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Pricing Bivariate Option under an Improved GARCH Process with Dynamic Copula Models

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Abstract **– This paper uses a modified GARCH process and dynamic copula function to study the price of bivariate options. Our model incorporates the time-dependent correlation between underlying assets, which is a common phenomenon in financial practice but be few considered in previous researches. Dynamic copula with time-dependent correlation is used to depict the dynamic nature of option price. In order to formulate the heavy-tailed characteristic of financial derivatives, we also improve the GARCH process by applying Tukey's H -distribution family. In numerical experiment, reformulated GARCH process expresses more precise curves than general GARCH process. And the rationality of new pricing method is illustrated by the numerical results, which show the differential pricing between improved and general methods for better-of-two-markets options on the Standard and Poor's 500 and NYSE indexes.**

Keywords **– Call-on-max option; GARCH process; H-transformation; Dynamic Copula.**

I. Introduction

As an excellent tool, multivariate options play a key role for hedging risk in finance field. Based on two or more underlying securities or indexes, multivariate options usually take the form of calls (or puts), which give right to buy (or sell) the best or worst performer of a number of underlying assets. Generally, the most important factor in the valuation of multivariate options is the dependence between two underlying assets.

For many years, multivariate option pricing was studied by the Brownian motion framework, which is the great work of Black and Scholes (1973) [1] and Merton (1973) [2]. Examples include Margrabe (1978) [3], Stulz (1982) [4], Reiner (1992) [5], and Shimko (1994) [6]. In these papers, the correlation was applied to measure the dependence between the assets. However, Embrechts et al. (2002) [7] and Forbes and Rigobon (2002) [8] pointed out that the correlation is not a unsatisfactory measure of dependence, because it may cause some confusion and misunderstanding. Therefore, more appropriate measures are called for the dependence structure. Copula function has been viewed as an ideal measure for its good properties that contain all the information about the dependence structure and captures nonlinear dependences. Rosenberg (1999) [9] and Cherubini and Luciano (2002) [10] recommended using Copulas to price bivariate options. But only static copulas are adopted in those work. However, it is a stylized fact of financial markets that the dependence structure between underlying assets would change greatly over time. As Boyer et al. (1999) [11] and Patton (2004) [12] suggested multivariate options should be priced by a dynamic model of the dependence structure of asset returns, the dependence structure between assets with time variation should prove helpful in providing a more realistic valuation of multivariate options. Therefore, a dynamic copula approach should be adopted to price the multivariate options.

Goorbergh et al. (2005) [13] applied the dynamic copula model with GARCH process to price the bivariate option. With the relationship between Kendall's tau and one-parameter copulas, Goorbergh used a particular time regression equation to evolve the dependence parameter (Kendall's tau), then the parameter of one-parameter copula are decided and the dynamic copula is described. The forcing variables in this equation are the conditional volatilities of the underlying assets. Zhang et al. (2008) [14] applied a new dynamic approach to price bivariate options with a GARCH process and time-varying copula. A series of the copulas are selected for different subsamples, which are divided by AIC criterion. It should be noted that in these method, the financial rate is expressed by GARCH process with normal innovations. But recent empirical studies of high frequency financial time series indicate that the tail behavior of GARCH models remains too short even with standardized Student-T innovations.

It is widespread that using the Gaussian distribution as statistical model for a given data set, especially in practice. However, departure from normality seems to be more the rule than the exception. In order to construct skew and heavy-tailed distributions, Tukey (1977) [15] suggested to transform a standard Gaussian variable Z with a specific non-linear transformation, which is composed by a skewness transformation G and a kurtosis transformation H and is called family of GH -transformations. The corresponding GH distribution has been successfully applied in financial, medical or environmental statistics. Badrinath and Chatterjee (1988) [16] did pioneer work applying it to stock returns of the New York stock exchange. Mills (1995) [17] applied them to FTSE index returns, Fischer et al. (2007) [18] to returns of aluminium and zinc, and Dutta and Babbel (2003) [19] as distributional model for US-Dollar London Inter Bank Offer Rates. In this paper, a new GARCH model approach to price bivariate options with time-varying copula is proposed. For the distribution of the innovations transfored to the generalized normality, the approach in the present paper makes the pricing more reasonable. The implement is done by Tukey's kurtosis transformation H .

The remainder of this paper is organized as followings. In Section 2, we introduce some notations for bivariate option and give GARCH process for financial assets under risk-neutral probability, the risk-neutral valuation of bivariate option and the H -transformation also be introduced in this section. Section 3 gives an explanation

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for the new idea to GARCH process. In Section 4, the numerical results are presented. Finally, conclusions are given in Section 5.

II. Preliminaries

A. Option Valuation

Consider a European-type bivariate option on the best (worst) performer of two assets. Four types of better-of-two-markets or worse-of-two-markets claims can be distinguished: call options on the better performer, put options on the worse per-former, call options on the worse performer, and put options on the better performer. These may be referred to as call-on-max, put-on-min, call-on-min, and put-on-max options, respectively. Using this convention, the payoffs of the four types are given by:

Call on max: $\max\{\max(S_{1,T}, S_{2,T}) - K, 0\}$,

Call on min: $\max\{\min(S_{1,T}, S_{2,T}) - K, 0\}$,

Put on min: $\max\{K - \min(S_{1,T}, S_{2,T}), 0\}$,

Put on max: $\max\{K - \max(S_{1,T}, S_{2,T}), 0\}$,

where $S_{i,t}$ is the price at time t of asset $i \in \{1,2\}$ and K is the strike price. This paper concentrates on bivariate option which is European-type call option on the best performer of two assets. The call option on the better performer can be referred to as call-on-max option. The payoff of a call-on-max option would be

$$\{\max(S_{1,T}, S_{2,T}) - K, 0\}.$$

Certainly, the initial asset prices should be close for the option to make sense. It is assumed that they are exactly equal to S_{T_0}. In the following, we use $r_{i,t}$ to denote the log-return on $i-th$ asset $(i = 1,2)$ from time $t-1$ to time t: $r_{i,t} = \log S_{i,t} / S_{i,t-1}$, and the corresponding exponential expression is $R_{i,t} = \exp(r_{i,t})$. Then, the payoffs of a call-on-max option of contracts are given by:

$$\{\max(R_{1,T}, R_{2,T}) - E, 0\},$$

where $R_{i,T} = S_{i,T} / S$ is the gross return at maturity on underlying $i \in 1,2$, and $E = K / S$ denotes the normalized exercise price of the option.

B. Sklar Theorem

After the great work of Sklar (1959) [20], it has been known that any multivariate continuous distribution function can be uniquely expressed by its marginals and a copula. Sklar's theorem illustrates the role that copulas play in the relationship between multivariate distribution functions and their univariate margins. Before declaring the Sklar's theorem, we show the definition of the copula function.

Definition 1 A two-dimensional copula is a function C from $[0,1]\times[0,1]$ to $[0,1]$ with the following properties:

1. For every u, v in $[0,1]$, $C(u,0) = C(0,v) = 0$ and $C(u,1) = u, C(1,v) = v$;

2. For every u_1, u_2, v_1, v_2 in $[0,1]$ such that $u_1 < u_2$ and $v_1 < v_2$,

$$C(u_2, v_2) - C(u_1, v_2) - C(u_2, v_1) + C(u_1, v_1) \geq 0.$$

Now we give the Sklar's theorem and Conditional Sklar's theorem.

Lemma 1 (Sklar's theorem.) Let H be a distribution function with margins F and G. There exists a copula C such that for all x, y in $\bar{R}$,

$$H(x,y) = C(F(x), G(y)). \tag{1}$$

If F and G are continuous, then C is unique. Otherwise, C is uniquely determined on $RanF \times RanG$. Conversely, if C is a copula and F and G are distribution functions, then the function H defined by (1) is a joint distribution function with margins F and G.

Lemma 2 (Conditional Sklar's theorem.) Let H be a conditional distribution function with margins F and G. And let $\mathcal{F}$ be some conditioning set, then there exists a conditional copula C such that for all x, y in $\bar{R}$,

$$H(x,y| \mathcal{F} \qquad \mathcal{F} \qquad \mathcal{F} \tag{2}$$

If F and G are continuous, then C is unique. Otherwise, C is uniquely determined on $RanF \times RanG$. Conversely, if C is a copula and F and G are distribution functions, then the function H defined by (2) is a joint distribution function with margins F and G.

This Lemma firstly appeared in Sklar (1959). The name “copula” was chosen to emphasize the manner in which a copula “copula” a joint distribution function to its univariate margins.

C. GARCH Process

The objective marginal distributions is chosen by the specification from Duan (1995) [21]. It provides a relatively easy transformation to risk-neutral distributions, while being general enough to capture volatility clustering. Instead of deriving the bivariate risk-neutral distribution directly, each marginal process is proposed to transform separately. Each of the objective marginal distributions of the asset returns is modeled by a $GARCH(1,1)$ process with Gaussian innovations under the objective probability measure P, that is, for $i = 1,2$:

$$\begin{cases} r_{i,t} = \mu_i + \varepsilon_{i,t}, \\ h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, n \\ \mathcal{L} \qquad {}_{,t-1}) = N(0, h_{i,t}) \end{cases} \tag{3}$$

where $\omega_i > 0, \alpha_i > 0$, and $\beta_i > 0$. μ_i is the drift, the set $\varphi_{i,t-1}$ corresponds to the information set of all information up to and including time $t-1$, and $\mathcal{L} \qquad {}_{-1})$ denotes the objective probability law conditional on the information set $\varphi_{i,t-1}$ under measure P. The marginal distributions are specified conditional on this common information set, so that copula theory can be used to construct a joint conditional distribution. To ensure covariance stationarity of the GARCH process, $\alpha_i + \beta_i$ is assumed to be less than 1.

Duan (1995) shows that, under certain conditions, the change of measure comes down to a change in the drift. In order to obtain the risk-neutral price, the conventional risk-neutral valuation relationship should be generalized to adapt the heteroscedasticity of the asset return process. Firstly,we give four assumptions for this purpose:

Assumption (Duan (1995)) A pricing measure Q is said to satisfy the locally risk-neutral valuation relationship if the following properties are all satisfied:

(1) measure Q is mutually absolutely continuous with respect to measure *P*.

(2) $R_{i,t} | \varphi_{i,t-1}$ distributes lognormally under measure Q.

(3) $E^Q(R_{i,t} | \varphi_{i,t-1}) = e^{r_f}$, where r_f is risk-free rate.

(4) $Var^Q(r_{i,t} | \varphi_{i,t-1}) = Var^P(r_{i,t} | \varphi_{i,t-1})$ almost surely with respect to the objective measure P.

In this assumption, under the two measures, the conditional variances are required to be equal. This is desirable because the conditional variance can be observed and estimated under measure *P*. This property and the fact that the conditional mean can be replaced by the risk-free rate yield a well-specified model that does not locally depend on preferences. Under these conditions, the locally risk-neutral valuation relationship holds (Duan,1995), and the log-return process in Equation (3) can be transformed into the one under the risk-neutral environment, the law of the returns under the risk-neutral probability measure *Q*, is given by:

$$\begin{aligned} r_{i,t} &= r_f - \frac{1}{2} h_{i,t} + \xi_{i,t}, \\ h_{i,t} &= \omega_i + \alpha_i (r_{i,t-1} - \mu_i)^2 + \beta_i h_{i,t-1}, \\ \mathcal{L} & \qquad {}_{,t-1}) = N(0, h_{i,t}), \end{aligned} \tag{4}$$

Equation (4) provides a relatively easy transformation from the objective model to the risk-neutral one. According to this equation, the terminal asset price is derived in the following corollary :

Lemma 3 (Duan (1995)) When the locally risk-neutral valuation relationship holds, the terminal price for the $i-th(i=1,2)$ asset can be expressed as:

$$S_{i,T} = S_{i,t} \exp[(T-t) r_f - \frac{1}{2} \sum_{s=t+1}^{T} h_{i,s} + \sum_{s=t+1}^{T} \xi_{i,s}]. \tag{5}$$

Therefore, under the locally risk-neutral probability measure Q, the call-on-max option with exercise price K maturing at time T has the t -time value :

$$COM_t = e^{-(T-t) r_f} E^Q[max(max(S_{1,T}, S_{2,T}) - K, 0)]. \tag{6}$$

D. H-transformation

Tukey (1977) [15] introduced the *g* and *h* family of distributions in 1977 and Hoaglin (1985) [22] discussed them in 1985. Tukey suggested to transform the standard Gaussian variable Z by

$$T_{g,h}(Z) = (\exp(gz) - 1) / g \cdot \exp(hZ^2 / 2) \tag{7}$$

to generate a family of distributions. Here g is a real constant and h is a nonnegative real constant.

When $h = 0$, we get the g distribution, which is given by Equation (8) is a nonlinear transform of a standard Gaussian random variable Z and is parameterized by g. The g distribution usually be used to measure the skewness of the distribution.

$$T_{g,0}(Z) = (\exp(gz) - 1) / g. \tag{8}$$

When $g > 0$, increased g implies greater skewness to the right. For $T_{-g,0}(Z) = -T_{g,0}(-Z)$, the same amount of skewness to the left can be obtained for $g < 0$. Therefore, g transformation is a skewness transformation.

For $(\exp(gZ) - 1) / g = Z + gZ^2 / 2! + gZ^3 / 3! + \cdots$, when $g = 0$, we will get the h distribution

$$T_{0,h}(Z) = Z \cdot \exp(hZ^2 / 2). \tag{9}$$

h transformation is a kurtosis transformation, there are no transformation takes place if $h = 0$. Positive values of h produce positive tail elongation and the distribution of $T_{g,h}(Z)$ is symmetric with increasingly heavy tails as h increases.

In brief, the skewness of $T_{g,h}(Z)$ depends on the factor g and the elongation of $T_{g,h}(Z)$ depends on the factor *h*. A linear transformation of $T_{g,h}(Z)$ given by Equation (7) accounts for the location (mean) and scale (variance) of the distribution :

$$X = A + B \cdot T_{g,h}(Z), \tag{10}$$

where X is the transformed variable and A and B are location and scale parameters. In the present paper, we only take account of the H -transformation. Therefore, the financial rate is expressed by Equation (11) :

$$r = A + B \cdot Z \cdot \exp(hZ^2 / 2). \tag{11}$$

III. GARCH Process with Generalized Normal Distribution

Using the Gaussian distribution as a statistical model for data set is widespread in practice, however, departure from normality seems to be more the rule than the exception. For example, the distribution of continuous returns of financial data displays more kurtosis than that permitted under the assumption of normality. Tukey introduced the *H*-distribution whose tails are heavier than those of the Gaussian distribution.

The subject of financial time series analysis has attracted substantial attention in recent years, especially the GARCH model. But recent empirical studies of high frequency financial time series indicate that the tail behavior of GARCH models remains too short even with standardized

Student-t innovations. For the expression of financial data is reasonable, we take the *H*-distribution to the GARCH model.

First, we shall estimate the expression of financial rate r by Equation (11). To estimate h, Equation (11) for the pth and (1–p)th percentiles of r is

$$r_p = A + B \cdot Z_p \cdot \exp(hZ_p^2/2), \qquad (12)$$

$$r_{(1-p)} = A + B \cdot Z_{(1-p)} \cdot \exp(hZ_{(1-p)}^2/2), \qquad (13)$$

For $Z_p = -Z(1-p)$, subtracting (12) from (13), we get

$$[r_p - r_{(1-p)}]/2Z_p = B \cdot \exp(hZ_p^2/2). \qquad (14)$$

Left side of Equation (14) is called the pseudosigma (p-sigma). Pseudosigmas can be used to measure the extent to which a distribution is more elongated than the Gaussian. A value for the p-sigma greater than one implies a distribution with thicker tails compare to the normal distribution. From (14), we shall find that $\ln(p - sigma)$ against $Z_p^2/2$ is the linear function to $1/Z_p^2$. Therefore, numerical estimates of h and $\ln B$ are obtained by regressing ln (p-sigma) against $Z_p^2/2$ for selected percentiles. For

$$r_{0.5} = A + B \cdot Z_{0.5} \cdot \exp(hZ_{0.5}^2/2) = A, \qquad (15)$$

the median of r provides a good estimate for A.

In practice, we use several selected letter values of p to obtain multiple estimates of h, take the median of the h_p estimates as an overall h value. Then, we get the expression of r. Next, for a numerical value of the financial time series r_t, we can get corresponding value of normal variable Z_t, we estimate the GARCH model by samples Z_t.

IV. Numerical Experiments

We use the data of NYSE composite index and S&P500 index, the sample contains 1508 daily observation from January 2, 2001 to January 3, 2007.

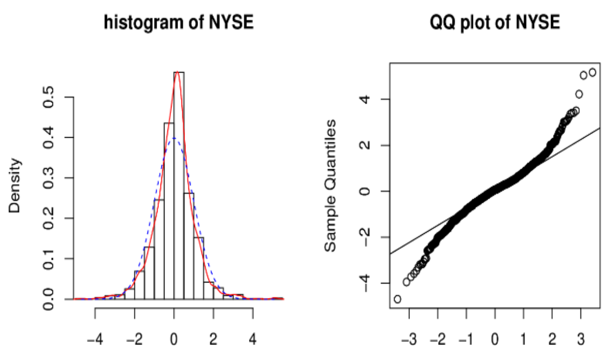

Fig. 1. Histogram and QQ plot of NYSE.

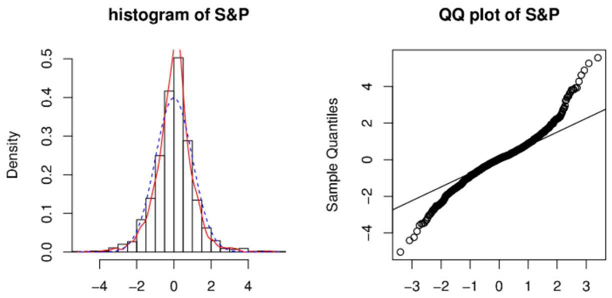

Fig. 2. Histogram and QQ plot of S&P.

Fig.1 and Fig.2 depict histogram and QQ plot of NYSE and S&P respectively. Note that the solid curve of histogram is empirical density function and the dashed curve of histogram is standard normal density function. From histogram, we can find that the returns of two indexes both have higher kurtosis than normal distribution. Checking via QQ plot, we also can get the above conclusions. So supposing the financial return obeys normal distribution is unreasonable.

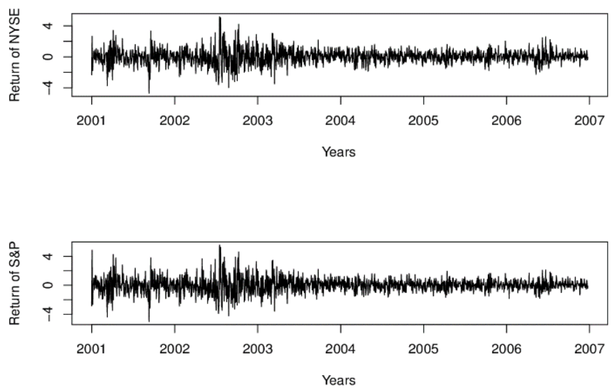

Fig. 3. Daily returns of NYSE and S&P500.

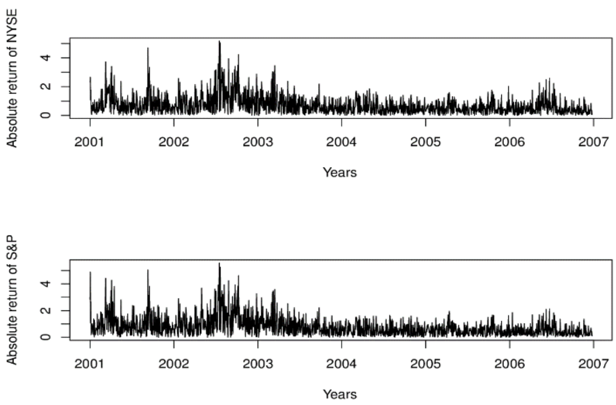

Fig. 4. Absolute daily returns of NYSE and S&P500.

Fig. 3 and Fig. 4 present the plots of return series and absolute return of series, the returns are in percentages. From the figures, we can find that there are a strong and obvious correlation between NYSE and S&P. We also can find that the returns of the indexes tend to have high excess kurtosis, and the mean of return series is close to zero, so the skewness is not a serious problem for daily returns. The phenomenon of volatility clustering can be found in the figures.

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Estimated by the method-of-percentiles, the expression of NYSE's rate is

$$r_1 = 0.0216 + 0.642Z \cdot \exp(0.2742Z^2 / 2), \quad (16)$$

and the expression of S&P500's rate is

$$r_2 = 0.0266 + 0.6231Z \cdot \exp(0.235Z^2 / 2). \quad (17)$$

We can see that the distribution of yield rate is high kurtosis. Then $Z_{i,t}(i=1,2,t=1,2,\cdots$ can be solved from Equation (16) and Equation (17). The parameter estimates of the improved GARCH (1,1) process for the marginal index return are listed in Table 1.

Table 1. Estimates of the improved GARCH parameters for marginal return process

Parameter	NYSE	S&P500
μ	0.0216 (0.0135)	0.0266 (0.0109)
$\omega \times 10^{-2}$	0.9164 (0.0515)	0.7626 (0.0423)
α	0.0558 (0.0149)	0.0596 (0.0306)
β	0.9340 (0.0254)	0.9327 (0.0173)

*Figures in brackets are standard errors.

The values for α and β nearly add up to one. These estimates are in line with previously reported values. It is noted that the outliers typically occur simultaneously and almost in the same direction. The parameter estimates of GARCH (1, 1) process whose innovations is normal for the marginal index return are listed in Table 2.

Table 2. Estimates of the GARCH parameters for marginal return process

Parameter	NYSE	S&P500
μ	0.0216 (0.0135)	0.0266 (0.0109)
$\omega \times 10^{-2}$	1.0497 (0.0515)	0.7229 (0.0423)
α	0.0839 (0.0237)	0.0851 (0.0340)
β	0.9047 (0.0135)	0.9087 (0.0220)

*Figures in brackets are standard errors.

Comparing Table 1 with Table 2, we found that β in improved models are obviously larger than in previous models, and α in improved models are obviously less than in previous models. It is noted that β will cause more volatility as it is larger.

For the dynamics in the dependence structure, Goorbergh (2005) [13] assumed that the dependence parameter evolves according to a particular time series regression equation. Specifically, let $\gamma(h_{1,t}, h2, t)$ be Kendall's tau at time t, Goorbergh supposes that $\gamma(h_{1,t}, h2, t)$ has a relationship with the conditional volatilities of the indexes The expression is:

$$\gamma(h_{1,t}, h_{2,t}) = \gamma_0 + \gamma_1 logmax(h_{1,t}, h_{2,t}), \quad (18)$$

The parameters γ_0 and γ_1 are estimated by regressing Kendall's tau, which calculated by the rolling window, on the estimated log maximum conditional volatility. Then, the estimated Kendall's tau is used to calculate the conditional copula parameter at time t, using the relationship between Kendall's tau and the copula's parameter:

$$\gamma = 4\int_0^1 \int_0^1 C(u,v) dC(u,v) - 1. \quad (19)$$

From Equation (19), we can find that only the copulas with one parameter can be used for Goorbergh's method. We use normal copula in this paper. The parameters γ_0 and γ_1 which estimated by improved GARCH and GARCH are listed in Table3:

Table 3. Estimates of the parameters for $\gamma(h_{1,t}, h2, t)$ }

Parameter	Improved GARCH	GARCH
γ_0	0.9075 (0.0297)	0.8947 (0.0253)
γ_1	0.0192 (0.0069)	0.0546 (0.0072)

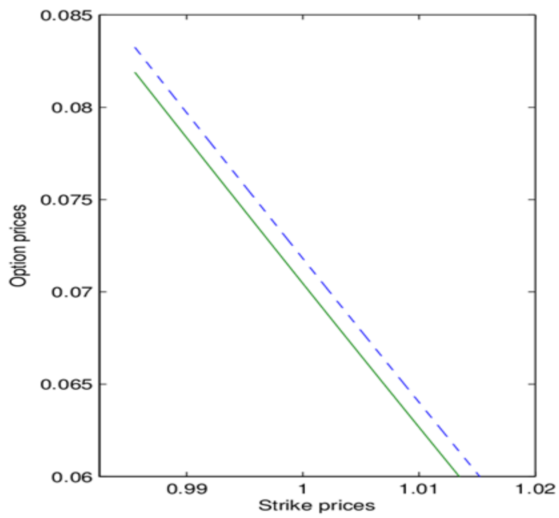

Fig. 5. One month maturity call-on-max option prices as a function of the strike using dynamic normal copulas

Then, the conditional normal copula can be fixed at time t by Equation (19). Return innovations are then generated from the dynamic normal copula and used to compute the bivariate option price. The initial volatility is defined as $\omega / (1 - \beta - \alpha)$, risk-free rate is assumed to be 4%, and the Monte Carlo study was based on 100,000 replications. Furthermore, 1 month maturity is assumed (20 trading days) and the strike price is set at levels from 0.98 to 1.02. In Fig.5 the upper curve is option pricing by improved normal GRACH process with dynamic normal copulas, the under curve is option pricing by normal GRACH process with dynamic normal copulas. The result for option pricing illustrates that the option prices obtained from improved normal GRACH process are higher than obtained from normal GRACH process.

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V. Conclusion

In this paper, an improved GARCH process for option pricing with dynamic copula model has been introduced. We introduce the theory of copula and Tukey's g and h distribution. Moreover, the drawbacks of GARCH process are summarized. From the empirical result, we can find that use the improved GARCH process to express financial marginal asset is reasonable. We use Goorbergh's time-varying copula model which are proposed for the valuation of claims on multiple assets. Contrary to earlier works on multivariate option pricing, the dependence structure was not taken as fixed, but rather as potentially varying with time. The time variation in the dependence structure was modeled using various parametric copulas by letting the copula parameter depend on the conditional volatilities of the underlying assets. Finally, we get option prices which are vary from pricing by normal GRACH process. Though Goorbergh's dynamic coupla models are novel, however there are some imitations: the accuracy of the regression Equation (18) and the restriction of the copula's family.

Appendix

According to Equation (3), since $R_{i,t}|\ \varphi_{t-1} = S_{i,t} / S_{i,t-1}|\ \varphi_{t-1} (i = 1, 2)$ is lognormally distributed under measure Q, it can be written as $\ln S_{i,t} / S_{i,t-1} = \omega_{i,t} + \xi_{i,t}$, where $\omega_{i,t}$ is the conditional mean and $\xi_{i,t}$ is a normal random variable with zero mean under the measure Q. And from the fourth condition of the Assumption, $h_{i,t} = Var^{p}(r_{i,t}|\ \varphi_{t-1}) = Var^{Q}(r_{i,t}|\ \varphi_{t-1})$. Therefore, we can obtain that

$$E^{Q}(S_{i,t} / S_{i,t-1}|\ \varphi_{t-1}) = E^{Q}(e^{\omega_{i,t}+\xi_{i,t}}|\ \varphi_{t-1})$$
$$= e^{\omega_{i,t}+Var^{Q}(r_{i,t})/2} = e^{\omega_{i,t}+h_{i,t}/2}.$$

From the third condition of Assumption, $E^{Q}(S_{i,t} / S_{i,t-1}|\ \varphi_{t-1}) = e^{r_f}$, then it follows that $\omega_{i,t} = r_f - h_{i,t} / 2$. By the preceding result and Equation (3), $\varepsilon_{i,t} = r_{i,t} - \mu_i$. Substituting $r_{i,t}$ and $\varepsilon_{i,t}$ into the conditional variance equation in Equation (3) yields the desirable result.

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