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Studies on Temperature Distribution in Hydromagnetic Flow through Porous Media with Slip Effects

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Abstract **– Effects of Hall current, slip parameter and heat transfer owing to stable hydromagnetic flow through a uniform channel bounded by porous media with measurable thickness is studied. The fluent in the course is pretended to be homogeneous, incompressible and Newtonian. Analytical solutions are constructed for the governing equations using BJR slip condition. Expressions for flow parameters, such as axial velocity, slip velocity, the temperature distribution, heat transfer coefficient and the wall shear stress have been obtained. Profiles of temperature distribution and the shear stress are computed for different values of porous parameter, Darcy velocity, Reynolds number, Magnetic number, Hall parameter and the coupling of Prandtl and Eckert numbers. It is observed that the flow constraints direct the role of altering the temperature distribution and the shear stress.**

Keywords **– BJR slip condition, Hall current, Magnetic field and Porous media.**

I. INTRODUCTION

The channel fluency in the former stages were prescribed on basis of Ohm's law without Hall effects. The Hall effects is deemed significant where the electron cyclotron frequency is much greater than electron collision frequency in due of robust magnetic fields. Hence, the Ohm's law is must in case of solving problems on channel flows. MHD channel runs with heat effects and the applications to electric power generator, electromagnetic flow meter, electromagnetic accelerators and propulsion systems are perceiving scientific recognition. The MHD aspect of heat transfer in channel flow has been synced by many researchers: Siegel **[1]**, Makinde **[2]**, Alpher **[3]**, Regirer **[4]** and Yen **[5]** examine the impact of magnetic field on heat transfer in a channel flow between conducting and non-conducting walls.

Tani **[6]** implied importance of the Hall current effect on the steady motion of electrically conducting viscous fluids in channels. Gupta and Gupta **[7]** has worked on heat and mass transfer of a viscous fluid on blowing or suction over an isothermal stretching sheet. Bhaskara and Bathaiah **[8]** have studied the outcome of Hall current on the viscous, incompressible marginally conducting flow through a straight porous channel under a uniform transverse magnetic field. Soundalgekar **[9]** observed the outcome of Hall current effect on the firm MHD couette flow with heat transfer.

In the case of flow past a porous midway, Beavers and Joseph **[10]** have displayed the archaic belief of slip condition at the porous boundaries is invalid and they have postulated the existence, called BJ slip conditions in due of momentum transfer. Khan *et al.* **[11]** studied the slip effects on shearing flows in porous medium. Jat and Santhosh Chaudhary **[12]** investigated MHD boundary layer flow precedent a porous substrate with BJ slip condition. The BJ slip condition is autonomous of the thickness of the porous layer and is valid when thickness of porous layer is much preponderant than the thickness of free flow.

As infrequent geophysical applications, Rudraiah **[13]** has impeded a new slip condition to overcome the former limitation which is now known as BJR slip condition. It is indicated that since the BJR slip **[13]** status involves the thickness of porous layer, H and hence as $H \rightarrow \infty$, it curtails to BJ slip condition. The validity of the condition is accepted, when the thickness of porous layer is diminutive with thickness of the free flow which is applicable to the bio mechanical problems. Recently, Ramakrishnan and Shailendhra **[14]** have studied the effects of hydromagnetic blood flow through a uniform channel bounded by porous media using BJR slip condition **[13]**. The prime subjective of the present task is to study the combined effects of magnetic field, porous parameter, Darcy velocity, Reynolds number, Hall parameter and the product of Prandle and Eckert numbers on hydromagnetic flow through a uniform channel bounded by porous media using Beavers Joseph-Rudraiah (BJR) slip condition.

II. FORMULATION OF THE PROBLEM

To investigate the Hall effects and heat transfer on hydromagnetic flow through a uniform channel covered by porous media, consider a physical configuration as shown in Fig.1. Consider an infinitely long channel of uniform width $2h$ through which a laminar, steady and viscous hydromagnetic fluid flows. The channel is bounded externally on either side by densely packed saturated porous layers each of width H at $y = \pm h$. It is assumed that the fluid behaves like a homogeneous conducting Newtonian fluid with constant density ρ, small electrical conductivity σ_e and viscosity μ.

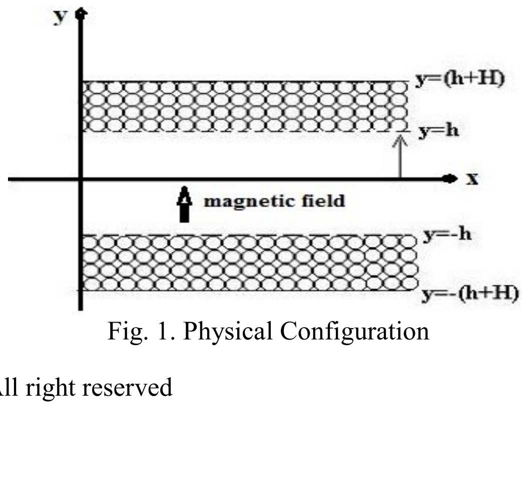

Fig. 1. Physical Configuration

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Let u and v be the velocity components of the fluid at a point (x, y). It is further assumed that the channel is symmetrical about the x-axis. A uniform magnetic field H_0 is applied in the y direction. $B_0 = \mu_e H_0$, is the electromagnetic induction where μ_e is the magnetic permeability. Since, the fluid is of small electrical conductivity with magnetic Reynolds number much less than unity so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The channel is infinitely long, all physical quantities (except pressure) depend only on y. The equation of continuity $\nabla.q = 0$ gives $v = 0$ where $q = (u, v, w)$.

Under the above assumptions, the equations governing the flow including Hall effects in dimensionless form are:

$$R\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial y^2} - Fu \quad (1)$$

$$v = 0 \quad (2)$$

$$R\frac{\partial p}{\partial y} = 0 \quad (3)$$

$$\frac{\partial^2 T}{\partial y^2} = -PE\left(\frac{\partial u}{\partial y}\right)^2 \quad (4)$$

where

$$Magnetic\ parameter\ (M) = \frac{\sigma_e \mu_e^2 H_0^2 h}{\rho U}$$

$$Reynolds\ Number\ (R) = \frac{Uh}{\nu}$$

$$Prandtl\ Number\ (P) = \frac{\mu C_p}{K_T}$$

$$Eckert\ Number\ (E) = \frac{U^2}{C_p (T_1 - T_0)}$$

$$Hall\ Parameter\ (m)\ and\ F = \frac{M^2}{1+m^2}$$

Here, U-Characteristic velocity, ν-Kinematic viscosity, C_p-Specific heat at constant pressure and K_T-the coefficient of thermal conductivity.

III. Solution of the Problem

Let us assume that the thickness of the porous layer (H) is much smaller or comparable than the width (h) of the flow in the channel i.e. $H \le h$. Since, the channel is of uniform width, use the approximation that the wall slope is everywhere negligible and the components of velocity and pressure gradient are approximately equal. After eliminating the pressure term between (1) and (3), the basic equation in non-dimensional form is as follows:

$$\frac{d^3 u}{dy^3} - F\frac{du}{dy} = 0 \quad (5)$$

since u is a function of y alone. The BJR slip conditions **[13]**, in dimensionless form, are as follows:

$$\frac{du}{dy} = \delta_1 Q_1 + \delta_2 (u_{b_1} - Q_1) \qquad at \quad y = -1 \quad (6)$$

$$\frac{du}{dy} = -\delta_1 Q_1 - \delta_2 (u_{b_2} - Q_1) \qquad at \quad y = 1 \quad (7)$$

$$v = \epsilon Q_y, \qquad Q_1 = -\frac{k}{\mu}\frac{\partial p}{\partial x} \quad (8)$$

$$\int_{-1}^{1} u(y)dy = N_f \quad (9)$$

where

$$\delta_1 = \frac{\lambda\sqrt{\lambda}\sigma_p \epsilon}{\delta_0 \sinh\left(\frac{\sigma_p}{\sqrt{\lambda}}\right)}, \qquad \delta_2 = \frac{\sqrt{\lambda}}{\delta_0}\sigma_p \coth\left(\frac{\sigma_p}{\sqrt{\lambda}}\right)$$

$$\sigma_p = \frac{H}{\sqrt{k}}, \qquad \delta_o = \frac{H}{h}$$

and λ is a positive constant, called viscosity factor, N_f is the net flux through the channel, u_{b1} and u_{b2} are the slip velocities, k is the permeability of the porous material.

Solving (5) subject to the conditions (6) to (9) we have,

$$u(y) = \frac{N_f}{2} + \frac{1}{2}\left[\frac{2Q_1(\delta_1 - \delta_2) + \delta_2 N_f}{Fsinh(\sqrt{F}) + \eta_1}\right] \times \left[sinh(\sqrt{F}) - \sqrt{F}cosh\left(\sqrt{F}y\right)\right] \quad (10)$$

The net flex is given by,

$$N_f = \frac{2}{F}\left[\frac{\eta_1 + \left(1 - \frac{\delta_0^2}{\sigma_p^2}(\delta_1 - \delta_2)\right)Fsinh(\sqrt{F})}{Fsinh(\sqrt{F}) + \delta_2\sqrt{F}cosh(\sqrt{F})}\right]\left(-R\frac{\partial p}{\partial x}\right) \quad (11)$$

and the slip velocity is

$$u_b = \frac{1}{2}\left[\frac{N_f Fsinh(\sqrt{F}) - \frac{2Q_1}{\delta_2}(\delta_1 - \delta_2)\eta_1}{Fsinh(\sqrt{F}) + \eta_1}\right] \quad (12)$$

where,

$$\eta_1 = \delta_2\left[\sqrt{F}cosh(\sqrt{F}) - sinh(\sqrt{F})\right]$$

Using (10) in (4), we get

$$\frac{d^2 T}{dy^2} = -\frac{PE(F^2)}{4}\left[\frac{2Q_1(\delta_1 - \delta_2) + \delta_2 N_f}{Fsinh(\sqrt{F}) + \eta_1}\right]^2 \times \left(\frac{cosh 2\sqrt{F}y - 1}{2}\right) \quad (13)$$

Solving (13) using the boundary conditions

$T = 1 \quad at \quad y = 1$
$T = 0 \quad at \quad y = -1$

we obtain,

$$T(y) = \frac{y+1}{2} + \frac{PE}{32}\left[\frac{2Q_1(\delta_1 - \delta_2) + \delta_2 N_f}{Fsinh(\sqrt{F}) + \eta_1}\right]^2 \times F\left[2F(y^2 - 1) + cosh(2\sqrt{F}) - cosh(2\sqrt{F}y)\right] \quad (14)$$

The rate of heat transfer coefficient at the lower wall $y = -h$ is given by

$$q_L = \frac{1}{2} + \frac{PE}{32}\left[\frac{2Q_1(\delta_1 - \delta_2) + \delta_2 N_f}{Fsinh(\sqrt{F}) + \eta_1}\right]^2 \times F\left[2\sqrt{F}sinh(2\sqrt{F}) - 4F\right] \quad (15)$$

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The rate of heat transfer coefficient at the upper wall $y = h$ is given by

$$q_U = \frac{1}{2} - \frac{PE}{32}\left[\frac{2Q_1(\delta_1 - \delta_2) + \delta_2 N_f}{F sinh(\sqrt{F}) + \eta_1}\right]^2 \times F\left[4F - 2\sqrt{F} sinh(2\sqrt{F})\right] \quad (16)$$

The shearing stress at the wall $y = -1$ is given by

$$\tau_L = \frac{1}{2}\left[\frac{2Q_1(\delta_1 - \delta_2) + \delta_2 N_f}{F sinh(\sqrt{F}) + \eta_1}\right] F sinh(\sqrt{F}) \quad (17)$$

The shearing stress at the wall $y = 1$ is given by

$$\tau_U = -\frac{1}{2}\left[\frac{2Q_1(\delta_1 - \delta_2) + \delta_2 N_f}{F sinh(\sqrt{F}) + \eta_1}\right] F sinh(\sqrt{F}) \quad (18)$$

IV. Results and Discussion

The purpose of the present discussion is to analyze and compare the effects of Reynolds number (R), magnetic parameter (M), porous parameter (σ_p), Hall parameter (m) and the product of Prandle and Eckert numbers on the temperature distribution (T) and the shear stress.

In the present study, numerical evaluations have been done for the following values of the parameters: Reynolds number R = 0.2, 0.4, 0.6, 0.8, the magnetic parameter M = 1, 2, 3, 4, 5, porous parameter = 10^1 to 10^6, Hall parameter m = 1, 2, 3, 4, 5 and the product of Prandle and Eckert numbers PE = 1, 2, 3, 4.

Figs. 2 - 9 illustrate the combined effects of R, m, M and PE on the temperature distribution using BJR slip condition. Note that Figs. 2-9 plotted for the value of the porous parameter $\sigma_p = 10^1$ and $\sigma_p = 10^2$. It is generally observed that the temperature distribution decreases with the increase of the porous parameter (σ_p). It is also seen that when $\sigma_p > 10^4$, there is no significant changes in these flow parameters, because in this case the walls behave like an impermeable walls.

Fig. 2 and 3, demonstrate that an increase in R resulting in an increment in the temperature distribution, when the other parameters m, PE and M assigned its minimum and maximum values. It is noted that from Fig. 3, when m, PE and M takes its maximum values and for R = 0.2, the distribution is almost a constant.

Fig. 4 and 5, projects that in the presence of magnetic field there is a reduction in the temperature distribution, when other parameters R, m and PE have fixed values.

From Fig. 6 and 7, it is seen that the temperature increases with increase in m, the Hall parameter. As reported in numerous MHD studies, this temperature component is the result of the Hall effect; therefore, it will respond positively to the increase in m values. On comparing Fig. 6 and 7, it is seen that the temperature distribution takes parabolic form when R, PE and M assigned fixed maximum values. This is due to the heat generated through viscous factor.

From Fig. 8 and 9, it is observed that the temperature increases with raise in PE, when other parameters R, m and M take its minimal and maximal values. It is noticed that the temperature distribution increases when the values of the parameters M, R and PE increase whereas the magnetic field (M) decreased it.

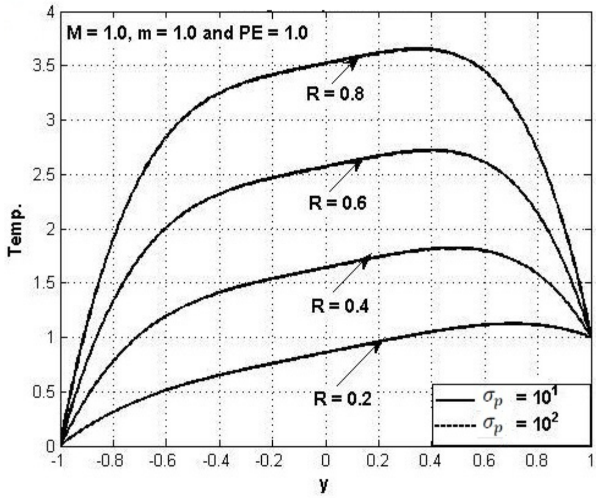

Fig. 2. Temperature distribution against y for different values of R and least value of other parameters

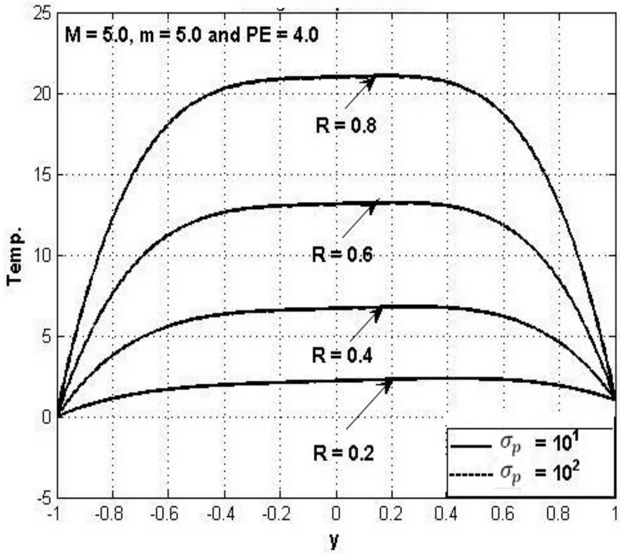

Fig. 3. Temperature distribution against y for different values of R and maximum value of other parameters

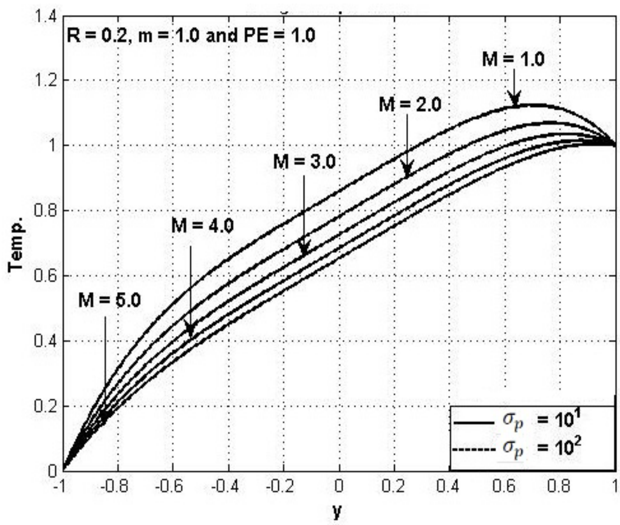

Fig. 4. Temperature distribution against y for different values of M and least value of other parameters

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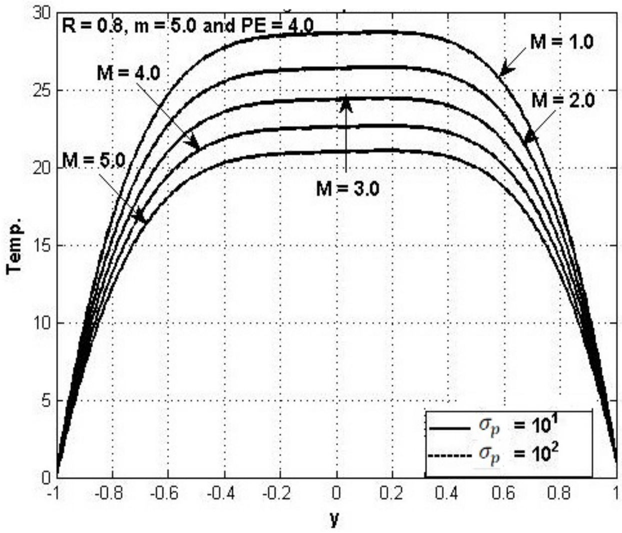

Fig. 5. Temperature distribution against y for different values of M and maximum value of other parameters

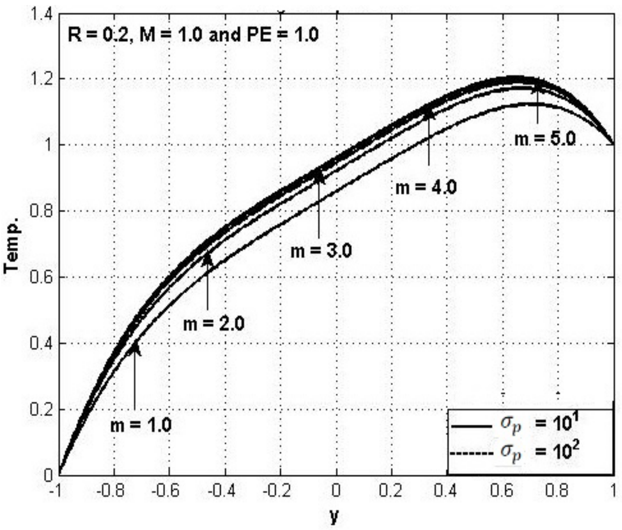

Fig. 6. Temperature distribution against y for different values of m and least value of other parameters

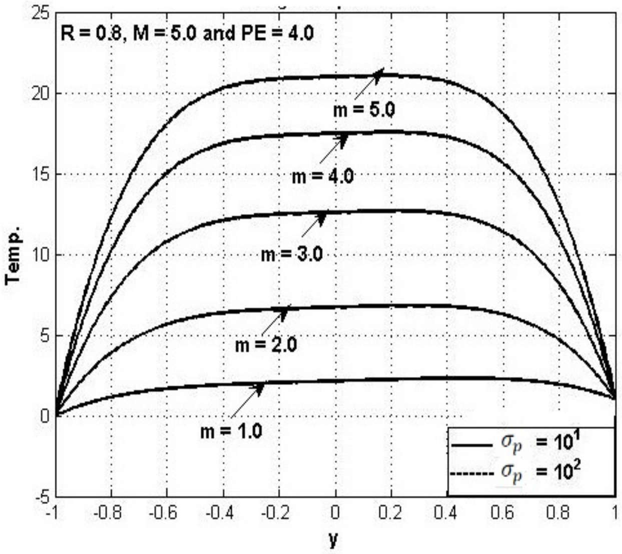

Fig. 7. Temperature distribution against y for different values of m and maximum value of other parameters

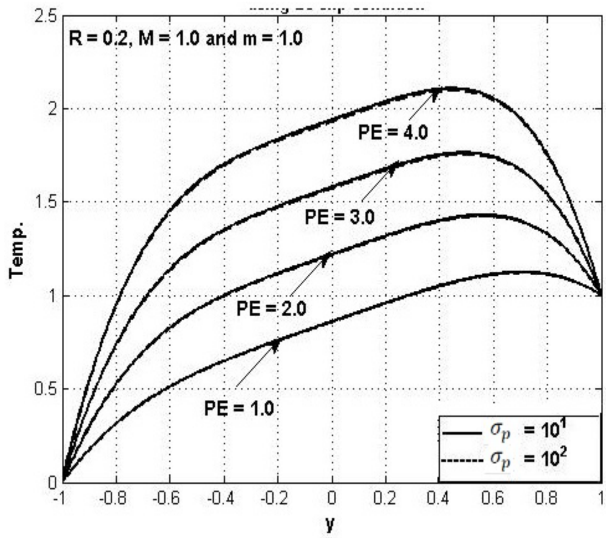

Fig. 8. Temperature distribution against y for different values of PE and least value of other parameters

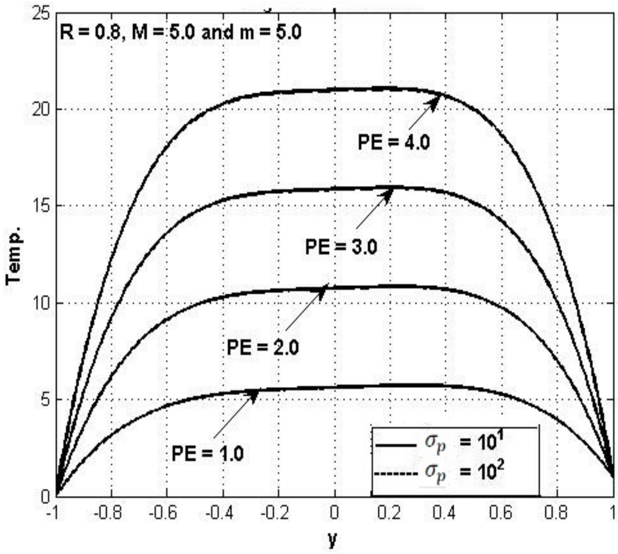

Fig. 9. Temperature distribution against y for different values of PE and maximum value of other parameters

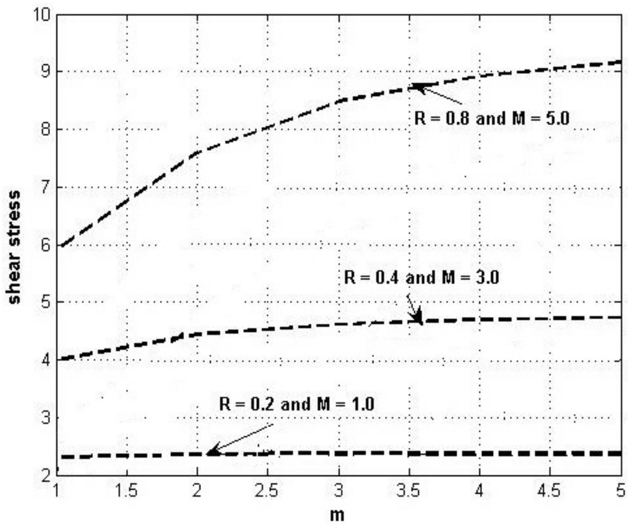

Fig. 10. Distribution of Shear stress against m at various values of other parameters

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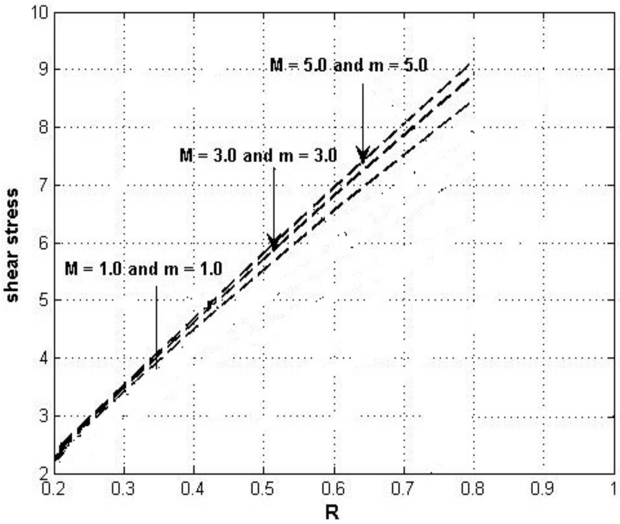

Fig. 11. Distribution of Shear stress against R at various values of other parameters

The shear stress for the lower wall q_L is shown in Figs. 10 - 11. It is observed that the shear stress have maximum, when the physical parameters have its possible maximum value. Shear stress starts decreasing when these parameters get reduced its values. From Fig 10, it is noticed that when $R = 0.2$ and $M = 1.0$, shear stress against the Hall parameter (m) indicates no significant change. Similar results can be obtained for the upper wall.

V. Conclusion

The effects of porous parameter, Reynolds number, Magnetic parameter, Hall parameter and the product of Prandtl and Eckert numbers on the steady hydromagnetic flow in a uniform channel bounded by porous media using BJR slip condition have been investigated. The results reveal that the porous parameter and the magnetic number produce a retarding effect on the temperature distribution and the shear stress. Other parameters like Hall parameter (m) and Product of Prandtl and Eckert Numbers (PE) have an accelerating effect on the temperature distribution. From these, it may be concluded that with a proper choice of porous parameter, Reynolds number, Magnetic parameter, Hall parameter, it is possible to achieve the control over the steady flow through a uniform channel covered by porous media.

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