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On the Analytic-Numeric Solution of System of Dynamics Drug Therapy and Harmonic Oscillator Models

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Abstract **– In this work, we present both analytical and numerical approaches to solve dynamics drug therapy and harmonic oscillator models. The procedures are being discussed and applied. The closed form numerical solutions obtained using Differential Transformation Method are compared with the analytical solutions of the models and are found to be very accurate and compatible. The results obtained have shown the ability of the methods for systems of differential equations.**

Keywords **– Analytic Numeric Approaches, Dynamics Drug Therapy Model, Harmonic Oscillator Model, Differential Transform Method.**

I. Introduction

A system of linear ordinary differential equations of the first order can be considered as

$$x_1^{/} = a_{11}x_1 + a_{12}x_2 + \dots\dots\dots\dots + a_{1n}x_n + g_1$$
$$x_2^{/} = a_{21}x_1 + a_{22}x_2 + \dots\dots\dots\dots + a_{2n}x_n + g_2$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \qquad (1)$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot$$
$$x_m^{/} = a_{m1}x_1 + a_{m2}x_2 + \dots\dots\dots\dots + a_{mn}x_n + g_m$$

Where $= d/dt$. Given are the functions $a_{ij}(t)$ and $g_t(t)$ on some interval a < t < b. The unknowns are the functions $x_1(t), x_2(t), \dots\dots x_n(t)$. The system is called homogeneous if all $g_j(t) = 0$, otherwise it is called non-homogeneous.

Matrix notation for systems. A non-homogeneous system of linear equations (1) is written as the equivalent vector-matrix system.

$$x^{/}(t) = A(t)x + g(t) \qquad (2)$$

Where

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{pmatrix}, \quad g(t) = \begin{pmatrix} g_1 \\ g_2 \\ \cdot \\ \cdot \\ g_n \end{pmatrix}, \quad A = \begin{pmatrix} a_{11} & a_{12} & \cdot & a_{1n} \\ & & & \\ a_{m1} & \cdot & \cdot\ \cdot & a_{mn} \end{pmatrix}$$

We restrict our study to the system of linear ordinary differential equations of the first order.

II. Analytical Approach

To demonstrate the analytical technique of solving first order system of differential equation, we consider matrix A of 2 × 2 constant element and X a 2 × 1 column vector of the form

$$F^{/} = Af \qquad (3)$$

Suppose we have two distinct real Eigen-values of A,

$$\frac{d}{dt}\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \qquad (4)$$

Let $F(t) = v\ell$ the first derivative is

$$F^{/}(t) = rv\ell^{rt} \qquad (5)$$

Where v and r are independent of t. Substitute (5) into (4), we obtained $rv\ell$ ℓ and upon cancelation of the exponential, we obtain the eigen-value problem,

$$Av = rv \qquad (6)$$

for eigenvalues r_k and corresponding eigenvectors v_k. We rewrite the eigenvalue equation (6) as

$$(A - \lambda I)v = 0 \qquad (7)$$

Where I is the $n \times n$ identity matrix. A nontrivial solution of (7) exists provided

$$\det(A - \lambda I)v \neq 0 \qquad (8)$$

Equation (8) is a $n\,th$ order polynomial equation in λ, and is called the characteristic equation of A. The characteristic equation can be solved for the eigenvalues and for each eigenvalue, a corresponding eigenvector can be determined directly from (6).

We can demonstrate how this works for the 2 × 2 matrix A of (3). We have

$$0 = (A - \lambda I)$$

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

$$(a-\lambda)(d-\lambda)-(cd)=0 \tag{9}$$
$$\lambda^2-(a+b)\lambda+(ad-cb)$$

This characteristic equation can be more generally written as

$$\lambda^2 - TrA\lambda + \det A = 0 \tag{10}$$

where TrA is the trace, or sum of the diagonal elements of the matrix A. If λ is an eigenvalue of A, then the corresponding eigenvector v may be found by solving

$$\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \tag{11}$$

where the equation of the second row will always be a multiple of the equation of the first row. The eigenvector v has arbitrary normalization, and we may always choose for convenience $v_1 = 1$ Using the principle of superposition, the general analytical solution of first order linear system ODE is

$$F(t) = c_1 v \ell^{\lambda_1 t} + c_2 v \ell^{\lambda_2 t} \tag{12}$$

In a scalar form, the general analytical solution is

$$\begin{aligned} f_1(t) &= c_1 v_1 \ell^{\lambda_1 t} + c_2 v_1 \ell^{\lambda_2 t} \\ f_2(t) &= c_1 v_2 \ell^{\lambda_1 t} + c_2 v_2 \ell^{\lambda_2 t} \end{aligned} \tag{13}$$

Here c_1 and c_2 are to be determined subject to initial conditions.

III. Numerical Approach

In this section, we consider analytic-numeric technique called Differential Transform Method. The differential transformation technique is one of the semi numerical analytical methods for system of ordinary and partial differential equations that use the form of polynomials as approximation solutions that are sufficiently differentiable. Its applicability for various kinds of differential equations are given in [1]-[5].

Suppose we consider an arbitrary function *f*(*t*) which can be expanded in Taylor series about a point t = 0 as

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(t)}{dt^k}\right]_{t=0} \tag{14}$$

Where $f(t)$ the original is function and $Y(k)$ is the transformation function. Here $\frac{d^k}{dt^k}$ means that k the derivate with respect to t.

The differential inverse transform of Y(k) is define

$$f(t) = \sum_{k=0}^{\infty} F(k)\, t^k \tag{15}$$

Substitute equations (14) in (15), we obtained

$$f(t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{d^k f(x)}{dt^k}\right]_{t=0} t^k \tag{16}$$

Equation (16) is called approximate solution of the function $f(t)$.

The fundamental mathematical operations performed by differential transform method are listed in Table 1

Table 1. One Dimensional Differential Transformation.

Functional Form	**Transformed Form**
f(t) = w(t) ± v(t)	F(k) = W(k) ± V(k)
f(t) = η v(t)	F(k) = η V(k) , η is a constant
f(t) = $\frac{d^n f(x)}{dt^n}$	F(k) = $\frac{(k+n)!}{k!} F(k+n)$
f(t) = ℓ^t	F(k) = $\frac{1}{k!}$
f(t) = ℓ^t	F(k) = $\frac{\lambda^k}{k!}$
f(t)= t	F(k) = $\delta(k-1)$
f(t) = t^m	F(k) = $\delta(k-n)$, δ is constant delta $\begin{cases}1, k=m \\ 0, k\neq m\end{cases}$
f(t) =sin $(ct+\beta)$	F(k) = $\frac{c^k}{k!}\sin\left(\frac{\pi k}{2}+\beta\right)$
f(t) = cos $(ct+\beta)$	F(k) = $\frac{c^k}{k!}\cos\left(\frac{\pi k}{2}+\beta\right)$

The Operation Properties of Differential Transformation

Suppose f(*t*), *y*(*t*),z(t) are three uncorrelated functions with time *t* and *F*(*k*), *Y* (*k*), Z(k) are the transformed functions corresponding to *f*(*t*), *y*(*t*), z(t) and the basic properties are shown as follows:

i. If *F*(*k*) = *D* [*f*(*t*)], *Y* (*k*) = *D* [*y* (*t*)], *Z* (*k*) = *D* [*z* (*t*) and c_1, c_2 and c_3 are independent of *t* and *k*, then

$$D\,[c_1\, f(t) + c_2\, y(t) + c_3\, z(t]] = c_1\, F(k) + c_2\, Y(k) + c_3\, Z(k) \tag{17}$$

(Symbol *D* denoting the differential transformation process).

ii. If $z(t) = f(t)y(t), f(t) = D^{-1}[F(k)], y(t) = D^{-1}[Y(k)]$ and $\otimes$ denote the convolution, the $D[z(t)] = D[f(t)y(t)] = F(k) \otimes Y(k) =$

$$\sum_{r=0}^{k} Y(r) - F(k-r) \tag{18}$$

iii. $f(t) = f_1(t) f_2(t) \ldots\ldots\ldots f_{n-1}(t) f_n(t)$ then

$$F(k) = \sum_{k_{n-1}}^{k} \sum_{k_{n-2}}^{k_{n-1}} \ldots\ldots \sum_{k_2=0}^{k=3} \sum_{k_1=0}^{k=2} F_1(k_1) F_2(k_2 - k_1) \ldots\ldots\ldots F_{n-1}(k_{n-1} - k_{n-2}) F_n(k - k_{n-1}) \tag{19}$$

IV. Application

Example 1. The Dynamics of the Drug Therapy Model

The human malady of ventricular arrhythmia or irregular heartbeat is treated clinically using the drug lidocaine. To be effective, the drug has to be maintained at a blood stream concentration of 1.5 milligrams per liter, but concentrations above 6 milligrams per liter are considered lethal in some patients. The actual dosage depends upon body weight. The adult dosage maximum for ventricular tachycardia is reported at 3 mg/kg 3. The drug is supplied in 0.5%, 1% and 2% solutions, which are stored at room temperature. A differential equation model for the dynamics of the drug therapy is.

$$\mathbf{F(t)} = \begin{cases} \dfrac{\mathbf{dx}}{\mathbf{dt}} = -\mathbf{0.09x(t)} + \mathbf{0.038y(t)} & \textbf{subject to } \mathbf{x(0) = 0} \\ \dfrac{\mathbf{dy}}{\mathbf{dt}} = \mathbf{0.066x(t)} - \mathbf{0.038y(t)} & \textbf{subject to } \mathbf{y(0) = 1} \end{cases} \tag{20}$$

Where $x(t)$ = Amount of lidocaine in the bloodstream,

$y(t)$ = Amount of lidocaine in body tissue.

1. *Analytical Approach*

We apply equation (9) to equation (20), which leads to

$$\begin{vmatrix} -0.09-\lambda & 0.038 \\ 0.066 & -0.038-\lambda \end{vmatrix} = 0$$

$$(-0.09-\lambda)(-0.038-\lambda) - (0.038)(0.066) = 0$$

$$\lambda^2 + 0.128\lambda + 0.000912,$$

$$\lambda_1 = -0.00757304$$

$$\lambda_2 = -0.12042694$$

The corresponding engen vectors v are

$$e_1 = \begin{pmatrix} -0.09+0.00757304 & 0.038 \\ 0.066 & -0.038+0.00757304 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$v_1 = 0.4610143018 \qquad v_2 = 1.0000000$$

Similarly, we have

$$e_2 = \begin{pmatrix} -0.09+0.12042694 & 0.038 \\ 0.066 & -0.038+0.12042694 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$v_1 = -0.248893076 \qquad v_2 = 1.0000000$$

e_1 and e_2 are Eigen-spaces for corresponding λ_1 and λ_2 respectively.

Consider the initial conditions, we obtained $c_1 =$ 0.7303863895 and $c_2 = 0.2696136105$.

Thus, the analytic solution is

$$x(t) = 0.3367185714\ell \qquad \ell$$

$$y(t) = 0.7303863895\ell \qquad \ell$$

2. *Numerical Approach*

Consider the fundamental operations of differential transformation method in Table 1, we obtained the following recurrence relation for the system of model equations (20).

$$\begin{cases} \mathbf{X(K+1)} = \dfrac{\mathbf{1}}{(\boldsymbol{k}+\mathbf{1})} [\![-\mathbf{0.09X(k)} + \mathbf{0.038Y(k)}]\!] \ \mathbf{X(0) = 0} \\ \mathbf{Y(K+1)} = \dfrac{\mathbf{1}}{(\boldsymbol{k}+\mathbf{1})} [\![\mathbf{0.066X(k)} - \mathbf{0.038Y(k)}]\!] \ \mathbf{Y(0) = 1} \end{cases} \tag{22}$$

The computational result is obtained to N = 19 **for example 1**

$$\begin{cases} X(0) = 0, \quad X(1) = \dfrac{19}{500}, \quad X(2) = -\dfrac{38}{15625} \\ X(3) = \dfrac{18373}{187500000}, \quad X(4) = -\dfrac{1729}{585937500} \\ X(5) = \dfrac{16657699}{2343750000000000} \quad X(6) = -\dfrac{15672169}{10986328125000000} \\ \vdots \\ X(19) = -\dfrac{44071037345894545312825607}{46583399713039398193359375000000000} \end{cases}$$

$$\begin{cases} Y(0) = 1, \quad Y(1) = -\dfrac{19}{500}, \quad Y(2) = \dfrac{247}{125000} \\ Y(3) = -\dfrac{589}{7500000}, \quad Y(4) = \dfrac{221521}{93750000000} \\ Y(5) = -\dfrac{13338019}{2343750000000000} \quad Y(6) = \dfrac{200781607}{17578125000} \\ \vdots \\ Y(19) = -\dfrac{70576156934783545121713883}{93166799426078796386718750} \end{cases}$$

Therefore, the closed form of the numerical solution can be written.

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$$\begin{cases} x(t) \approx 0 + \frac{19}{500}t \\ -\frac{38}{15625}t^2 \\ +\frac{18373}{187500000}t^3 \\ -\frac{1729}{585937500}t^4 \\ +\frac{16657699}{23437500000}t^5 \\ -\frac{15672169}{109863281250000}t^6 \\ +\frac{15098812213}{615234375000000}t^7 \\ -\frac{15098812213}{6152343750000000}t^8 \\ +\frac{12876084830719}{2162933349609300}t^{10} \\ -\frac{1240502036641 8853}{190338134765625000}t^{11} \\ +\frac{166729764729541}{2549171447753906250}t^{12} \\ -\frac{1124410337492762 4259}{1855796813964843750}t^{13} \\ +\frac{1057885161365675 5669}{20297777652740478 5156}t^{14} \\ -\frac{1019183016001364 3222293}{2435733318328857 4218750}t^{15} \\ +\frac{2634295499460222019}{8364468812942504882 81250}t^{16} \\ -\frac{9238033353747275481069139}{4140746641159057617 1875000}t^{17} \\ +\frac{5112629249128981539 15707}{3425249978899955749 5117187500}t^{18} \\ -\frac{440710373458945453128 25607}{465833997130393981933593750 0000000}t^{19} \end{cases} \tag{23}$$

$$\begin{cases} y(t) \approx 1 - \frac{19}{500}t \\ +\frac{247}{125000}t^2 \\ -\frac{589}{7500000}t^3 \\ +\frac{221521}{937500000}t^4 \\ -\frac{13338019}{23437500000}t^5 \\ +\frac{200781607}{1757812500000}t^6 \\ -\frac{2417951153}{123046875000}t^7 \\ +\frac{181991540641}{615234375000}t^8 \\ -\frac{10958342522419}{27685546875000}t^9 \\ +\frac{164959962542167}{346069335937500}t^{10} \\ -\frac{1986562415784 9361}{3806762695312500}t^{11} \\ +\frac{149522275397829361}{2855072021484375000}t^{12} \\ -\frac{9003255336962593219}{1855796813964843750000}t^{13} \\ +\frac{104253319764404013 79}{2498188018788281250 0000}t^{14} \\ -\frac{1632138130038309226769}{4871466366577148437500000}t^{15} \\ +\frac{122845879408522400719681}{48714666365771484375 0000000}t^{16} \\ -\frac{739697691508451551643 0419}{4140746641159057617875000000}t^{17} \\ +\frac{856533965500511318629 7299}{7166676878929138183593750 0000}t^{18} \\ -\frac{705761569347835451217 13883}{931667994260787963867 187500000}t^{19} \end{cases} \tag{24}$$

Example 2. Damped Harmonic Oscillator Model

When the mass slides over the table there will be a frictional force applied to the mass in the opposite direction of motion. Assuming it is proportional to the velocity of the mass, we obtain the following equation. $\frac{d^2x}{dt^2} + \frac{dx}{dt}\mu + \omega^2 x = 0$ which governs the motion of the mass.

Here $\omega = \sqrt{\frac{k}{m}}$, k is the constant of spring, m is the mass. The term $\frac{dx}{dt}\mu$ models the friction with the table where μ is the damping coefficient. One can convert this linear second order differential equation into a system of two first order differential equations by letting $v = \frac{dx}{dt}$, v is the velocity, thus we have

$$\boldsymbol{F(t)} = \begin{cases} \frac{dx}{dt} = v(t) & \text{subject to } x(0) = 0 \\ \frac{dv}{dt} = -\mu v(t) + \omega^2 x(t) & \text{subject to } v(0) = 1 \end{cases} \tag{25}$$

Where $\mu = 0.5$ and $\omega = \sqrt{\frac{k}{m}} = 0.8101405285$

3. *Analytical Approach*

$$\begin{vmatrix} 0-\lambda & 1 \\ 0.8101405285 & -0.5-\lambda \end{vmatrix} = 0$$

$$(-\lambda)(-0.5-\lambda) - (0.8101405285)(1.0) = 0$$
$$\lambda^2 + 0.5\lambda - 0.8101405285 = 0$$
$$\lambda_1 = 0.6841523048$$
$$\lambda_2 = -1.184152305$$

The corresponding engen vectors v are

$$e_1 = \begin{pmatrix} -0.6841523048 & 1 \\ 0.8101405285 & -1.184152305 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$
$$v_1 = 1.461662840 \qquad v_2 = 1.0000000$$

Similarly, we have

$$e_2 = \begin{pmatrix} 1.184152305 & 1 \\ 0.8101405285 & 0.68415230 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$
$$v_1 = -0.8444859576 \qquad v_2 = 1.0000000$$

e_1 and e_2 are Eigen-spaces for corresponding λ_1 and λ_2 respectively.

Thurs, the analytic solution of example 2 is

$$v(t) = 1.461662840\, C_1\, e^{0.6841523048\, t} - 0.8444859576\, C_2\, e^{-1.184152305\, t}$$

$$x(t) = C_1\, e^{0.6841523048\, t} + C_2\, e^{-1.184152305\, t}$$

Consider the initial value conditions to determine c_1 = 0.3661888421 and c_2 = 0.6338111579.

Thus, the analytic solution is

$$v(t) = 0.5352446229\, e^{0.6841523048\, t} - 0.5352446226\, e^{-1.184152305\, t} \tag{26}$$

4. *Numerical Approach*

Consider the fundamental operations of differential transformation method in Table 1, we obtained the following recurrence relation to the system of model equations (25),

$$\begin{cases} X(k+1) = \dfrac{V(k)}{(k+1)} & X(0) = 0 \\ V(k+1) = \dfrac{-(0.5)V(k) + (0.8101405285)\, X(k)}{(k+1)} & V(0) = 1 \end{cases} \tag{27}$$

We obtained the following:

$$\begin{cases} X(0) = 0 \\ X(1) = 1 \\ X(2) = -\dfrac{1}{4} \\ X(3) = \dfrac{212081057}{1200000000} \\ X(4) = -\dfrac{623427019}{1600000000} \\ X(5) = \dfrac{5305732289173037249}{48000000000000000000} \\ X(6) = -\dfrac{11366494225019111747}{5760000000000000000000} \\ \vdots \\ \vdots \\ \vdots \\ X(19) = \dfrac{68010961309620264983381571163152134263660}{622822914093219840000000000000000000000} \end{cases}$$

$$\begin{cases} V(0) = 1 \\ V(1) = -\dfrac{1}{2} \\ V(2) = \dfrac{2120281057}{4000000000} \\ V(3) = -\dfrac{623427019}{4000000000} \\ V(4) = \dfrac{5305732289173037249}{96000000000000000000} \\ V(5) = -\dfrac{11366494225019111747}{96000000000000000000} \\ V(6) = \dfrac{142800246341698743232100921 93}{5760000000000000000000000000000} \\ \vdots \\ \vdots \\ \vdots \\ V(19) = -\dfrac{25687897721667801106967731463842475 5}{198667596201984000000000000000000000} \end{cases}$$

Therefore, the closed form of the numerical solution can be written as

$$\begin{cases} x(t) \approx t - \dfrac{1}{4}t^2 \\ + \dfrac{212081057}{1200000000}t^3 \\ - \dfrac{623427019}{1600000000}t^4 \\ + \dfrac{53057322891730}{4800000000000000}t^5 \\ - \dfrac{11366494225019111}{576000000000000000}t^6 \\ + \dfrac{1428002463416987432323}{403200000000000000000000}t^7 \\ - \dfrac{389249284660336149402 86}{76800000000000000000000000}t^8 \\ + \dfrac{39486123363972920560888 1}{5806080000000000000000000000}t^9 \\ - \dfrac{18492871144878432259957915 6}{232243200000000000000000000000}t^{10} \\ + \dfrac{110210795563206520095667741959 0}{127733760000000000000000000000000}t^{11} \\ - \dfrac{144460577550638139938336274 12}{1703116800000000000000000000000000}t^{12} \\ + \dfrac{23737460317195195192048800224 3}{30656102400000000000000000000000000}t^{13} \\ - \dfrac{72990711124297868216124602916155}{1115882127360000000000000000000000000}t^{14} \\ + \dfrac{78631927312146990427867911882205}{15216574464000000000000000000000000000}t^{15} \\ - \dfrac{12188130155655993914685933747 2}{318823644960000000000000000000000000000}t^{16} \\ + \dfrac{242526697836710312932880016250 0}{910559815925760000000000000000000000000}t^{17} \\ - \dfrac{574296397549236101186524611175 1}{327801533733273600000000000000000000000}t^{18} \\ + \dfrac{68010961309620264983381571163152 13}{62282291409321984000000000000000000000}t^{19} \end{cases} \tag{28}$$

$$\begin{cases} v(t) \approx 1 - \dfrac{1}{4}t \\ + \dfrac{212081057}{1200000000}t^2 \\ - \dfrac{623427019}{4000000000}t^3 \\ + \dfrac{5305732289173037249}{96000000000000000000}t^4 \\ - \dfrac{11366494225019111747}{96000000000000000000}t^5 \\ + \dfrac{1428002463416987432321009 2}{576000000000000000000000000000}t^6 \\ - \dfrac{38924928466033614940286 1}{9600000000000000000000000000}t^7 \\ + \dfrac{394861233639729205608881921 91}{645120000000000000000000000000}t^8 \\ - \dfrac{18492871144878432256088819 2}{23224320000000000000000000000000}t^9 \\ + \dfrac{110210795563200956677419539498 2}{116121600000000000000000000000000}t^{10} \\ - \dfrac{144460577550638139938336274 12}{1419264000000000000000000000000000}t^{11} \\ + \dfrac{30858698412353749663440291677}{3065610240000000000000000000000000}t^{12} \\ - \dfrac{72990711124297868216155594444}{79705866240000000000000000000000000}t^{13} \\ + \dfrac{78631927312146990427869118822050}{1014438297600000000000000000000000000}t^{14} \\ - \dfrac{121881301556599391468593374727384}{19926466560000000000000000000000000000}t^{15} \\ + \dfrac{2425266978367103129321800162500705}{53562342113228000000000000000000000000}t^{16} \\ - \dfrac{5742963975492361011865246111751597 8}{182111963185152000000000000000000000000}t^{17} \\ + \dfrac{68010961309620264983338157116315213 7}{3278015337332760000000000000000000000000}t^{18} \\ - \dfrac{25687897721667801106967731463842 47}{1986675962019840000000000000000000000000}t^{19} \end{cases} \tag{29}$$

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Table 2. Numerical and Analytical results Dynamics of the drug therapy model

t	$x(t)$ Amount of lidocaine in the bloodstream		
Sec.	**Analytical solution**	**Numerical solution**	$E = \lvert x_A - x_N \rvert$
0.00	0.0000000000	0.0000000000	0.000000
4.00	0.1186712203	0.1186712204	1.10E-10
8.00	0.1884367893	0.1884367894	1.10E-10
12.0	0.2280976437	0.2280976436	1.10E-10
16.0	0.2492644878	0.2492644878	0.000000
20.0	0.2591065152	0.2591065151	1.10E-10
24.0	0.2620493671	0.2620493673	2.10E-10
28.0	0.2608240052	0.2608240085	3.310E-09
32.0	0.2571146270	0.2571146883	6.1310E-08
36.0	0.2490490992	0.2490509408	8.141610E-06
40.0	0.2459951859	0.2460002737	5.087810E-06

Table 3. Numerical and Analytical results Dynamics of the drug therapy model.

t	$y(t)$ Amount of lidocaine in body tissue		
Sec.	**Analytical solution**	**Numerical solution**	$E = \lvert y_A - y_N \rvert$
0.00	1.0000000000	1.0000000000	0.000000
4.00	0.8751408670	0.8751408670	0.000000
8.00	0.7903312387	0.7903312387	0.000000
12.0	0.7304904698	0.7304904698	0.000000
16.0	0.6862959921	0.6862959920	1.10E-10
20.0	0.6519823638	0.6519823640	2.10E-10
24.0	0.6239816770	0.6239816766	4.10E-10
28.0	0.6000836921	0.6000836894	2.710E-09
32.0	0.5789170257	0.5789169767	4.910E-08
36.0	0.5505135916	0.5505121192	1.472410E-06
40.0	0.5416859579	0.5416818853	4.072610E-06

Table 4. Numerical and Analytical results Damped harmonic oscillator model.

t	$x(t)$ Amount of Deformation (distance)		
Sec.	**Analytical solution**	**Numerical solution**	$E = \lvert x_A - x_N \rvert$
0.00	0.0000000000	0.0000000000	0.000000
0.60	0.5438918877	0.5438918872	5.10E-10
1.20	1.087218706	1.087218703	3.10E-09
1.80	1.770382839	1.770382838	1.10E-09
2.40	2.733494934	2.733494934	0.000000
3.00	4.152618071	4.152618090	1.910E-09
3.60	6.275898981	6.275899703	7.2210E-07
4.20	9.468942567	9.468957786	1.521910E-05
4.80	14.27874719	14.27896120	2.140110E-04
5.40	21.52789447	21.53009173	2.1972610E-03
6.00	32.45546672	32.47307500	1.76082810E-02

Table 5. Numerical and Analytical results Damped harmonic oscillator model

t	v(t) velocity covered		
Sec.	**Analytical solution**	**Numerical solution**	$E = \lvert y_A - y_N \rvert$
0.00	1.0000000000	1.0000000000	0.000000
0.60	0.8635075598	0.8635075598	3.10E-10
1.20	0.9852997685	0.9852997676	9.10E-10
1.80	1.3298737390	1.3298737370	2.10E-09
2.40	1.9284378010	1.9284377970	4.10E-09
3.00	2.8696773760	2.8696773520	2.410E-08
3.60	4.3077514790	4.3077506290	8.5010E-07
4.20	6.485118188	6.485100157	1.80310--05
4.80	9.772237963	9.771984495	2.5346810E-04
5.40	14.73002946	14.72742743	2.6020310E-03
6.00	22.20530342	22.18445153	2.08518910E-2

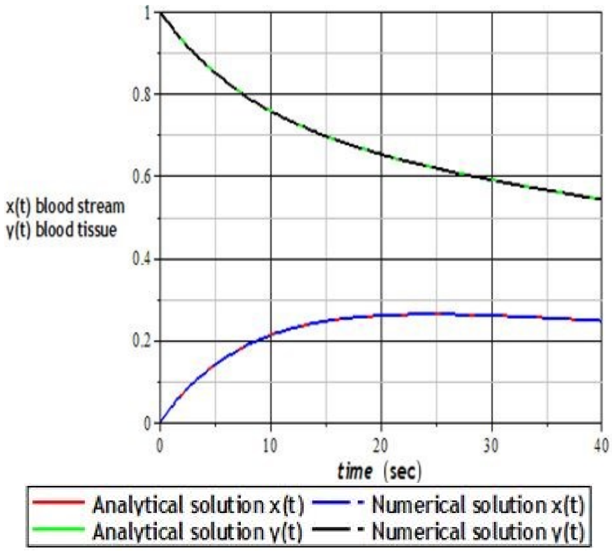

Fig. 1. Analytical vs Numerical solutions irregular heartbeats and lidocaine in the blood.

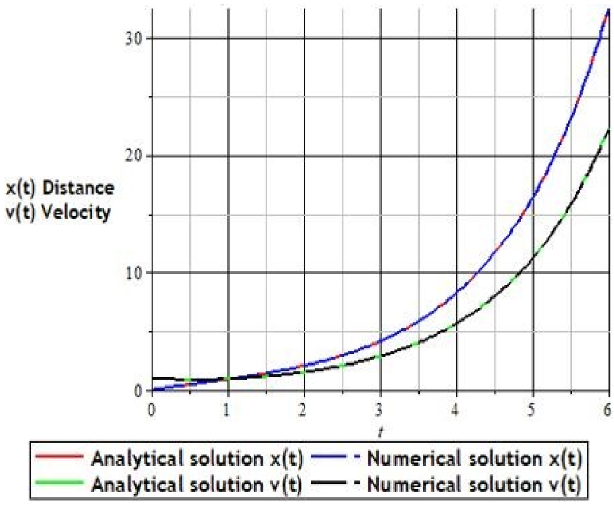

Fig. 2. Analytical vs Numerical solutions Damped harmonic oscillator model.

V. Conclusion

In this paper, we presented a reliable two approaches to solve the well-known dynamics of the drug therapy and damped harmonic oscillator models. The DTM was used in a direct way without using perturbation or restrictive assumptions. The numerical technique provides a closed-form approximation solution while the analytical approach proves a general form.

We conclude that both techniques are promising tool for solving linear systems of ordinary differential equations.

References

[1] J.K. Zhou, Differential Transformation and Its Applications for Electrical Circuits, Huazhong University Press, Wuhan, China, 1986. (In Chinese).

[2] C.L. Chen, S.H. Lin, C.K. Chen, Application of Taylor transformation to nonlinear predictive control problem, Applied Mathematical Modeling 20 (1996) 699–710.

[3] C.K. Chen, S.H. Ho, Application of differential transformation to eigenvalue problems, Applied Mathematics and Computation 79 (1996) 173–188.

[4] C. L. Chen, Y. C. Liu. Solution of two-point boundary value problems using the differential transformation method. J Opt Theory Appl., 99(1998):23–35.360 *A.-M. Batiha and B. Batiha*

[5] F. Ayaz. Applications of differential transform method to differential algebraic equations. Applied Mathematics and Computation, 152(2004): 649–657.

[6] Lecture notes on mathematical model training workshop at Centre for Energy Research and Development Obafemi Awolowo University, Ile-Ife February 5-8, 2013.

[7] Biazar E. Babolian b, R. Islam (2004) Solution of the system of ordinary differential equations by Adomian decomposition method Applied Mathematics and Computation 147: 713–719

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