.

Proof. (1)

$$
\begin{aligned}
x \in (T^{+}(\mu))_t &\Leftrightarrow T^{+}(\mu)(x) \geq t \\
&\Leftrightarrow \bigwedge_{a \in T(x)} \mu(a) \geq t \\
&\Leftrightarrow \forall a \in T(x),\ \mu(a) \geq t \\
&\Leftrightarrow T(x) \subseteq \mu_t \\
&\Leftrightarrow x \in T^{+}(\mu_t).
\end{aligned}
$$

(2).

$$
\begin{aligned}
x \in (T^{-1}(\mu))_t &\Leftrightarrow T^{-1}(\mu)(x) \geq t \\
&\Leftrightarrow \bigvee_{a \in T(x)} \mu(a) \geq t \\
&\Leftrightarrow \text{for some } a \in T(x),\ \mu(a) \geq t \\
&\Leftrightarrow T(x) \bigcap\ \iota \\
&\Leftrightarrow x \in T^{-1}(\mu_t).
\end{aligned}
$$

Definition 3.13. [17]

Let f be a mapping from a set X to a set Y. Let μ be a fuzzy set in X and λ be a fuzzy set in Y. Then the image $f(\mu)$ of μ is the fuzzy set in Y defined by

$$f(\mu)(y) = \bigvee_{x \in f^{-1}(y)} \mu(x) \quad \text{for all } y \in Y.$$

And the inverse image $f^{-1}(\lambda)$ of λ defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$ for all $x \in X$.

Theorem 3.14

Let $f: R \to R'$ be a Boolean algebra homomorphism. If f is surjective and μ is a fuzzy subalgebra of R, then $f(\mu)$ is a fuzzy subalgebra of R'.

Proof.

Given $f(x), f(y) \in f(\mu)$, let $x' \in f^{-1}(f(x))$ and $y' \in f^{-1}(f(y))$ be such

$$
\begin{aligned}
\mu(x') &= \bigvee_{t \in f^{-1}(f(x))} \mu(t),\quad \mu(y') \\
&= \bigvee_{t \in f^{-1}(f(y))} \mu(t)
\end{aligned}
$$

Respectively. Then

$$\mu(f(x)+f(y)) = \bigvee_{z \in f^{-1}(f(x)+f(y))} \mu(z)$$
$$\geq \mu(x') \wedge \mu(y')$$
$$= \mu(f(x)) \wedge \mu(f(y)).$$

And

$$\mu(f(x)f(y)) = \bigvee_{z \in f^{-1}(f(x)f(y))} \mu(z)$$
$$\geq \mu(x') \wedge \mu(y')$$
$$= \mu(f(x)) \wedge \mu(f(y)).$$

Also, $\mu(\overline{f(x)}) \geq \mu(f(x))$. Therefore $f(\mu)$ is a fuzzy subalgebra of R'.

Theorem 3.15

Let $f: R \to R'$ be a Boolean algebra homomorphism. If λ is a fuzzy subalgebra of *R*', then $f^{-1}(\lambda)$ is a fuzzy subalgebra of *R*.

Proof.

For any $x, y \in R$, we have

$$f^{-1}(\lambda)(x+y) = \lambda(f(x+y))$$
$$= \lambda(f(x)+f(y))$$
$$\geq \lambda(f(x)) \wedge \lambda(f(y)).$$

And

$$f^{-1}(\lambda)(xy) = \lambda(f(xy))$$
$$= \lambda(f(x)f(y))$$
$$\geq \lambda(f(x)) \wedge \lambda(f(y)).$$

Also, $f^{-1}(\lambda)(\bar{x}) = \lambda(f(\bar{x})) \geq \lambda(f(x))$. Therefore $f^{-1}(\lambda)$ is a fuzzy subalgebra of *R*.

IV. Conclusion

Fuzzy set theory and rough set theory take into account two different aspects of uncertainty that can be encountered in real-world problems in many fields. Fuzzy sets deal with possibilities uncertainty, connected from ambiguity of information. The combination of fuzzy set and rough set lead to various models. In this paper, we substituted a universe set by a Boolean algebra, and introduced the set-valued homomorphism and *T*-rough fuzzy subalgebra in a Boolean algebra based on definitions in [3, 4, 8, 15, 16]. The concept of *T*-rough fuzzy subalgebra in a Boolean algebra is a extension of fuzzy subalgebra in a Boolean algebra. We hope that this extended research many provide a powerful tool in approximate reasoning. Also, we believe, this paper offered here will turn out to be more useful in the theory and applications of rough sets and fuzzy sets.

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