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Application of Differential Equation in Heat Conduction Model

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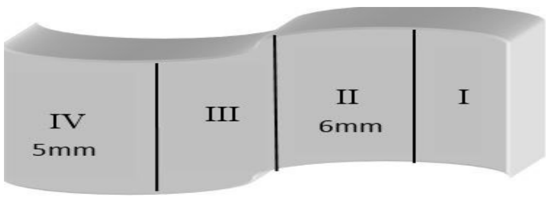
***Abstract* – Heat conduction is a phenomenon of heat transfer without macroscopic motion in the medium, and heat transfer will occur as long as there is temperature difference between the medium and the medium. As a main way of heat transfer, heat transfer theory is widely used, such as heating vulcanization of rubber products, heat treatment of steel forgings and so on. In the study of this kind of problem, most literatures use the method of partial differential equation to solve this kind of problem, but it is difficult to get the analytical solution and numerical solution. In this paper, based on the design of a certain professional garment, the establishment of ordinary differential equations as a theoretical basis, aimed at analyzing the heat conduction process in different locations of the temperature changes with time. When considering this model, the energy loss per unit time heat source can be correlated with the parameters of the medium by Fourier heat conduction law from microscopic point of view, and the total heat loss of the heat source can be correlated with temperature by energy conservation law from macroscopic point of view. In this paper, the relationship between these two aspects is established by using the ordinary differential equations, and the curves of temperature variation with time at different locations are obtained.**

***Keywords* – Differential Equation, Friyege Conduction Law, Four Order Runge Kutta Method, Heat Conduction.**

I. Introduction

With the development of the economy, a large amount of fields need to involve high temperature working environment. Under such conditions, people need to wear special clothes to avoid being burned. Among them, a professional clothing consists of three layers, [1]namely, I, II, III layer, known that I layer and the outside direct contact, there is still a gap between III layer and skin. Therefore, this gap is recorded as IV layer. In order to design such special clothing to control the temperature of human body's surface at the lowest cost and the shortest development cycle, a corresponding model is established to solve the problem [1] [2] [3].



Environment temperature 75℃
Skin temperature 37℃
Fig. 1 special garment structure drawing.

To this end, the dummy in vivo temperature controlled at 37 C is placed in a high-temperature environment. Some parameters (density, specific heat, heat conductivity and thickness of different layers) of special clothing materials are known. Experiments were carried out at ambient temperature of 75 C, thickness of layer II of 6 mm, thickness of layer IV of 5 mm and working time of 90 minutes. The temperature of the outer side of the dummy skin was measured [4].

In order to solve this problem, a simple model is first considered: the relationship between temperature and time of a single heat source when it radiates heat to the environment. Under this model, the environment temperature.

(θ) is stable.

Fig. 2. Variation of system temperature when a single heat source dissipates to the environment.

II. Model Building

According to the microscopic Friyege conduction law,

$$q = A\lambda \frac{T - T_0}{\delta} \qquad (1)$$

From a macroscopic point of view, the energy loss of a heat source results in the decrease of its temperature. Therefore, the energy conservation law and the heat calculation formula can be obtained [5].

$$\begin{cases} dQ = qdt = A\lambda \dfrac{T-\theta}{\delta} dt = -cmdT \\ T(0) = T_0 \end{cases} \qquad (2)$$

To study this problem, we assume that the ambient temperature is a function of time, let $T_0(t)$ be a function of environmental temperature over time, and $T_0(0) = 75^{\circ} C$. At the same time, let $T_i(t)$, $i = 1, 2, 3, 4$ be a function of the temperature of layer I, II, III and IV with time, and take the

[1] The unit of temperature is expressed by ℃

temperature function at the end of layer IV as the surface temperature function of human skin [5] [6].

The relationship between heat transfer of each part in Δt time is investigated. The equivalent dissipation area of the environment is assumed to be A_0, and its heat conductivity is λ_0. Since the heat transfer is assumed to be only one layer at a time, the energy dissipation of the environment in Δt time is as follows:

$$dQ = A_0\lambda_0 \frac{T_0(t)-T_1(t)}{\delta_1} dt \qquad (3)$$

From the law of conservation of energy, the temperature of the environment decreases accordingly.

$$dQ = A_0\lambda_0 \frac{T_0(t)-T_1(t)}{\delta_1} dt = -c_0 m_0 dT_0(t) \quad (4)$$

The energy dissipated by the environment is not entirely used to raise the temperature of the I layer, and some of it remains in the environment. Therefore, the equation listed here is about the temperature change of the heat source. However, when the heat is transferred to the inside of the high temperature protective clothing, without considering the heat radiation and convection, it can be considered that the heat loss of the front layer is all in the time of Δt. It is used to enhance the temperature of the back layer. Therefore, according to the law of conservation of energy, the following equation of state is established:

$$A_i\lambda_i \frac{T_i(t)-T_{i+1}(t)}{\delta_{i+1}} dt = c_{i+1}\ m_{i+1}\ dT_{i+1}\ (t),\ i=1,2,3 \qquad (5)$$

The left end of the equation represents the heat loss in layer i. The reason for the heat transfer is that there is a temperature difference between layer i and layer i+1. It is expressed by the temperature distribution function $T_i(t)-T_{i+1}(t)$. In this closed system, all the heat of the i layer at the left end of the equation is transferred to the i + 1 layer, resulting in the change of the temperature of the i+ 1 layer at the right end of the equation, which is calculated by the heat calculation formula.

At the same time, because the thickness of protective clothing is small, it can be approximately regarded as a cuboid, so the equivalent volume of its heating can be determined by the following formula:

$$A_0\delta_0 = V_0 \qquad (6)$$

Thus, the equation of state of heat conduction can be obtained by simplifying (6) into (4) and paralleling (5):

$$\begin{cases} \lambda_0 \dfrac{T_0(t)-T_1(t)}{\delta_0\delta_1} dt = -c_0\rho_0 dT_0(t) \\ A_i\lambda_i \dfrac{T_i(t)-T_{i+1}(t)}{\delta_{i+1}} dt = \\ c_{i+1}m_{i+1}dT_{i+1}(t),\ i=1,2,3 \\ 0 \le t \le 5400 \\ T_0(0) = 75^\circ, \\ T_0(5400) = T_1(5400) = \ldots = 48.08^\circ \end{cases} \qquad (7)$$

This is a system of ordinary differential equations with initial value problems, in which there are five unknown functions, four equations and some parameters. It is difficult to solve directly. Therefore, we still need further analysis to solve this equation, mainly using the quantitative relationship given by the data in the appendix.

Visual analysis of the data given in the annex is made. The scatter plot is as follows:

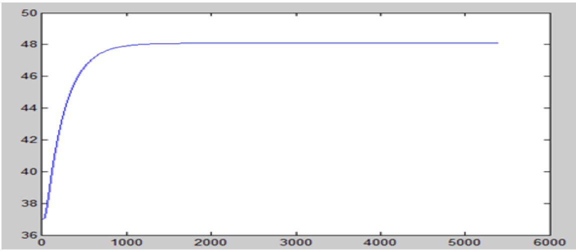


Fig. 3. Curves of temperature change with time in layer IV.

Qualitative analysis shows that the curve shows a trend of rising first and then stable. In the initial stage, with the temperature rising, the rate of temperature change with time is also rising, and then the temperature gradually becomes stable. The heat conduction process is caused by the temperature difference between different objects. In the initial stage of the fourth layer, the heat accumulation from the third layer is higher than that from the fourth layer, so the heat conduction rate is higher. When the temperature reaches a stable state, the heat conduction is no longer carried out.

Therefore, a retardation factor $\left(1-\frac{T}{T'}\right)$ can be added to the temperature change of IV layer, T' is the limit temperature of the system. Obviously, the higher the temperature T of IV layer is, the smaller the rate of temperature change is.

The rate of temperature change with time can be expressed as follows:

$$\begin{cases} \dfrac{dT}{dt} = \dfrac{q}{cm}\left(1-\dfrac{T}{T'}\right) \\ T(0) = 37^\circ \end{cases} \qquad (8)$$

It is also known by (1) that: $\begin{cases} \dfrac{dT}{dt} = KT\left(1-\dfrac{T}{T'}\right) \\ T(0) = 37^\circ \end{cases}$

K is related to dissipative area, specific heat capacity, medium thickness and thermal conductivity.

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$$K = \frac{A_3 \lambda_4}{c_4 \delta_4 m_4} \qquad (9)$$

Using variable separation method to solve the constant coefficient differential equation with initial condition, the following results can be obtained:

$$T_4(t) = \frac{T^{'}}{1 + \frac{T^{'} - T(0)}{T(0)} e^{-Kt}} \qquad (10)$$

III Solving Differential Equations

This is a set of ordinary differential equations with initial value problems, in which the change of layer IV (human skin surface temperature) has been obtained by the above theoretical analysis and data fitting, so we start from the inner layer to solve the differential equation, using the MATLAB toolbox to solve, the specific steps are as follows:

1) *According to the Fitting of the Above*

data, the value of $K = \frac{A_3 \lambda_4}{c_4 \delta_4 m_4}$ can be obtained. The value of parameter $\frac{A_3}{m_4}$ can be obtained, so that the unknown parameter $\frac{A_i}{m_{i+1}}$ of equation group (7) can be changed to known.

2) *From the Innermost Function, the First Four*

Sets of equations can be solved analytically, but the form is more complex. After solving the analytic solution, we discretize it, get the same form of data as the appendix and regression fitting. Using the above analysis method, we get the same form of solution as the IV function, and R^2 statistical quantity are all above 0.99. The model can explain the above dependent variables and has better fitting effect.

3) *In Order to Solve the Outermost Layer*

(Environment) temperature distribution function, it is difficult to obtain the analytic solution because the right-hand function of this ordinary differential equation is complex. Under the given initial conditions, we use the fourth-order Runge Kutta method to obtain the numerical solution of this function. The numerical solution of the method is as follows [7] [8] [9]:

$$\begin{cases} k_1 = f(x_i, y_i) \\ k_2 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2} k_1) \\ k_3 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2} k_2) \\ k_4 = f(x_i + h, y_i + h\, k_3) \\ y_{i+1} = y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \end{cases} \qquad (11)$$

4) *Discretization of the Outermost Functions*

And regression fitting, form and procedure, such as 3) (when the thickness of protective clothing is small, the equivalent dissipation area can be regarded as a constant). After fitting, the curves we get are shown below.

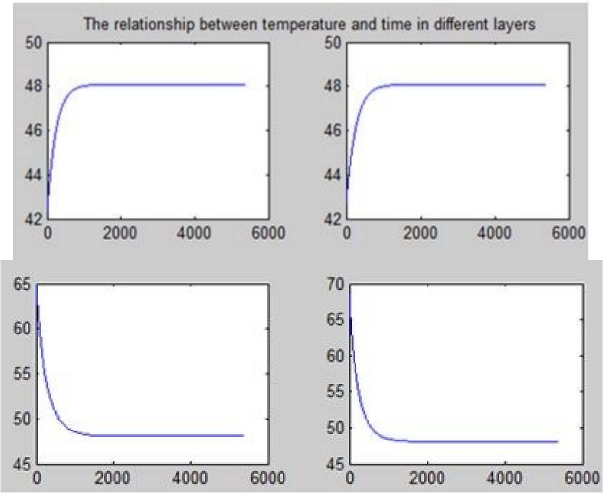

Fig. 4. Relationship between temperature and time in different layers.

IV. Conclusion

Fig. 4 is the relationship between time and temperature of layer III, II, I and outermost.

Analysis of the above figure is available.

1. Compared with the temperature changes on the surface of human skin, we have got two conclusions:

a) The initial value of layer III temperature begins at 42 degrees.

b) The time needed to achieve stable temperature in layer III is shorter than that in IV layer (human skin layer).

This result is reasonable, because the material of protective clothing and dummy material are different, so the initial value is different, because the layer III protective clothing is closer to the heat source than the human body, so the time needed to achieve stability is less than the human body temperature to achieve stability.

2. The temperature variation of layer II is similar to that of layer III, except that the initial value of temperature begins at 43 degrees and the time to reach a stable state is slightly less than that of layer II.

3. The temperature variation of layer I is quite different from that of layers II and III. The initial value changes around 64 degrees and reaches a stable state around 1000 seconds. The stable value is consistent with that of layers II, III and IV.

This indicates that the temperature of the protective clothing on the first layer reaches 64 degrees when it contacts the heat source, because the heat source conducts heat directly to it, so its initial temperature is higher.

4. The outermost temperature variations are shown in the diagram above. The initial temperature is 75℃, reaching a stable value around 1000 seconds, which is in good agreement with the actual situation.

The above is the relationship between the temperature of each layer and time. The fitting results of the above equations are given below.

$$
\begin{aligned}
T_0(t) &= \frac{48.08}{-0.2966\times e^{-0.003543t}+1} \\
T_1(t) &= \frac{48.08}{-0.2577\times e^{-0.003186t}+1} \\
T_2(t) &= \frac{48.08}{0.12545\times e^{-0.00422t}+1} \\
T_3(t) &= \frac{48.08}{0.1356\times e^{-0.004542t}+1} \\
T_4(t) &= \frac{48.08}{0.2995\times e^{-0.004204t}+1}
\end{aligned} \qquad (12)
$$

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