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Fuzzy Soft Multiset Approach to Decision Making Problems

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***Abstract* – Molodtsov formulated soft set theory as a general mathematical tool for dealing with uncertainties about vague concepts. In this paper, we used the adjustable approach to fuzzy soft set based decision making. Using concrete and illustrative example, we presented an adjustable approach to fuzzy soft multiset based decision making problem by the use of level soft sets of fuzzy soft multisets. We also, introduced the weighted fuzzy soft multisets and examined its application to decision making problems.**
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***Keywords* – Soft Set, Fuzzy Set, Fuzzy Soft Multisets, Level Soft Set, Decision Making, Choice Value and Threshold.**

I. Introduction

Most of the complicated problems we are confronted with in real life, such as engineering, economics, environmental science, medical and social sciences have various levels of uncertainties and imprecision embedded in them. The solutions of such problems involve the use of mathematical principles based on uncertainties and imprecision. In order to solve these problems different theories were developed, such as theory of probability [1], theory of fuzzy set [2], theory of interval mathematics [3], theory of rough and vague sets [4], [5], respectively, which were considered as mathematical tools for dealing with uncertainties. But all these theories have their limitations in dealing with the uncertainties. The major problem associated with these theories is the inadequacies of the parameterization tools. To surmount these limitations, Molodtsov [6] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties and imprecision that is free from the difficulties that have troubled the traditional mathematical approaches. Molodtsov pointed out the application of soft set in several directions, such as game theory, operation research, perron integration among others. This theory has proven useful in many different fields such as decision making [7], data analysis [8], forecasting and so on.

Research on soft sets has been progressing, since its introduction by Molodtsov in 1999 up to the present time and several results have been achieved both in theory and applications.

In this paper, we used the adjustable approach to fuzzy soft set based decision making introduced by Feng *et al.* [10] and applied it to decision making in fuzzy soft multisets.

II. Preliminaries

2.1. *Fuzzy Set*

We recall the definition of the notion of fuzzy set by Zadeh [2]:

Definition 2.1.1.

Let A be a subset of X, μ_A called **indicator function** or **characteristic function** and is define as, $\mu_A: X \longrightarrow \{1, 0\}$ such that $\mu_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$.

This correspondence between a set and its indicator function is obviously a one-to-one correspondence.

Let U be a universe. A Fuzzy set X over U is a set defined by a function μ_X representing a mapping, $\mu_X: U \longrightarrow [0, 1]$.

μ_X is called the membership function of X, and the value $\mu_X(u)$ is called the grade of **membership** of $u \in U$and represents the degree of u belonging to the fuzzy set X. Thus a fuzzy set X over U, can be represented as follows:

$X = \left\{\frac{u}{\mu_X(u)} : u \in U, \mu_X(u) \in [0, 1]\right\}$ or
$X = \left\{\frac{\mu_X(u)}{u} : u \in U, \mu_X(u) \in [0, 1]\right\}$ or
$X = \{\langle u, \mu_X(u) \rangle : u \in U, \mu_X(u) \in [0, 1]\}$.

Example 2.1.2.

Let $U = \{h_1, h_2, h_3, h_4\}$. A fuzzy set X over U can be represented by
$X = \left\{\frac{h_1}{0.4}, \frac{h_2}{0.6}, \frac{h_3}{0.2}, \frac{h_4}{0.7}\right\}$.

2.2. *Soft Set*

We first recall some basic notions in soft set theory. Let U be an initial universe set, E be a set of parameters or attributes with respect to U, $P(U)$ be the power set of U and $A \subseteq E$.

Definition 2.2.1 [6]

A pair (F, A) is called a **soft set** over U, where F is a mapping given by $F: A \rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $x \in A$, $F(x)$ may be considered as the set of x-elements or as the set of x-approximate elements of the soft set (F, A).

The soft set (F, A) can be represented as a set of ordered pairs as follows:

$$(F, A) = \{(x, F(x)): x \in A, F(x) \in P(U)\}.$$

Example 2.2.1

Let $U = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ consisting of six students and $A = \{a_1, a_2, a_3\}$ be the set of parameters under consideration, where each parameter e_i, $i = 1,2,3$ stands for, brilliant, average, healthy, respectively. In this case to define a soft set means to point out brilliant students, aver-

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-age students and healthy students.

Such that $F(a_1) = \{S_1, S_2, S_5\}$, $F(a_2) = \{S_3, S_4, S_6\}$ and $F(a_3) = \{S_1, S_4, S_5, S_6\}$. Then the soft set (F, A) over U is given by

$(F, A) = \{(a_1, \{S_1, S_2, S_5\}), (a_2, \{S_3, S_4, S_6\}), (a_3, \{S_1, S_4, S_5, S_6\})\}$.

2.3 Fuzzy Soft Set

Let U be an initial universe set and E be a set of parameters (which are fuzzy words or sentences involving fuzzy words). Let $P(U)$ denotes the set of all fuzzy subsets of U, and $A \subseteq E$.

Definition 2.3.1 [7].

A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F: A \longrightarrow P(U)$. $\tilde{\Gamma}$ is called fuzzy approximation function of the fuzzy set (F, A) and the values $F(x)$ are fuzzy subsets of U, $\forall\, x \in A$. Therefore, a fuzzy soft set (F, A) over U can be represented by the set of ordered pairs $(F, A) = \{(x, F(x)) : x \in A, F(x) \in P(U)\}$.

Example 2.3.1.

Suppose that $U = \{h_1, h_2, h_3, h_4, h_5\}$ be a universe set and $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. $A = \{x_1, x_2, x_3\} \subseteq E$, $F(x_1) = \left\{\frac{h_2}{0.8}, \frac{h_4}{0.6}\right\}$, $F(x_2) = U$ and $F(x_3) = \left\{\frac{h_1}{0.3}, \frac{h_4}{0.4}, \frac{h_5}{0.9}\right\}$, then the fuzzy soft set (F, A) is written as, $(F, A) = \left\{\left(x_1, \left\{\frac{h_2}{0.8}, \frac{h_4}{0.6}\right\}\right), (x_2, U), \left(x_3, \left\{\frac{h_1}{0.3}, \frac{h_4}{0.4}, \frac{h_5}{0.9}\right\}\right)\right\}$.

2.4 Soft Multiset

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \biguplus_{i \in I} P(U_i)$, where $P(U_i)$ denotes the power sets of $U_i's$, $E = \biguplus_{i \in I} E_{U_i}$ and $A \subseteq E$.

Definition 2.4.1[9].

A pair (F, A) is called a soft multiset over U, where F is a mapping given by $F: A \longrightarrow U$.

In other words, a soft multiset over U is a parameterized family of subsets of U.

For $a \in A, F(a)$ may be considered as the set of a −approximate elements of the soft multiset (F, A). Based on the definition, any change in the order of the universes will produce a different soft multiset.

Example 2.4.1

Suppose that there are three universes U_1, U_2 and U_3. Let us consider a soft multiset (F, A) which describes the "attractiveness of houses", "cars" and "hotels" that Mr. X is considering for accommodation purchase, transportation purchase, and venue to hold a wedding celebration respectively.

Let $U_1 = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, $U_2 = \{c_1, c_2, c_3, c_4, c_5\}$ and $U_3 = \{v_1, v_2, v_3, v_4\}$.

Let $E_U = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \left\{\begin{matrix} e_{U_1},1 = expensive, e_{U_1},2 = cheap, e_{U_1},3 = beautiful, \\ e_{U_1},4 = wooden, e_{U_1},5 = in\ green\ surroundings \end{matrix}\right\},$$

$$E_{U_2} = \left\{\begin{matrix} e_{U_2},1 = expensive, e_{U_2},2 = cheap, e_{U_2},3 = Model\ 2000 \\ and\ above, \\ e_{U_2},4 = Black, e_{U_2},5 = Made\ in\ Japan, e_{U_2},6 = Made \\ in\ Malaysia \end{matrix}\right\},$$

$$E_{U_3} = \left\{\begin{matrix} e_{U_3},1 = expensive, e_{U_3},2 = cheap, e_{U_3},3 = majestic, \\ e_{U_3},4 = in\ Kuala\ Lumpur, e_{U_3},5 = in\ Kajang \end{matrix}\right\}.$$

Let $U = \biguplus_{i=1}^{3} P_i(U_i)$, $E = \biguplus_{i=1}^{3} E_{U_i}$ *and* $A \subseteq E$, such that

$$\begin{aligned} A = \{a_1 &= (e_{U_1},1, e_{U_2},1, e_{U_3},1), a_2 = (e_{U_1},1, e_{U_2},2, e_{U_3},1), \\ a_3 &= (e_{U_1},2, e_{U_2},3, e_{U_3},1), a_4 = (e_{U_1},5, e_{U_2},4, e_{U_3},2), \\ a_5 &= (e_{U_1},4, e_{U_2},3, e_{U_3},3), a_6 = (e_{U_1},2, e_{U_2},3, e_{U_3},2), \\ a_7 &= (e_{U_1},3, e_{U_2},1, e_{U_3},1), a_8 = (e_{U_1},1, e_{U_2},3, e_{U_3},2)\}. \end{aligned}$$

Suppose that

$F(a_1) = (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\})$,
$F(a_2) = (\{h_2, h_3, h_6\}, \{c_1, c_3, c_4, c_5\}, \{v_2\})$,
$F(a_3) =$
$(\{h_1, h_4, h_5\}, \{c_1, c_3\}, \emptyset)$,

$F(a_4) = (\{h_1, h_4, h_6\}, \emptyset, \{v_1, v_4\})$,
$F(a_5) = (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\})$,
$F(a_6) = (\{h_1, h_4, h_5\}, \{c_1, c_3\}, U_3)$,
$F(a_7) = (\{h_1, h_4\}, \emptyset, \{v_3\})$,
$F(a_8) = (\{h_2, h_3, h_6\}, \{c_1, c_3\}, \{v_1, v_4\})$.

Then we can view the soft multiset (F, A) as consisting of the following collection of approximations:

$$\begin{aligned} (F, A) = \{&(a_1, (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\})), \\ &(a_2, (\{h_2, h_3, h_6\}, \{c_1, c_3, c_4, c_5\}, \{v_2\})), \\ &(a_3, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, \emptyset)), (a_4, (\{h_1, h_4, h_6\}, \emptyset, \{v_1, v_4\})), \\ &(a_5, (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\})), (a_6, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, U_3)), \\ &(a_7, (\{h_1, h_4\}, \emptyset, \{v_3\})), (a_8, (\{h_2, h_3, h_6\}, \{c_1, c_3\}, \{v_1, v_4\}))\}. \end{aligned}$$

III. FUZZY SOFT MULTISET

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of set of parameters. Let $U = \biguplus_{i \in I} FS(U_i)$, where $FS(U_i)$ denotes the set of all fuzzy subsets U_i, $E = \biguplus_{i \in I} E_{U_i}$ and $A \subseteq E$.

Definition 3.1[11].

A pair (F, A) is called a fuzzy soft multiset over U, where F is a mapping given by $F: A \longrightarrow U$.

In other words, a fuzzy soft multiset over U is a parameterized family of fuzzy subsets of U. For $a \in A, F(a)$ may be considered as the set of a −approximate elements of the fuzzy soft multiset (F, A). Based on the definition, any change in the order of the universes will produce a different fuzzy soft multiset.

Example 3.1

Suppose that there are three universes U_1, U_2 and U_3. Let us consider a fuzzy soft multiset (F, A) which describes the "attractiveness of houses", "cars" and "hotels" that Mr. X with a budget is considering for accommodation purchase, transportation purchase, and venue to hold a wedding celebration respectively.

Let $U_1 = \{h_1, h_2, h_3, h_4, h_5\}$, $U_2 = \{c_1, c_2, c_3, c_4\}$ and $U_3 = \{v_1, v_2, v_3\}$.

Let $E_U = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \left\{\begin{matrix} e_{U_1},1 = expensive, e_{U_1},2 = cheap, \\ e_{U_1},3 = wooden, e_{U_1},4 = in\ green\ surroundings \end{matrix}\right\},$$

$$E_{U_2} = \{e_{U_2},1 = expensive, e_{U_2},2 = cheap, e_{U_2},3 = Sporty\},$$

$$E_{U_3} = \left\{\begin{matrix} e_{U_3},1 = expensive, e_{U_3},2 = cheap, e_{U_3},3 = in\ Kuala \\ Lumpur, \\ e_{U_3},4 = Majestic \end{matrix}\right\}.$$

Let $U = \uplus_{i=1}^{3} P_i(U_i),\ E = \uplus_{i=1}^{3} E_{U_i}\ and\ A \subseteq E,$ such that

$$A = \{a_1 = (e_{U_1},1, e_{U_2},1, e_{U_3},1), a_2 = (e_{U_1},1, e_{U_2},2, e_{U_3},1), a_3 = (e_{U_1},2, e_{U_2},3, e_{U_3},1), a_4 = (e_{U_1},5, e_{U_2},4, e_{U_3},2), a_5 = (e_{U_1},4, e_{U_2},3, e_{U_3},3), a_6 = (e_{U_1},2, e_{U_2},3, e_{U_3},2),$$

Suppose that

$$F(a_1) = \left(\left\{\frac{h_1}{0.2},\frac{h_2}{0.4},\frac{h_3}{0.8},\frac{h_4}{0.5},\frac{h_5}{0}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.5},\frac{c_3}{0.4},\frac{c_4}{0..6}\right\},\left\{\frac{v_1}{0.8},\frac{v_2}{0.7},\frac{v_3}{0.7}\right\}\right),$$
$$F(a_2) = \left(\left\{\frac{h_1}{0.2},\frac{h_2}{0.4},\frac{h_3}{0.8},\frac{h_4}{0.5},\frac{h_5}{0}\right\},\left\{\frac{c_1}{0.4},\frac{c_2}{0.5},\frac{c_3}{0.8},\frac{c_4}{0.5}\right\},\left\{\frac{v_1}{0.4},\frac{v_2}{0.4},\frac{v_3}{0.3}\right\}\right),$$
$$F(a_3) = \left(\left\{\frac{h_1}{0.7},\frac{h_2}{0.7},\frac{h_3}{0.1},\frac{h_4}{0.8},\frac{h_5}{0.7}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6},\frac{c_3}{0.3},\frac{c_4}{0.5}\right\},\left\{\frac{v_1}{0.5},\frac{v_2}{0.4},\frac{v_3}{0.2}\right\}\right),$$
$$F(a_4) = \left(\left\{\frac{h_1}{0.9},\frac{h_2}{0.5},\frac{h_3}{0.5},\frac{h_4}{0.2},\frac{h_5}{0.7}\right\},\left\{\frac{c_1}{0},\frac{c_2}{0.2},\frac{c_3}{0.7},\frac{c_4}{0.6}\right\},\left\{\frac{v_1}{0.8},\frac{v_2}{0.7},\frac{v_3}{0.9}\right\}\right),$$
$$F(a_5) = \left(\left\{\frac{h_1}{0.9},\frac{h_2}{0.5},\frac{h_3}{0.5},\frac{h_4}{0.2},\frac{h_5}{0.7}\right\},\left\{\frac{c_1}{0.7},\frac{c_2}{0.8},\frac{c_3}{0.5},\frac{c_4}{0.4}\right\},\left\{\frac{v_1}{0.5},\frac{v_2}{0.5},\frac{v_3}{0.7}\right\}\right),$$
$$F(a_6) = \left(\left\{\frac{h_1}{0.7},\frac{h_2}{0.7},\frac{h_3}{0.1},\frac{h_4}{0.8},\frac{h_5}{0.7}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6},\frac{c_3}{0.3},\frac{c_4}{0.5}\right\},\left\{\frac{v_1}{0.8},\frac{v_2}{0.7},\frac{v_3}{0.6}\right\}\right).$$

Then we can view the fuzzy soft multiset (F, A) as consisting of the following collection of approximations:

$$(F,A) = \left\{\begin{matrix}
\left(a_1\left(\left\{\frac{h_1}{0.2},\frac{h_2}{0.4},\frac{h_3}{0.8},\frac{h_4}{0.5},\frac{h_5}{0}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.5},\frac{c_3}{0.4},\frac{c_4}{0.6}\right\},\left\{\frac{v_1}{0.8},\frac{v_2}{0.7},\frac{v_3}{0.7}\right\}\right)\right), \\
\left(a_2\left(\left\{\frac{h_1}{0.2},\frac{h_2}{0.4},\frac{h_3}{0.8},\frac{h_4}{0.5},\frac{h_5}{0}\right\},\left\{\frac{c_1}{0.4},\frac{c_2}{0.5},\frac{c_3}{0.8},\frac{c_4}{0.5}\right\},\left\{\frac{v_1}{0.4},\frac{v_2}{0.4},\frac{v_3}{0.3}\right\}\right)\right), \\
\left(a_3\left(\left\{\frac{h_1}{0.7},\frac{h_2}{0.7},\frac{h_3}{0.1},\frac{h_4}{0.8},\frac{h_5}{0.7}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6},\frac{c_3}{0.3},\frac{c_4}{0.5}\right\},\left\{\frac{v_1}{0.5},\frac{v_2}{0.4},\frac{v_3}{0.2}\right\}\right)\right), \\
\left(a_4\left(\left\{\frac{h_1}{0.9},\frac{h_2}{0.5},\frac{h_3}{0.5},\frac{h_4}{0.2},\frac{h_5}{0.7}\right\},\left\{\frac{c_1}{0},\frac{c_2}{0.2},\frac{c_3}{0.7},\frac{c_4}{0.6}\right\},\left\{\frac{v_1}{0.8},\frac{v_2}{0.7},\frac{v_3}{0.9}\right\}\right)\right), \\
\left(a_5\left(\left\{\frac{h_1}{0.9},\frac{h_2}{0.5},\frac{h_3}{0.5},\frac{h_4}{0.2},\frac{h_5}{0.7}\right\},\left\{\frac{c_1}{0.7},\frac{c_2}{0.8},\frac{c_3}{0.5},\frac{c_4}{0.4}\right\},\left\{\frac{v_1}{0.5},\frac{v_2}{0.5},\frac{v_3}{0.7}\right\}\right)\right), \\
\left(a_6\left(\left\{\frac{h_1}{0.7},\frac{h_2}{0.7},\frac{h_3}{0.1},\frac{h_4}{0.8},\frac{h_5}{0.7}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6},\frac{c_3}{0.3},\frac{c_4}{0.5}\right\},\left\{\frac{v_1}{0.8},\frac{v_2}{0.7},\frac{v_3}{0.6}\right\}\right)\right)
\end{matrix}\right\}.$$

Each approximation has two parts: a predicate name and an approximate value set.

We can logically explain the above example as follows: we know that

$a_1 = (e_{U_1},1, e_{U_2},1, e_{U_3},1)$, where $e_{U_1},1 = expensive\ house$, $e_{U_2},1 = expensive\ car$ and $e_{U_3},1 = expensive\ hotel$. Then

$$F(a_1) = \left(\left\{\frac{h_1}{0.2},\frac{h_2}{0.4},\frac{h_3}{0.8},\frac{h_4}{0.5},\frac{h_5}{0}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.5},\frac{c_3}{0.4},\frac{c_4}{0.6}\right\},\left\{\frac{v_1}{0.8},\frac{v_2}{0.7},\frac{v_3}{0.7}\right\}\right).$$

We can see that, the membership value for house h_1 is 0.2, so this house is not expensive for Mr. X; also we can see that the membership value for house h_3 is 0.8, this means that the house h_3 is expensive and since the membership value for house h_5 is 0, then this house is absolutely not expensive. Now, since the first set is concerning expensive houses, then we can explain the second set as follows: the membership value for car c_1 is 0.8, so this car is expensive (however, this car may not be expensive if the first set is concerning cheap houses), also we can see that the membership value for car c_3 is 0.4, this means that, this car is not very expensive for him and since the membership value for car c_4 is 0.6, then this car is quite expensive. Now, since the first set is concerning expensive houses and the second set is concerning expensive cars, then we can also explain the third set as follows: since the membership value for v_1 is 0.8, so this hotel is expensive (but this hotel may not be expensive if the first set is concerning cheap houses and the second set is concerning cheap cars), also we can see that, the membership value for venue v_2 and v_3 is 0.7, this means that these venues are almost expensive. Therefore, depending on the previous explanation we can say the following.

If the $\left\{\frac{h_1}{0.2},\frac{h_2}{0.4},\frac{h_3}{0.8},\frac{h_4}{0.5},\frac{h_5}{0}\right\}$ is the fuzzy set of expensive houses, then the fuzzy set of relatively expensive cars is $\left\{\frac{c_1}{0.8},\frac{c_2}{0.5},\frac{c_3}{0.4},\frac{c_4}{0.6}\right\}$ and if $\left\{\frac{h_1}{0.2},\frac{h_2}{0.4},\frac{h_3}{0.8},\frac{h_4}{0.5},\frac{h_5}{0}\right\}$ is the fuzzy set of expensive houses and $\left\{\frac{c_1}{0.8},\frac{c_2}{0.5},\frac{c_3}{0.4},\frac{c_4}{0.6}\right\}$ is the fuzzy set of relatively expensive cars, then the fuzzy set of relatively expensive hotels is $\left\{\frac{v_1}{0.8},\frac{v_2}{0.7},\frac{v_3}{0.7}\right\}$. it is obvious that, the relation in fuzzy soft multiset is conditional relation.

IV. Application of Fuzzy Soft Multiset in Decision Making Problems

Like most of the decision making problems, fuzzy soft multiset based decision making involves the evaluation of all the objects which are decision alternatives. Most of these problems are essentially humanistic and therefore subjective in nature (that is based on human understanding and vision system). In general, there actually does not exist a unique or uniform criterion for the evaluation of decision options.

4.1 Level Soft Sets of Fuzzy Soft Multiset

In this subsection, we present an approach to fuzzy soft multiset based decision making problems. This is based on the following concept called level soft set.

Definition 4.1.1.

Let $\mathcal{G} = (F, A)$ be a fuzzy soft multiset over U, where $A \subseteq E$ and E is the parameter set. For $t \in [0,1]$, the $t-level$ soft set of the fuzzy soft multiset $\mathcal{G}$ is a crisp soft set $L(\mathcal{G}; t) = (F_t, A)$ defined by

$F_t(a) = L(F(a); t) = \{x \in U: F(a)(x) \geq t\}$, for all $a \in A$.

In the definition above, $t \in [0,1]$ can be viewed as a given threshold on membership values. For real life applications of fuzzy soft multiset based decision making, usually these thresholds are chosen in advance by the decision makers and represent their requirements on membership levels.

In the definition of $t-level$ soft set, the level (or threshold) assigned to each parameter is always a constant value $t \in [0,1]$. But in some decision making problems, it may happen that decision makers would like to impose different thresholds on different decision parameters. To cope with such problems, we can use a function instead of a constant number as the threshold on membership values.

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Definition 4.1.2.

Let $\mathcal{G} = (F, A)$ be a fuzzy soft multiset over U, where $A \subseteq E$ and E is the parameter set. Let $\lambda: A \to [0,1]$ be a fuzzy set in A which is called **threshold fuzzy set.** The level soft set of the fuzzy soft multiset $\mathcal{G}$ with respect to the fuzzy set λ is a crisp soft set $L(\mathcal{G}; \lambda) = (F_\lambda, A)$ defined by

$$F_\lambda(a) = L(F(a); \lambda(a)) = \{x \in U: F(a)(x) \geq \lambda(a)\},$$

for all $a \in A$.

It is obvious that level soft set with respect to a fuzzy set generalize t −level soft sets by substituting a function on the parameter set A, namely a fuzzy set $\lambda: A \to [0,1]$, for a constant $t \in [0, 1]$.

Definition 4.1.3.

(The mid-level soft set of a fuzzy soft multiset). Let $\mathcal{G} = (F, A)$ be a hesitant fuzzy soft multiset over U, where $A \subseteq E$ and E is the parameter set. Based on the hesitant fuzzy soft multiset $\mathcal{G} = (F, A)$, we can define a fuzzy set $\widetilde{mid}_\mathcal{G}: A \to [0, 1]$ by $\widetilde{mid}_\mathcal{G}(a) = \frac{1}{|U|}\sum_{x\in U} F(a)(x)$, for all $a \in A$. The fuzzy set $\widetilde{mid}_\mathcal{G}$ is called the mid-threshold of the fuzzy soft multiset $\mathcal{G}$. In addition, the level soft set of $\mathcal{G}$ with respect to the mid-threshold fuzzy set $\widetilde{mid}_\mathcal{G}$, namely $L(\mathcal{G}; \widetilde{mid}_\mathcal{G})$ is called the mid-level soft set of ϖ and simply denoted by $L(\mathcal{G}; mid)$. In what follows the mid-level decision rule will mean using the mid-threshold and considering the mid-level soft set in fuzzy soft multiset based decision making.

Definition 4.1.4.

(The Top-level soft set of a fuzzy soft multiset). Let $\mathcal{G} = (F, A)$ be a fuzzy soft multiset over U, where $A \subseteq E$ and E is the parameter set. Based on the fuzzy soft multiset $\mathcal{G} = (F, A)$, we can define a fuzzy set $\widetilde{max}_\mathcal{G}: A \to [0, 1]$ by $\widetilde{max}_\mathcal{G}(a) = \max_{x\in U} F(a)(x)$, for all $a \in A$. The fuzzy set $\widetilde{max}_\mathcal{G}$ is called the max-threshold of the fuzzy soft multiset $\mathcal{G}$. In addition, the level soft set of $\mathcal{G}$ with respect to the max-threshold $\widetilde{max}_\mathcal{G}$, namely $L(\mathcal{G}, \widetilde{max}_\mathcal{G})$ is called the **Top-level decision rule** will mean using the max-threshold and considering the top-level soft set in fuzzy soft multiset based decision making.

Algorithm 1

(I) Input the fuzzy soft multiset (F, A).

(II) Input a threshold fuzzy set $\lambda: A \to [0, 1]$, (or give a threshold value $t \in [0, 1]$; or choose the mid-level decision rule; or choose the top-level decision rule) for decision making.

(III) Compute the level soft set $L(\mathcal{G}; \lambda)$ of $\mathcal{G}$ with respect to the threshold fuzzy set λ (or the t −level soft set $L(\mathcal{G}; t)$; or the mid-level soft set $L(\mathcal{G}; mid)$ or the top-level soft set $L(\mathcal{G}; max)$).

(IV) Present the level soft set $L(\mathcal{G}; \lambda)$(or $L(\mathcal{G}; t)$; or $L(\mathcal{G}; mid)$ or $L(\mathcal{G}; max)$ in tabular form and compute the choice c_i for all i.

(V) The optimal decision is to select o_k if $c_k = max_i c_i$, from each U_i.

(VI) If there are more than one k, then any one of o_k may be chosen, from each U_i.

Remark:

In order to get a unique optimal choice according to the algorithm, the decision makers can go back to the second step and change the threshold (or decision criteria) in case that there is more than one optimal choice that can be obtained in the last step. Moreover, the final optimal decision can be adjusted according to the decision makers preferences.

Example 4.1.

Consider example 3.1

Table 4.1. Tabular representation of fuzzy soft multiset (F, A).

U/A	a_1	a_2	a_3	a_4	a_5	a_6
h_1	0.2	0.2	0.7	0.9	0.9	0.7.
h_2	0.4	0.4	0.7	0.5	0.5	0.7.
h_3	0.8	0.8	0.1	0.5	0.5	0.1.
h_4	0.5	0.5	0.8	0.2	0.2	0.8.
h_5	0	0	0.7	0.7	0.7	0.7.
c_1	0.8	0.4	0.8	0	0.7	0.8.
c_2	0.5	0.2	0.6	0.2	0.8	0.6.
c_3	0.4	0.8	0.3	0.7	0.5	0.3.
c_4	0.6	0.5	0.5	0.6	0.4	0.5.
v_1	0.8	0.4	0.5	0.8	0.5	0.8.
v_2	0.7	0.4	0.4	0.7	0.5	0.7.
v_3	0.7	0.3	0.2	0.9	0.7	0.6.

Let us take $t = 0.7$, then we obtain the $0.7 - level$ set of the fuzzy set $F(a_1)$, $F(a_2)$, $F(a_3)$, $F(a_4)$, $F(a_5)$, $F(a_6)$ as follows:

$L(F(a_1); 0.7) = \{\{h_3\}, \{c_1\}, \{v_1, v_2, v_3\}\}$,
$L(F(a_2); 0.7) = \{\{h_3\}, \{c_3\}, \emptyset\}$,
$L(F(a_3); 0.7) = \{\{h_1, h_2, h_4, h_5\}, \{c_1\}, \emptyset\}$,
$L(F(a_4); 0.7) = \{\{h_1, h_5\}, \{c_3\}, \{v_1, v_2, v_3\}\}$,
$L(F(a_5); 0.7) = \{\{h_1, h_5\}, \{c_1, c_2\}, \{v_3\}\}$,
$L(F(a_6); 0.7) = \{\{h_1, h_2, h_4, h_5\}, \{c_1\}, \{v_1, v_2\}\}$.

Table 4.2: Tabular representation of $t-level$ soft set $L((F,A);t)$ with choice value.

U/A	a_1	a_2	a_3	a_4	a_5	a_6	Choice Value.
h_1	0	0	1	1	1	1	4.
h_2	0	0	1	0	0	1	2.
h_3	1	1	0	0	0	0	2.
h_4	0	0	1	0	0	1	2.
h_5	0	0	1	1	1	1	4.
c_1	1	0	1	0	1	1	4.
c_2	0	0	0	0	1	0	1.
c_3	0	1	0	1	0	0	2.
c_4	0	0	0	0	0	0	0.
v_1	1	0	0	1	0	1	3.
v_2	1	0	0	1	0	1	3.
v_3	1	0	0	1	1	0	3.

From table 4.2, the maximum choice value from U_1 is 4 and the optimal decision is to select either house h_1 or h_5; the maximum choice value from U_2 is 4 and the optimal decision is to select car c_1. Also, the maximum choice value from U_3 is 3 and the optimal decision is to select either venues v_1, v_2 or v_3.

Therefore, Mr X should purchase house h_1 or h_5; car c_1 and venue v_1 or v_2 or v_3 as the best house, car and venue respectively for his wedding celebration after specifying weights for different parameters.

Example 4.2.

Consider example 3.1

$\widetilde{mid}_{(F,A)}$
$= \{(a_1, 0.53), (a_2, 0.41), (a_3, 0.53), (a_4, 0.6), (a_5, 0.58), (a_6, 0.61)\}$

Table 4.3. Tabular representation of $mid-level$ soft set $L((F,A);mid)$ with choice value.

U/A	a_1	a_2	a_3	a_4	a_5	a_6	Choice Value.
h_1	0	0	1	1	1	1	4.
h_2	0	0	1	0	0	1	2.
h_3	1	1	0	0	0	0	2.
h_4	0	1	1	0	0	1	3.
h_5	0	0	1	1	1	1	4.
c_1	1	0	1	0	1	1	4.
c_2	0	0	1	0	1	0	2.
c_3	0	1	0	1	0	0	2.
c_4	1	1	0	1	0	0	3.
v_1	1	0	0	1	0	0	2.
v_2	1	0	0	1	0	1	3.
v_3	1	0	0	1	1	1	4.

According to table 4.3, the maximum choice value from U_1 is 4 and the optimal decision is to select either house h_1 or h_5; the maximum choice value from U_2 is 4 and the optimal decision is to select car c_1. Also, the maximum choice value from U_3 is 4 and the optimal decision is to select venue v_3.

Hence, Mr X should purchase house h_1 or h_5; car c_1 and venue v_3 as the best house, car and venue respectively for his wedding celebration.

Example 4.3.

Consider example 3.1.

$Top_{(F,A)} =$
$\{(a_1, 0.8), (a_2, 0.8), (a_3, 0.8), (a_4, 0.9), (a_5, 0.9), (a_6, 0.8)\}$.

Table 4.4. Tabular representation of $top-level$ soft set $L((F,A);top)$ with choice value.

U/A	a_1	a_2	a_3	a_4	a_5	a_6	Choice Value.
h_1	0	0	0	1	1	0	2.
h_2	0	0	0	0	0	0	0.
h_3	1	1	0	0	0	0	2.
h_4	0	0	1	0	0	1	2.
h_5	0	0	0	0	0	0	0.
c_1	1	0	1	0	0	1	3.
c_2	0	0	0	0	0	0	0.
c_3	0	1	0	0	0	0	1.
c_4	0	0	0	0	0	0	0.
v_1	1	0	0	0	0	1	2.
v_2	0	0	0	0	0	0	0.
v_3	0	0	0	1	0	0	1.

From table 4.4, the maximum choice value from U_1 is 2 and the optimal decision is to select either house h_1 or h_2 or h_3. Also, the maximum choice value from U_2 is 3 and the optimal decision is to select car c_1. Similarly, the maximum choice value from U_3 is 2 and the optimal decision is to select venue v_1.

So, Mr X should purchase either house $h_1$or h_2 or h_3; car c_1 and venue v_1 as the best house, car and venue respectively for his wedding celebration.

V. Weighted Fuzzy Soft Multiset based Decision Making

Lin in 1996 [12] defined a new theory of mathematical analysis, namely the weighted soft sets (W-soft sets). In accordance with Lin's style, Maji *et al* [13] defined the weighted table of a soft set. A weighted table of a soft set is presented by having $d_{ij} = w_j \times h_{ij}$ instead of 0 and 1 only, where h_{ij} are entries in the table of the soft set and w_j are the weights of the attributes e_j. The weighted choice value of an object o_i is $\bar{c}_i$, given by $\bar{c}_i = \sum_j d_{ij}$. By imposing weights on choice parameters, a revised algorithm for arriving at the final optimal decisions was established in [9]. In line with this idea, we introduce the notion of weighted fuzzy soft multisets and present its application to decision making problems.

Let $\{U_i: i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \emptyset$ and let $\{E_{U_i}: i \in I\}$ be a collection of sets of parameters or attributes related to the universes. Let $U = \biguplus_{i\in I} HFS(U_i)$, where $HFS\ (U_i)$ denotes the set of all fuzzy submultisets of the U_i`s, $E = \biguplus_{i\in I} E_{U_i}$ and $A \subseteq E$.

Definition 5.1 [13]

A weighted fuzzy soft multiset is a triple $\Xi = (F, A, w)$ where (F, A) is a **fuzzy soft multiset** over U, and $w: A \longrightarrow [0, 1]$ is a **weight function** specifying the weight $w_j = w(e_j)$ for each attribute $e_j \in A$.

By definition, every fuzzy soft multiset can be considered as a weighted fuzzy soft multiset. Obviously, the notion of weighted fuzzy soft multiset provides a mathematical framework for modeling and analyzing the decision making problems in which all the choice parameters may not be of equal importance. The difference between the importance of parameter are characterize by the weight function in a weighted fuzzy soft multiset.

Algorithm 1 can be revised to deal with decision making problems based on weighted fuzzy soft multisets (see algorithm 2). In the revised algorithm, we take the weights of parameters in to consideration and compute the weighted choice values $\bar{c}_i$ instead of choice valuesc_i. Note that for a weighted fuzzy soft multiset $\Xi = (F, A, w)$ the weight function $w: A \longrightarrow [0, 1]$ can be used as a threshold fuzzy set, which implies that one can consider the level soft set $L((F, A); w\,)$. This will be called decision making based on the **weight function decision rule** in what follows. Sometimes it is much reasonable to use this decision rule since the decision maker may need higher membership levels on the parameters he puts on more emphasis.

Algorithm 2.

(I) Input the weighted fuzzy soft multiset $\Xi = (F, A, w)$.

(II) Input a threshold fuzzy set $\lambda: A \to [0, 1]$, (or give a threshold value $t \in [0, 1]$; or choose the mid-level decision rule; or choose the top-level decision rule or choose the weight function decision rule) for decision making.

(III) Compute the level soft set $L((F, A); \lambda\,)$ of Ξ with respect to the threshold fuzzy set λ (or the t −level soft set $L((F, A); t\,)$; or the mid-level soft set $L((F, A); mid\,)$ or the top-level soft set $L((F, A); max\,) or\ L((F, A), w)$).

(IV) Present the level soft set $L((F, A); \lambda\,)$(or $L((F, A); t\,)$; or $L((F, A); mid)$ or $L((F, A); max) or\ L((F, A), w)$ in tabular form and compute the weighted choice value $\bar{c}_i$ of o_i for all i.

(V) The optimal decision is to select o_k if $\bar{c}_k = max_i \bar{c}_i$, from each U_i.

(VI) If k has more than one value, then any one of o_k may be chosen, from each U_i.

Note that in the last step of algorithm 2, if too many optimal choices are obtained, one can go back to the second step and change the threshold (or decision rule) previously used so as to adjust the final optimal decision.

Example 5.1

Consider example 3.1, Suppose that Mr X has impose the following weights for the parameters in A: for parameter expensive, $w_1 = 0.7$; for the parameter cheap, $w_2 = 0.5$; for the parameter majestic, $w_3 = 0.6$; for the parameter in green surrounding, $w_4 = 0.8$; for the parameter in Kuala Lumpur, $w_5 = 0.7$; for the parameter sporty, $w_6 = 0.8$.

Table 5.1: Tabular representation of soft set $L((F, A); w)$ with choice value.

U/A	a_1	a_2	a_3	a_4	a_5	a_6	*Choice Value.*
h_1	0	0	1	1	1	0	**2.1.**
h_2	0	0	1	0	0	0	0.6.
h_3	1	1	0	0	0	0	1.2.
h_4	0	1	1	0	0	1	1.9.
h_5	0	0	1	0	1	0	1.3.
c_1	1	0	1	0	1	1	**2.8.**
c_2	0	0	1	0	1	0	1.3.
c_3	0	1	0	0	0	0	0.5.
c_4	0	1	0	0	0	0	0.5.
v_1	1	0	0	1	0	1	**2.3.**
v_2	1	0	0	0	0	0	0.7.
v_3	1	0	0	1	1	0	2.2.

From table 5.1, it follows that, the maximum choice value from U_1 is 2.1 and the optimal decision is to select house h_1. Also, the maximum choice value from U_2 is 2.8 and the optimal decision is to select car c_1. Similarly, the maximum choice value from U_3 is 2.3 and so the optimal decision is to select venue v_1.

Therefore, Mr X should purchase house h_1; car c_1 and venue v_1 as the best house, car and venue respectively for his wedding celebration after specifying weights for different parameters.

VI. Conclusion

In this paper, we have used the adjustable approach introduced by Feng et al. [10]. We have used concrete and illustrative examples to present an approach to fuzzy soft multiset based decision making, using level soft sets of fuzzy soft multisets. We also, introduced the weighted fuzzy soft multiset and investigated its application to decision making.

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