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Optimal Solution of Differential Equations for Heat Conduction in Infinitely Large Flat Wall Models

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***Abstract* – Thermal protective clothing in high temperature environment is usually composed of multiple layers of insulation materials. It is of great significance to study the establishment of thermal conductivity model of multilayer protective clothing and the optimal solution of thickness. In this paper, based on the infinite large-wall heat conduction model, the Fourier heat conduction law is transformed to derive the heat conduction differential equation, and the segmented partition analysis method is adopted for the established combined model. The multi-layer infinitely large flat-wall heat conduction model in solids and the theoretical model of convective heat transfer in gas are respectively established and combined into a change function of tempera--ture and distance at a single moment. Temperature-based logistic regression was performed using MATLAB based on available time and skin surface temperature statistics. Combined with two independent variables affecting the time and distance of temperature, the analytical solution of the "high temperature environment-clothing-air layer-skin" heat conduction partial differential model is solved.**

***Keywords* – Fourier Heat Conduction Law, Infinite Large Multi-Layer Flat Wall Heat Conduction, Logistic Regression.**

I. INTRODUCTION

Thermal protection is a very important and widely studied subject. Thermal protective clothing, as an intermediate between human and environment, is the most effective and economical means of protection in the production of substances, equivalent to human second skin. In general, the test of thermal protective clothing is based on the actual environment of many high-temperature operations for testing and testing, which consumes huge financial resources and time. Therefore, it is necessary to establish a thermal conductivity model of high temperature thermal protective clothing to express the specific heat conduction process and give a mathematical expression of heat transfer.

The thermal protective suit is usually composed of three layers of fabric material, which are recorded as layers I, II and III. The layer I is in contact with the external environment, and there is a gap between the layer III and the skin. The air layer is recorded as an IV layer. This paper establishes the thermal conductivity model under the specified conditions as an example, focusing on the thermal conduction process in the following experimental conditions, that is, the surface temperature is controlled at 37ºC, the ambient temperature is 75ºC, the thickness of the II layer is 6mm, the thickness of the IV layer is 5mm, and the exposure time is 90min.

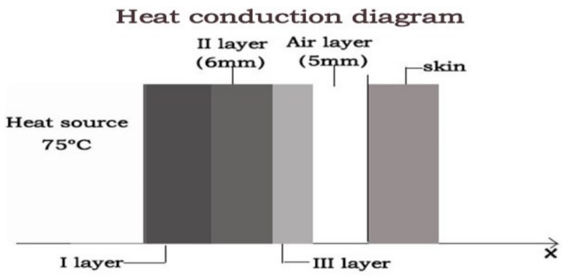

Fig. 1. Schematic diagram of the thermal protective clothing structure

In recent years, research on thermal protective clothing has always been a hot issue in the world. In the thermal protection of heat and low heat for a long time, Torvi [1] proposed the heat conduction of thermal protective outer fabric. Song G [2] et al. also studied the heat conduction mode in a timely manner. This paper hopes to integrate the two aspects of time and space, use the model combination method, and adopt the segmentation and partitioning process to make a mathematical description of the constant heat conduction system at a certain time.

II. MODELING OF HEAT CONDUCTION

1) *Multi-layer Infinite Large Flat Wall Heat Conduction Model (Solid)*

It is assumed that the protective clothing object is isotropic, the physical parameter density and the specific heat capacity are constant, the heat source distribution in the object is uniform, and the interval temperature distribution is continuously changed [3]. Fourier's law of heat conduction is a basic expression describing the heat transfer heat transfer density in a thermally conductive manner. In thermal conduction heat transfer, the heat transfer heat flux in the x direction is written as

$$q_x = -\lambda \frac{\partial T}{\partial x} \tag{1}$$

In this paper, the protective clothing is transmitted in the same direction in all directions. The heat conduction is carried out in the direction perpendicular to the skin, that is, the normal direction. The three-dimensional heat conduction problem is transformed into a one-dimensional problem, and a one-dimensional infinite large flat wall heat transfer model is established.

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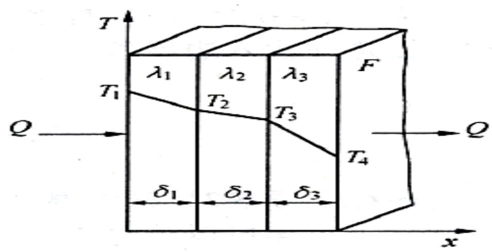

Fig. 2. Multi-layer infinite large flat wall heat conduction method

Known by (1), the thermal conductivity λ is numerically equal to the heat flux density produced by the unit temperature gradient in the material. [3] According to the law of conservation of energy：

$$cm\Delta T = \lambda \frac{\partial T}{\partial x} S \cdot dt \qquad (2)$$

Modeling the diffusion problem, assuming that the environment in which the object is located is constant, the heat transfer equation is modified to:

$$c\rho T_t - \frac{\partial(\lambda T_x)}{\partial x} = 0 \qquad (3)$$

c is the specific heat capacity, ρ is the density. For uniform objects λ,c,ρ are constants. T_t and T_x respectively represent the first-order partial conductance of temperature versus time and distance. The above formula simplifies the one-dimensional heat conduction differential equation:

$$T_t - a^2 T_{xx} = 0 (a^2 = \frac{\lambda}{c\rho}) \qquad (4)$$

There are time conditions for steady state heat conduction as follows:

$$\left.\frac{\partial T}{\partial t}\right|_{x=0} = 0 \qquad (5)$$

Let $T_i(t), i = 1,2,3,4$ be the heat source, layer I, II, III, IV, skin temperature time function, and obtain initial conditions:

$$T_5|_{t=0} = 37^{\circ}C$$

$$T_0|_{t=0} = 75^{\circ}C \qquad (6)$$

Let δ = 1, 2, 3, 4 be the thicknesses of layers I, II, III and IV, respectively, and analyze the boundary conditions of the first and second types:

$$T_i|_{x=\sum_{n=1}^{i}\delta_n} = T_{i+1}|_{x=\sum_{n=1}^{i+1}\delta_n}$$

$$-\left.\frac{\partial T_i}{\partial x}\right|_{x=\sum_{n=1}^{i}\delta_n} = \frac{q_i}{\lambda} \qquad (7)$$

2) *Gas Membrane Theoretical Model*

There is mainly thermal convection between the solid and the gas. It is assumed that the heat transfer resistance between the fluid and the solid is concentrated in an effective film with a thickness of δ near the wall surface. [4] This model is called convective heat transfer film. Theoretical model.

$$Q = \alpha A \Delta T \qquad (8)$$

$$q_x = \alpha \Delta T$$

h is the convection coefficient, and T_1 and R are known. Combine (7) to get:

$$q_x = \frac{T_1 - T_4}{R} = h(T_4 - T_5) \qquad (9)$$

The factors affecting the convection coefficient α have two parts, one is the nature of the fluid. The second is that the fluid is affected. According to the experimental law and theoretical research, the number of dimensionless feature numbers is 8-4=4, and each feature number is represented by the corresponding symbol[5], which can be written as:

$$Nu = C\, Re^a\, Pr^k\, G\, r^b \qquad (10)$$

Nu, Re, Pr, Gr are the Nusselt number, the Reynolds number, the Plant number, and the Grace number, respectively[6]. The coefficients C, a, b, k are the coefficients in the empirical formula and can be calculated experiment-tally [7]. The convection coefficient is expressed by the following formula:

$$h = C\frac{\lambda}{l}\left(\frac{c_p \mu}{\lambda} \times \frac{\beta g \Delta t l^3 p^2}{\mu^2}\right)^n \qquad (11)$$

According to the hypothetical experimental structure and theoretical derivation, the final convection coefficient h = 80W/(m^2·°C)

III. Model Solution

Establish one-dimensional thermal differential equation based on model：

$$\begin{cases} T_t - a^2 T_{xx} = 0\left(a^2 = \frac{\lambda}{c\rho}\right) \\ T_5|_{t=0} = 37^{\circ}C \\ T_0|_{t=0} = 75^{\circ}C \\ \left.\frac{\partial T}{\partial t}\right|_{x=0} = 0 \\ \boldsymbol{T_i}|_{x=\sum_{n=1}^{i}\delta_n} = \boldsymbol{T_{i+1}}|_{x=\sum_{n=1}^{i+1}\delta_n} \\ -\left.\frac{\partial T_i}{\partial x}\right|_{x=\sum_{n=1}^{i}\delta_n} = \frac{q_i}{\lambda} \end{cases} \qquad (12)$$

According to the time condition, the differential equation can be simplified:

$$\frac{d^2 T}{dx^2} = 0 \qquad (13)$$

Get a special solution for:

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$$T_i(x) = \frac{T_i - T_{i+1}}{\delta_i} x + D \qquad (14)$$

It can be found that the temperature is linear when the heat is stable in the large flat wall [8]. Simultaneous boundary conditions and special solutions of the equation show that the heat flux is the same

$$q_x = \lambda_1 \frac{T_1 - T_2}{\delta_1} = \lambda_2 \frac{T_2 - T_3}{\delta_2} = \lambda_3 \frac{T_3 - T_4}{\delta_3} \qquad (15)$$

Add them together:

$$q_x = \frac{T_1 - T_4}{\frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \frac{\delta_3}{\lambda_3}} \qquad (16)$$

Set:$R_1 = \frac{\delta_1}{\lambda_1}, R_2 = \frac{\delta_2}{\lambda_2}, R_3 = \frac{\delta_3}{\lambda_3}, R = R_1 + R_2 + R_3$
The above formula can be written as:

$$q_x = \frac{T_1 - T_4}{R} \qquad (17)$$

According to the theoretical model of convective heat transfer, the equation of T_4 can be established:

$$q_x = \frac{T_1 - T_4}{R} = h(T_4 - T_5) \qquad (18)$$

Logistic regression model was used to solve the corresponding function of skin surface temperature with respect to time. In this paper, using MATLAB toolbox to discretize the data and logistic regression fitting, the following results can be obtained:

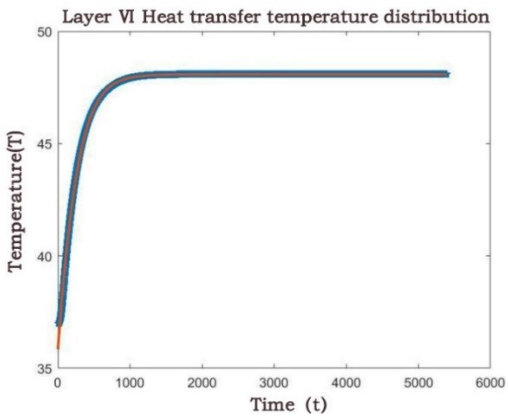

Fig. 3. Logistic regression image of skin surface temperature

According to the program results logistic regression R = 0.9988, indicating that the degree of fit under this condition is very good.

$$T_5(t) = \frac{k}{a + e^{-bt}} \qquad (19)$$

Parameter Description:
a = 2.931(2.926,2.935)
b = 0.004687(0.004678,0.004696)
k = 140.9(140.7,141.1)

According to the multi-layer infinite flat wall heat conduction model established by I, II and the membrane theoretical model of convective heat transfer[9], the logistic regression results are brought into the equation, and the simultaneous equations are solved:

$$\begin{cases} T_5(t) = \frac{k}{a + e^{-bt}} \\ q_x = \frac{T_1 - T_4}{R} = \alpha(T_4 - T_5) \end{cases} \qquad (20)$$

$$T_4(t) = \frac{T_1(a + e^{-bt}) + \hbar Rk}{(\hbar R + 1)(a + e^{-bt})} \qquad (21)$$

In the same way, the formula (21) is brought into the formula (16), which can be obtained separately:

$$T_3(t) = \frac{T_1(a + e^{-bt})(R_3\hbar + 1) + (R - R_3)\hbar Rk}{R(\hbar R + 1)(a + e^{-bt})} \qquad (22)$$

$$T_2(t) = \frac{T_1(a + e^{-bt})[(R\hbar + 1)(R - R_1) + 1] + \hbar Rk}{R(\hbar R + 1)(a + e^{-bt})} \qquad (23)$$

Table 1 parameters of each layer

Layers	R	Specific heat capacity	Heat Conduction	Thickness
I Layer	7.31	1377	0.082	0.6
II Layer	16.21	2100	0.37	6
III Layer	80	1726	0.045	3.6
IV Layer	178.57	1005	0.028	5

According to the above table, the numerical solution of the model under experimental conditions can be calculated:

$$T(x,t) = \begin{cases} 65.0505 + \dfrac{9.949}{2.931 + e^{-0.004687t}} & x = 0.6mm \\ 54.0919 + \dfrac{31.981}{2.931 + e^{-0.004687t}} & x = 6.6mm \\ 0.00845 + \dfrac{140.882}{2.931 + e^{-0.004687t}} & x = 10.2mm \\ \dfrac{140.9}{2.931 + e^{-0.004687t}} & x = 15.2mm \end{cases}$$

The numerical solution can be used to calculate the temperature distribution of each sub-level plane at different times. The temperature profile of each layer is depicted by MATLAB as shown below:

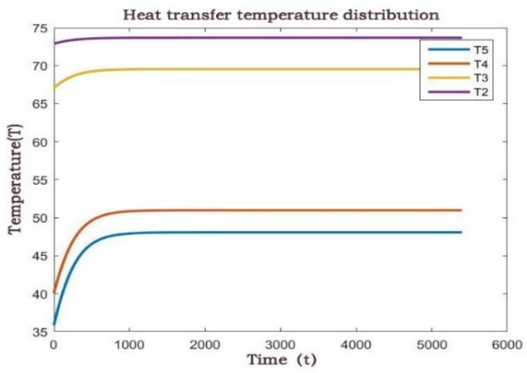

Fig. 4. Heat transfer temperature distribution at different levels

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IV. CONCLUSION

The final calculation results of the above segmentation partition discussion are given below, where a and b are empirical constants. The protective clothing-air-skin temperature model is obtained:

$$T(x,t)=\begin{cases}\dfrac{T_1(a+e^{-bt})[(R\hbar+1)(R-R_1)+1]+\hbar Rk}{R(\hbar R+1)(a+e^{-bt})} & x=\sigma_1\\ \dfrac{T_1(a+e^{-bt})(R_3\hbar+1)+(R-R_3)\hbar Rk}{R(\hbar R+1)(a+e^{-bt})} & x=\sigma_1+\sigma_2\\ \dfrac{T_1(a+e^{-bt})+\hbar Rk}{(\hbar R+1)(a+e^{-bt})} & x=\sigma_1+\sigma_2+\sigma_{31}\\ \dfrac{k}{a+e^{-bt}} & x=\sigma_1+\sigma_2+\sigma_3+\sigma_4\end{cases}$$

According to the analysis results, the temperature variation characteristics of the I, II, III and IV layers can be analyzed:

(1) T3 and T1, T2 have huge differences in expression, although they all show logistic regression form, but there are huge differences in parameters, which is the expression of thermal isolation ability of thermal protective clothing.

(2) The trend of temperature changes is roughly the same, and the magnitude of the change is different. The amplitudes of the I and II layers are relatively small, while the amplitudes of the III and IV layers are relatively large, which can be considered proportional to the difference between the temperature and the temperature.

(3) There is a difference in the initial value of the temperature. The initial temperature of the I and II layers is a high level of 60 degrees or higher, and the temperature of the III and IV layers is a low level of 50 degrees. The initial value condition is determined by the potential function of the environment.

(4) The stable temperature is different. The stable temperature of the I layer is up to 74 degrees, and the stable temperature of the II, III, IV layers are successively decreased. The difference of the stable temperature is the change of the thermal conductivity and the position of the layer.

(5) The time to reach a stable temperature varies. The stabilization time of the I layer is as short as 550 seconds, and the stability time of the II, III, and IV layers is sequentially increased. The settling time is approximately inversely proportional to the level distance and inversely proportional to the temperature difference.

In this paper, the clothing-air-skin temperature model is obtained by the processing method of segmentation and partitioning. After calculation, the temperature distribution of each sub-level surface at different times can be obtained. The temperature distribution map of each layer is better drawn by MATLAB. The model in this paper provides a reference for the future research of thermal protective clothing, and has great significance for improving the current situation of the production and use of thermal protective clothing.

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