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Modelling GDP in Nigeria using Bayesian Model Averaging

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Abstract **– Statistical modelling has played significant roles in various field of endeavors and this has helped policy makers and planners to take decisive decision. This study attempts to model Gross Domestic Product (GDP) in Nigeria using Bayesian Model Averaging (BMA). It sought to know factors that have contributed to the growth of GDP as well as best model for modelling GDP in Nigeria. Essentially, empirical growth research faces a high degree of model uncertainty. Therefore, this study adopts the use of Bayesian Model Averaging in a bid to overcome the model uncertainty in a single model selection process. We obtained estimates of the posterior probabilities via Markov chain Monte Carlo (MCMC) which was further used as weights to model averaged estimates and predictions. Each model was weighted accordingly with a model prior and a parameter prior. Model uncertainty and posterior inclusion probability of each predictor was determined. The result shows that Exchange Rate is the most important variable affecting the GDP of the Nigerian economy followed by Interest Rate. The best model consists of an average of two predictors with Exchange Rate as one of the major contributor. The top five models were checked and it explained 40.16% uncertainty, which reveals the risk of depending on single model prediction. Every predictor, irrespective of its minute contribution provides some information which may be lost if the predictor is not considered.**

Keywords **– Posterior Inclusion Probabilities, Posterior Model Probabilities, Model Uncertainty, Gross Domestic Product.**

I. Introduction

Economic growth is a fundamental requisite to economic development. This informs why in Nigeria, growth continuously dominates the main policy thrust of government's development objectives. Essentially, economic growth is associated with policies aimed at transforming and restructuring the real economic sectors. Nevertheless, the lack of sufficient domestic resources, Savings and investment to support and sustain the sectors is a major impediment to economic development in the country because of the gap between savings and investment (Imimole and Imoughele (2012)).

After independence in 1960, the immediate challenge that faced the Nigerian economy was how to increase economic growth in order to reduce extreme poverty, improve health care, overcome illiteracy, strengthen democratic and political stability, improve the quality of the natural environment, diminish the incidence of crime and violence, and become an investment end of choice for international capital, ceteris paribus (Ismaila and Imoughele (2015)).

Ullah and Rauf (2013) noted that whenever there is increase in real GDP of a country it will boosts up the overall output and we called it economic growth. The economic growth is helpful to increase the incomes of the society, help the nation to bring the unemployment at low level and also helpful in the deliveries of public services. Macroeconomic policy refer to those policy of Government aimed at the aggregate economy, usually to promote the macro goals of full employment, stability, and growth. Common macroeconomic policies are fiscal and monetary. Fiscal policy is the macroeconomic policy where the government makes changes in government spending or tax to stimulate economic growth while monetary policy deals with changes in money supply or changes with the parameters that affects the supply of money in the economy. The objectives of this policy include the achievement of sustainable economic growth and development, stable price and full employment.

Raftery (1996) noted that Bayesian Model Averaging (BMA) is an empirical tool to deal with model uncertainty in various milieus of applied science. Shortly later, Fernandez et al (2001b) applied BMA procedure to examine the model uncertainty imbeded in the GDP across 72 Countries but with similar predictors. Olubusoye et al (2015) elicited a g - parameter prior and compared it with Fernandez et al (2001a) g - priors which it performed better in terms of it predictive ability. They also applied the BMA approech using this elicited g - parameter prior to the Nigerian Inflation process. In general, BMA is employed when there exist a variety of models which may all be statistically reasonable but most likely result in different conclusions about the key questions of interest to the researcher. In this situation, the standard approach of selecting a single model and basing inference on it underestimates uncertainty about quantities of interest because it ignores uncertainty about model form. Typically, though not always, BMA focuses on which regressors to include in the analysis, it also help to determine models, or more specifically sets of explanatory variables, which possess high likelihoods. Hoeting *et al*. (1999) introduced several methods for implementing BMA, discussed these methods and presented a number of examples.

In the recent time, the variables that contribute to gross domestic product in Nigeria have been on the increase. Single model equation could not estimate adequately the contribution of these factors to gross domestic product because of model uncertainty problem. With increase in these factors, there is the need to estimate the significant factors that contribute to GDP. As a result this study will employ BMA that has overcome the problem of model uncertainty. Also less attention has been given to using

BMA to model Nigeria GDP and this study intends to fill this gap.

II. Materials And Methods

A. Bayesian Model Averaging

When faced with model uncertainty, a formal Bayesian approach is to treat the model index as a random variable, and to use the data to conduct inference on it. Let us assume that in order to describe the data *y* we consider the possible models; M_j, j = 1 , . . . , J grouped in the model space *M*.

By averaging across a large set of models one can determine those variables which are relevant to the data generating process for a given set of priors used in the analysis. Each model (a set of variables) receives a weight and the final estimates are constructed as a weighted average of the parameter estimates from each of the models. BMA includes all of the variables within the analysis, but shrinks the impact of certain variables towards zero through the model weights.

These weights are the key feature for estimation via BMA and will depend on a number of key features of the averaging exercise including the choice of prior specified.

B. Bayesian Model Averaging for a Regression Model

Consider a linear regression model with a constant term, β_0, and k potential explanatory variables $X_1, X_2, \ldots, X_k$.

$$Y = \beta X + \varepsilon \tag{1}$$

where β is a nx1 parameter vector, *X* is a matrix of predictors n x p, $\varepsilon = \varepsilon_1, \ldots, \varepsilon_T$ is the disturbance vector and T is the sample size.

C. Posterior Inclusion Probabilities

The Posterior distribution of each coefficient of interest β_i, given the data D is

$$\Pr(\beta_i|D) = \sum_{i=1}^{2^k} \Pr(\beta_i|M_i)\Pr(M_i|D) \tag{2}$$

D. Posterior Model Probabilities

The Posterior model probability $\Pr(M_i|D)$ of M_i is the ratio of its marginal likelihood to the sum of marginal likelihoods over the entire model space which is given by

$$\Pr(M_i|D) = \frac{\Pr(D|M_i)\Pr(M_i)}{\Pr(D)} = \frac{\Pr(D|M_i)\Pr(M_i)}{\sum_{j=1}^{2^k}\Pr(D|M_j)\Pr(M_j)} \tag{3}$$

E. The Integrated Likelihood

The likelihood function of model, M_i , $\Pr(D|\beta_i, M_i)$, summarizes all the information about β_i that is provided by the data, D. The integrated likelihood (also known as the marginal likelihood) is the probability density of the data, conditional on the model M_i, which equals the likelihood times the prior density, $\Pr(\beta_i|M_i)$, integrated over the parameter space, so that we have equation (10) which follows from the law of total probability.

$$\Pr(D|M_i) = \int \Pr(D|\beta_i, M_i)\Pr(\beta_i|M_i)d\beta_i \tag{4}$$

The integrated likelihood is the crucial ingredient in deriving the model weight for model averaging.

β_i is the vector of parameters from model M_i,
$\Pr(\beta_i|M_i)$ is the prior density of β_i under model M_i,
$\Pr(D|\beta_i, M_i)$ is the likelihood
$\Pr(M_i)$ is the prior probability that M_i is the true model.

F. Priors Specification

According to Clyde (2000), "assigning the prior distributions on both the parameters and model space is perhaps the most difficult aspect of BMA."

F1. Assigning Model Prior

The obvious difficulty with assigning priors for the model parameters in BMA is that using non-informative (improper) priors will result in the improper predictive distributions Pr(D|M) and we cannot interpret them as model probabilities, nor can we interpret their ratios as Bayes Factors which is the key tool in comparing the models. Here we primarily focus on the simple case of normal linear regression, where a uniform prior probability is used for each model to explain the lack of prior information and this follows from the rule of thumb. Although many authors proposed various kinds of informative model priors but for this study uniform model prior is used which is given as:

$$\Pr(M_i) = 1/2^K \tag{5}$$

where $\Pr(M_i) > 0$ and $\sum_{i=1}^{M}\Pr(M_i)$

F2. Assigning the Parameters Prior

Theo S Eicher *et al*. (2011) addressed the issue of which default prior to use for BMA in linear regression, compared 12 candidate parameter priors: the unit information prior (UIP) to the integrated likelihood, a proper data-dependent prior, and 10 priors considered by Fern´andez *et al*. (2001a) and concluded that from the literature UIP with uniform model prior generally outperformed the other priors considered. Hence, the choice of Unit Information Prior (UIP) will be used for this study.

G. Posterior Mean and Variance

The estimated posterior mean and variance of β_i are given as

$$E(\beta|D) = \sum_{i=1}^{2^k}\hat{\beta}_i\Pr(M_i|D) \tag{6}$$

$$V(\beta|D) = \sum_{i=1}^{2^k}\left(var[\beta|D, M_i] + \hat{\beta}_i^{\,2}\right)Pr\,(M_i|D) - E[\beta|D]^2 \tag{7}$$

where $\hat{\beta}_i = E[\beta|D, M_i]$

H. BMA Predictive Performance

One of the main arguments for using the BMA is based on its ability to improve our predictions, as measured by the out-of-sample prediction error. In Hoeting et al. (1999) there are several examples of BMA applications, each equipped with a convincing out-of-sample validation in terms of its predictive performance. The cross-validation was performed by splitting each data set into two parts, training set, D^T and prediction set, D^P. The training set is used for model selection and the second set for prediction. Two measures of predictive ability were used, the coverage for 90% predictive interval, measured by proportion of observation of the second set falling within the 90% of the

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corresponding posterior prediction interval (Hoeting *et al.*, 1999).

H1. Log Predictive Score Rule

The second measure is the logarithmic scoring rule of Good (1952). Specifically we measure the predictive ability of a single model M as

$$-\sum_{\beta \in D^P} log\{pr(\beta | M, D^T)\} \qquad (8)$$

And compare it with predictive ability of BMA as measured by

$$-\sum_{\beta \in D^P} \left[\log\{\sum_{i=1}^{I} pr(\beta | M_i, D^T) pr(M_i | D^T)\}\right] \qquad (9)$$

The smaller the predictive log score for a given model or model average, the better the predictive performance.

III. Experimental Work

A. Presentation of the Posterior Inclusion Probabilities

This section presents the result of analysis as well as some discussions. Annual data spanning from 1985 to 2015 obtained from Central Bank of Nigeria (CBN) statistical bulletin and World Development Indicators (WPI) were used for the study. Real Gross Domestic Product (RGDP) is the endogenous variable while Stock Exchange (SE), Capital Expenditure (CE), Import (IMP.), Export (EXP.), Exchange Rate (EXC), Interest Rate (IR), Inflation (INF.), Money Demand (M1), Money Supply (M2), Balance of Trade (BOT), Real Effective Exchange Rate (REER), Net Income from Abroad (NIFA), Crop Production (CRP), Foreign Direct Investment (FDI) and Interest Paid on external Debt (IPEDT) are the exogenous variables.

Table 1: Summary of Posterior Probabilities for each Regressors.

Regressors	Prior Inclusion Probability	Posterior Mean	Posterior Standard Deviation
Exchange Rate	0.61856	2.455297e-02	2.480879e-02
Interest Rate	0.38142	1.345672e-01	2.344887e-01
Net Income from Abroad	0.29468	-5.942437e-13	1.398141e-12
Real Effective Exchange Rate	0.28894	3.088992e-03	7.371407e-03
Money Supply	0.28832	-1.281109e-04	4.157401e-04
Import	0.27174	-1.726761e-04	5.389241e-04
Crop Production	0.26602	7.858175e-03	1.989121e-02
Stock Exchange	0.24032	1.723884e-05	4.955044e-05
Money demand	0.23952	9.618409e-05	1.142948e-03
Inflation	0.23364	-8.906617e-03	2.505230e-02
Export	0.21976	1.276587e-05	3.060463e-04
Foreign Direct Investment	0.20090	-1.324803e-01	5.216353e-01
Capital Expenditure	0.18948	-4.126355e-04	2.416113e-03
Balance of Trade	0.16896	-1.408354e-08	1.488392e-07
Interest Paid on External Debt	0.16038	2.831251e-11	2.892761e-10

Table 1 contains the Posterior Inclusion Probabilities, the posterior means and the posterior standard deviations of each regressor in the model. These parameter estimates and standard deviations directly incorporate model uncertainty. This table shows that the Exchange Rate with PIP 62% is very important in modeling Nigerian GDP. It indicates that for any (Nigerian) GDP model selection, Exchange Rate should always be present. All other PIPs are less than 50% although they have some explanatory power as well.

Table 2: Summary of the Model Averaging

Mean no. of Regressors 4.0603	Draws 50000	Burnins 10000
No. of Models Visited 28454	Model Space 2^k 32768	Correlation PMP 0.9653
Model Prior uniform / 7.5	g-Prior UIP	Shrinkage-Statistic Average =0.9688
% visited 87	% Top Models 60	No. Obs. 31

B. Selected Top Models

Table 3: Posterior Model Probabilities of some selected Top Models

Models	SE	CE	IMP	EXP	EXC	IR	INF	M1	M2	BOT	REER	FDI	NIFA	IPEDT	CRP	PMP (Exact)	PMP (MCMC)
1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0.01133	0.00968
2	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0.01130	0.01050
3	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0.01109	0.01308
4	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0.00736	0.00494
5	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0.00512	0.00468

Looking into which models actually perform better we check the top models which is a binary representation of all the models included. The table 4.3 above shows the important predictors (covariates) of the first five best models and their posterior probabilities among the 28454 models visited for both MCMC and Exact sampler. The model with the highest posterior probabilities represents only 1% of the total posterior probability, indicating that there is a fair amount of model uncertainty (99%). Thus selecting the best model (Model 1) as the true model will result in the loss of about 99% of information on GDP. This shows the risk of explaining GDP by a single model which is characterized by ignored model uncertainty and discarded covariates which leads to the loss of information.

C. Posterior Model Probabilities for the model averaging

Table 4: Posterior Model Probabilities of the five Top Models

PMP (Exact)	PMP (MCMC)
0.59836	0.59836

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In table 4 above the average values of PMP (Exact) and PMP (MCMC) are the same 0.59836, which is the probability weight of the posterior distributed across all models visited comprising of Exchange Rate, Import, Interest rate, money demand and Money supply which explain 59.84% of the Nigerian GDP.

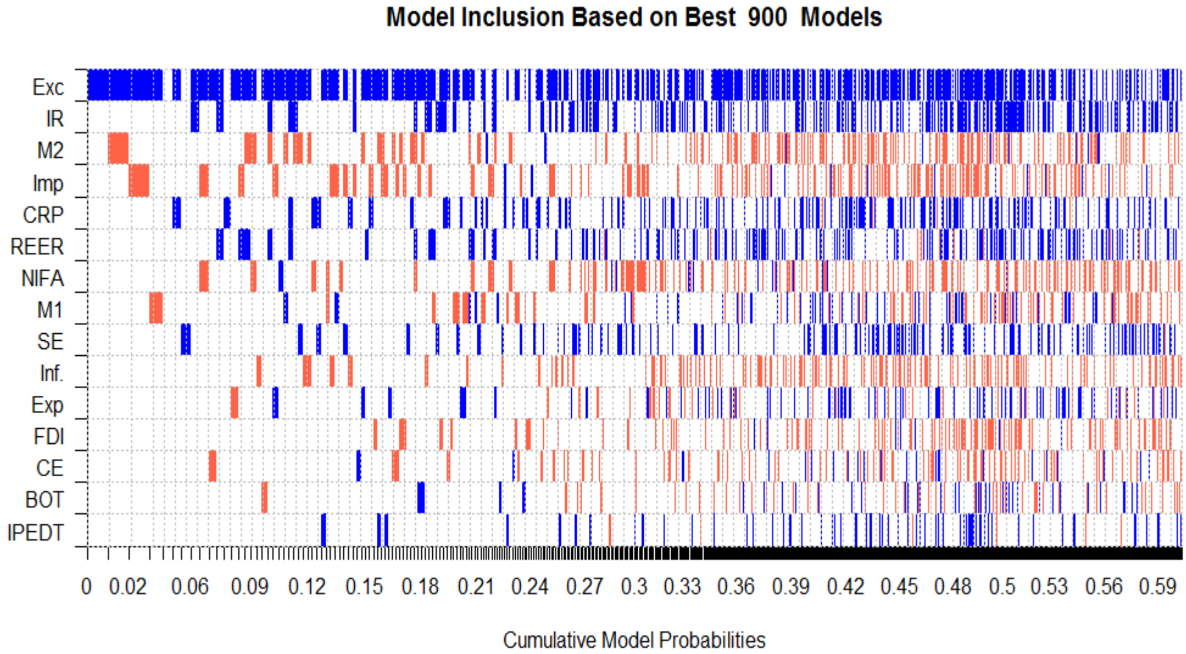

Fig. 1: Chart Showing Cumulative Probabilities on the best 900 Model

In order to get a more comprehensive overview of the models, we check the plot of the models. Fig. 1 shows the Cumulative Model Probabilities of inclusion where the blue colour implies a positive coefficient, red colour is a negative coefficient, and white colour implies non-inclusion (a zero coefficient). The horizontal axis shows the best models, scaled by their PMPs. This graphical representation of the inclusion or non-inclusion shows that Exchange Rate (Exc) is the most important variable followed by Interest Rate (IR) which are positively signed as indicated by the blue colour which imply a positive impact on the GDP that is, an increase in the variable will lead to an increase in the GDP. While M2 and Import (Imp) are also important but negatively signed which implies a negative impact on the Nigerian GDP. The white spaces indicate the non-inclusion of the corresponding variable implying that they have little or no significance.

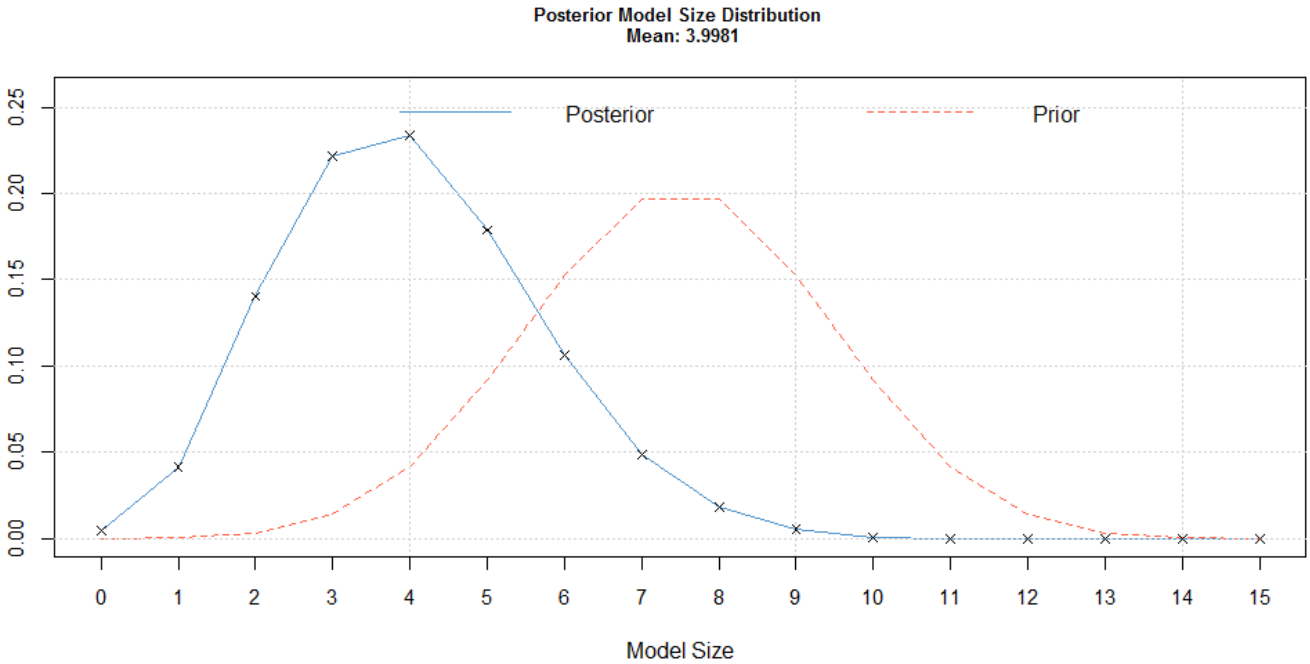

Fig. 2: Posterior Model Size Distribution with Uniform Prior

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Fig. 2 shows that the model prior is a symmetric distribution around K/2 = 7.5, updating it w the data yield a posterior that puts more importance on the mean models. Hence, the posterior model size distribution remains quite close to the model prior.

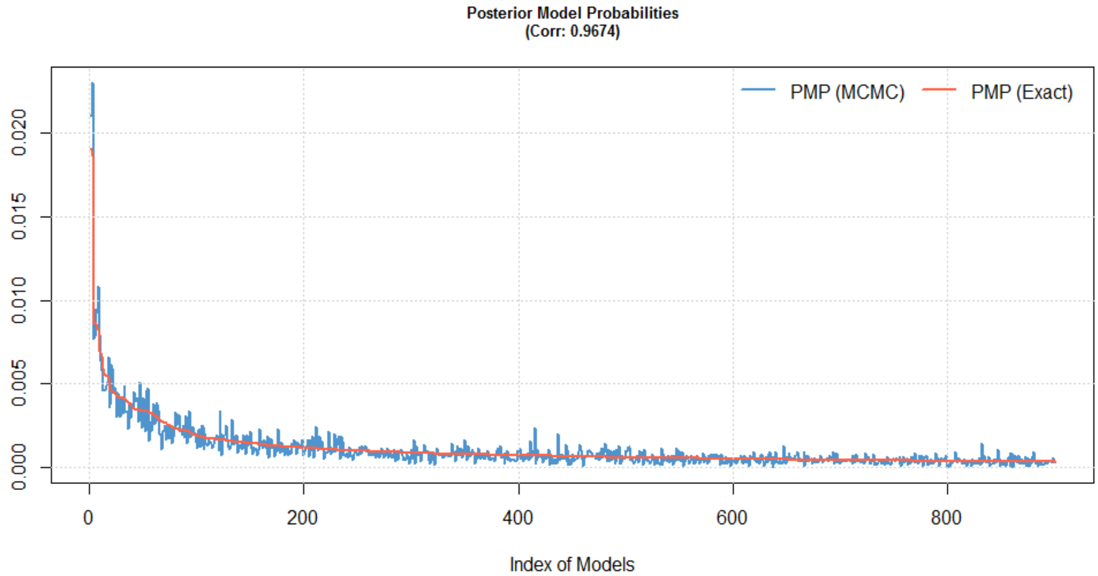

Fig. 3: Posterior Model Probabilities with Uniform Prior

Fig. 3 above displays the best 900 models ordered by their analytical PMP (the red line) and its MCMC iteration counts (the blue line) plot. The more complicated the distribution of the marginal likelihoods are the more difficult it is for the sampler to converge at a good approximation of PMPs and the quality of approximation may be inferred from the number of times a model got drawn versus their actual marginal likelihoods. Thus, the figure above shows the similarity between iteration count frequencies and analytical PMPs which converges at 0.9674. It indicates a good degree of convergence.

D. Predictive Performance of the Model

Since we are more impressed with a modelling strategy that consistently assigns higher probabilities to the event that actually occur. Thus, measuring how well a model predicts future observations is one way to judge the efficiency of BMA strategy. For the in sample prediction we have that the table below shows the distribution of the forecast for the last three years, conditional on what we know from other years, with their explanatory variables.

In the table below we assess the predictive performance specifically using the Log Predictive Score (LPS) which measures the predictive ability of an individual model, using the sum of the logarithms of the observed ordinates of the predictive density for each observation in the analysis. The smaller the predictive log score for a given model or model average, the better the predictive performance and as it is shown in the table below we have the predictive values to be small which gives a better predictive performance of the analysis.

Table 6: Predictive values for the Real GDP

Actual (RGDP)	5.49	6.22	2.79
Predicted (RGDP)	3.022642	2.383653	2.893194

IV. Conclusion

Bayesian Model Averaging was used to control the model uncertainty in this study. It serves as an efficient tool for discovering promising regressors and models by obtaining estimates of their posterior probabilities via Markov chain Monte Carlo (MCMC). BMA provides a better average predictive performance that takes account of important source of uncertainty in the selected model. The results indicate that the degree of convergence of iteration count frequencies and analytical PMPs is very high (0.97). The best model for Nigerian GDP has a posterior probability value of 0.11 which shows that the weight or probability spreads accross the entired 28454 models visited. In contrast, there are about 89% uncertainty that the chosen model is the true one. Exchange rate is the most robust auxiliary regressor affecting the economic growth, it implies that an increase in Exchange Rate, that is, when the rate appreciates (when the amount of Naira required to purchase a foreign currency reduces) brings a positive development in the Nigerian economy, followed by Interest Rate. It was found that it gives a 95% credible interval which is better calibrated than single-model confidence interval. Also, the BMA model gives good predictive performance when forecasting for the last three observations of the data.

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