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Application of the Second Kind Chebyshev Polynomials to Coefficient Estimates of Certain Class Analytic Functions

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Abstract **– In this work, making the second kind Chebyshev polynomials, we introduce and investigate new subclasses of analytic functions on the open unit disk in the complex plane. We obtain upper bound estimates for the initial three coefficients of the function belonging to these classes.**

Keywords **– Analytic Function; Univalent Function; Coefficient Bound; Chebyshev Polynomials.**

I. Introduction and Preliminaries

Let $U = \{z \in \mathbb{C} : |z| < 1\}$ is open unit disc in the complex plane and A is the class of all complex valued analytic functions on U such that $f(z)$ is given by the following series

$$f(z) = z + a_2 z^2 + a_3 z^3 + \cdots = z + \sum_{n=2}^{\infty} a_n z^n,\ a_n \in \mathbb{C}, \tag{1}$$

consequently normalized by $f(0) = 0 = f'(0) - 1$.

Also, let S be the class of all functions in A which are univalent in U. Some of the important and well-investigated subclasses of S are well known classes S^* and C given below (see [1- 3])

$$S^* = \left\{ f \in S : \operatorname{Re}\left(\frac{zf'(z)}{f(z)} \right) > 0,\ z \in U \right\} \text{ and } C = \left\{ f \in S : \operatorname{Re}\left(\frac{(zf'(z))'}{f'(z)} \right) > 0,\ z \in U \right\}$$

such they are said to be starlike and convex functions in U, respectively.

An analytic function f is subordinate to an analytic function ϕ and written $f(z) \prec \emptyset(z)$, provided that there is an analytic function $\omega(z)$ defined on U with $\omega(0) = 0$ and $|\omega(z)| < 1$ satisfying $f(z) = \phi(\omega(z))$. Ma and Minda [16] unified various subclasses of starlike and convex functions for which either of the quantity $\frac{zf'(z)}{f(z)}$ or $\frac{(zf'(z))'}{f'(z)} = 1 + \frac{zf''(z)}{f'(z)}$ is subordinate to a more general function. For this purpose, they considered an analytic function $\phi(z)$ with positive real part in U, $\phi(0) = 1$, $\phi'(0) > 0$. The class of Ma-Minda starlike and Ma-Minda convex functions consists of the functions $f \in A$ satisfying the subordination $\frac{zf'(z)}{f(z)} \prec \emptyset(z)$ and $1 + \frac{zf''(z)}{f'(z)} \prec \emptyset(z)$, respectively.

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It is well known that in analytic functions theory, the coefficient problem is significant. In particular, several results on coefficient estimates for the initial coefficients $|a_2|$, $|a_3|$ and $|a_4|$ were obtained for various subclasses of analytic functions (see, for example, [3, 5, 6, 9, 10, 13, 17-19, 21, 22]). Recently, new methods have been applied to the solution of coefficient problems (see, [1, 4, 12]).

As known that Chebyshev polynomials play a considerable role in numerical and mathematical physics and are four kinds. The well-known kinds of the Chebyshev polynomials are the first and second kinds. The most of research articles related to specific orthogonal polynomials of Chebyshev family, contain essentially results of Chebyshev polynomials of first and second kinds $T_n(x)$ and $U_n(x)$, and their numerous uses in different applications (see [7, 14]).

In this paper, we will use the second kind Chebyshev polynomials to investigation of the coefficient and Fekete-Szegö problems of certain subclass of analytic functions.

It is well known that in the case of real variable x on the interval $(-1,1)$, the second kind Chebyshev polynomial is defined by

$$U_n(x)=\frac{\sin\left[(n+1)\arccos x\right]}{\sin(\arccos x)}=\frac{\sin\left[(n+1)\arccos x\right]}{\sqrt{1-x^2}}. \tag{2}$$

It follows from (2) that

$$U_{n+1}(t)=2tU_n(t)-U_{n-1}(t),\ n=1,2,3,\dots. \tag{3}$$

Since $U_0(t)=1,\ U_1(t)=2t$, from (3) we obtain $U_2(t)=4t^2-1$ and $U_3(t)=4t\left(2t^2-1\right)$.

We consider the function $G(t,z)=\dfrac{1}{1-2tz+z^2},\ t\in(0,1),\ z\in U$.

It is well known that $G(t,z)=1+2\cos\alpha\cdot z+\left(3\cos^2\alpha-\sin^2\alpha\right)z^2+\left(8\cos^3\alpha-4\cos\alpha\right)z^3+\cdots,\ z\in U$ for $t=\cos\alpha,\ \alpha\in\left(0,\frac{\pi}{2}\right)$. Thus, the function $G(t,z),\ t\in(0,1),\ z\in U$ is analytic respect to parameter z in U and has the following series expansion

$$G(t,z)=1+U_1(t)z+U_2(t)z^2+U_3(t)z^3+\cdots,\quad z\in U, \tag{4}$$

where $U_n(t),\ n=1,2,3,\dots$ is second kind Chebyshev polynomial.

It can be easily verified that $G(t,0)=1$ and $G_z'(t,0)>0$.

So that, taking $G(t,z)$ instead of the aforementioned subordination function $\phi(z)$ we will define a subclass of univalent functions as follows:

Definition 1.1:

A function $f\in S$ given by (1) is said to be in the class $S^*C(G;\beta,t)$, $\beta\geq 0$, $t\in(0,1)$, where G is the function given by (4), if the following condition is satisfied

$$\frac{zf'(z)+\beta z^2 f''(z)}{\beta zf'(z)+(1-\beta)f(z)} \prec G(t,z), z \in U.$$

Definition 1.2:

A function $f \in S$ given by (1) is said to be in the class $S^*(G;t)$, $t \in (0,1)$, where G is the function given by (4), if the following condition is satisfied

$$\frac{zf'(z)}{f(z)} \prec G(t, z), z \in U.$$

Definition 1.3:

A function $f \in S$ given by (11) is said to be in the class $C(G;t)$, $t \in (0,1)$, where G is the function given by (4), if the following condition is satisfied

$$1+\frac{zf''(z)}{f'(z)} \prec G(t, z), z \in U.$$

Note 1.1:

As you can see that the class $S^*C(G;\beta,t)$, defined by Definition 1.1 is a generalization of the Ma-Minda starlike and Ma-Minda convex functions such that in the special cases for $\beta=0$ and $\beta=1$ the classes $S^*(G;t)$ and $C(G;t)$ are Ma-Minda starlike and Ma-Minda convex functions, respectively, when there function is $G(t,z)$ for fixed value $t \in (0,1)$.

In this paper, making use of the second kind Chebyshev polynomial, we introduce and investigate new subclasses $S^*C(G;\beta,t)$, $\beta \geq 0$, $S^*(G;t)$ and $C(G;t)$ for $t \in (0,1)$ of the analytic functions on the open unit disk U. We obtain upper bound estimates for the initial three of the function belonging to these classes.

To prove our main results, we shall require the following well known lemmas.

Lemma 1.1[8] *:*

Let P be the class of all analytic functions $p(z)$ of the form

$$p(z)=1+p_1 z+p_2 z^2+p_3 z^3+\cdots=1+\sum_{n=1}^{\infty} p_n z^n$$

with $\operatorname{Re} p(z)>0$, for $z \in U$ and $p(0)=1$. Then, $|p_n| \leq 2$ for every $n=1,2,3,\ldots$. These inequalities are sharp for each $n=1,2,3,\ldots$. Moreover, $2p_2=p_1^2+(4-p_1^2)x$, $4p_3=p_1^3+2(4-p_1^2)p_1x-(4-p_1^2)p_1x^2+2(4-p_1^2)(1-|x|^2)z$ for some complex x, z with $|x| \leq 1$, $|z| \leq 1$.

Lemma 1.2[1,15] :

Let P be the class of all analytic functions $p(z)$ of the form $p(z)=1+p_1z+p_2z^2+p_3z^3+\cdots=1+\sum_{n=1}^{\infty}p_nz^n$ with $\operatorname{Re} p(z)>0$, for $z\in U$ and $p(0)=1$. Then,

$$\left|p_2-\frac{\nu}{2}p_1^2\right|\le 2\max\{1,|\nu-1|\}=2\begin{cases}1 & \text{if } \nu\in[0,2],\\ |\nu-1| & \text{elswhere.}\end{cases}$$

Lemma 1.3[1,15] :

Let P be the class of all analytic functions $p(z)$ of the form $p(z)=1+p_1z+p_2z^2+p_3z^3+\cdots=1+\sum_{n=1}^{\infty}p_nz^n$ with $\operatorname{Re} p(z)>0$, for $z\in U$ and $p(0)=1$. If $B\in[0,1]$ and $B(2B-1)\le D\le B$, then, $\left|p_3-2Bp_1p_2+Dp_1^3\right|\le 2.$

II. Upper Bound Estimates for the Coefficients

In this section, we give bound estimates for the first three coefficient of the function belonging to defined function classes in the first section.

Firstly we will prove the following theorem for the first three coefficients of the function belonging to the class $S^*(G;t)$.

Theorem 2.1:

Let the function $f(z)$ given by (1.1) be in the class $S^*(G;t)$, $t\in\left(\frac{1}{2},1\right)$. Then,

$$|a_2|\le 2t\ ,\ |a_3|\le\frac{8t^2-1}{2}\ \text{ and }\ |a_4|\le\frac{1}{3}\begin{cases}8t^3+12t^2+t-2 & \text{if } t\in\left(\frac{1}{2},\frac{\sqrt{2}}{2}\right],\\ 24t^3+12t^2-7t-2 & \text{if } t\in\left[\frac{\sqrt{2}}{2},1\right).\end{cases}$$

Proof:

Let $f\in S^*(G;t)$, $t\in\left(\frac{1}{2},1\right)$. Then, there is an analytic function $\omega:U\to U$ with $\omega(0)=0$, $|\omega(z)|<1$ satisfying the following condition

$$\frac{zf'(z)}{f(z)}=G(t,\omega(z)),\ z\in U. \qquad (5)$$

Let the function $p\in P$ be defined as follows

$$p(z):=\frac{1+\omega(z)}{1-\omega(z)}=1+p_1z+p_2z^2+p_3z^3+\cdots\ .$$

It follows that

$$\omega(z)=\frac{p(z)-1}{p(z)+1}=\frac{1}{2}\left[p_1 z+\left(p_2-\frac{p_1^2}{2}\right)z^2+\left(p_3-p_1p_2+\frac{p_1^3}{4}\right)z^3+\cdots\right]. \tag{6}$$

Taking $z\equiv\omega(z)$ in (4), we get

$$\begin{aligned}G(t,\omega(z))=1+\frac{U_1(t)}{2}p_1 z+\left[\frac{U_1(t)}{2}\left(p_2-\frac{p_1^2}{2}\right)+\frac{U_2(t)}{4}p_1^2\right]z^2\\+\left[\frac{U_1(t)}{2}\left(p_3-p_1p_2+\frac{p_1^3}{4}\right)+\frac{U_2(t)}{2}p_1\left(p_2-\frac{p_1^2}{2}\right)+\frac{U_3(t)}{8}p_1^3\right]+\cdots.\end{aligned} \tag{7}$$

Substituting this expression of the function $G(t,\omega(z))$ in (5) with sample computation, we write

$$\begin{aligned}&z+2a_2z^2+3a_3z^3+4a_4z^4+\cdots=z+\left[a_2+\frac{U_1(t)}{2}p_1\right]z^2+\left\{\begin{aligned}&a_4+a_3\frac{U_1(t)}{2}p_1+a_2\left[\frac{U_1(t)}{2}\left(p_2-\frac{p_1^2}{2}\right)+\frac{U_2(t)}{4}p_1^2\right]\\&+\frac{U_1(t)}{2}\left(p_3-p_1p_2+\frac{p_1^3}{4}\right)+\frac{U_2(t)}{2}p_1\left(p_2-\frac{p_1^2}{2}\right)+\frac{U_3(t)}{8}p_1^3\end{aligned}\right\}z^4.\\&+\left[a_3+a_2\frac{U_1(t)}{2}p_1+\frac{U_1(t)}{2}\left(p_2-\frac{p_1^2}{2}\right)+\frac{U_2(t)}{4}p_1^2\right]z^3\end{aligned}$$

Comparing the coefficients of the like power of z in both sides of the last equality with sample computation, we get

$$a_2=\frac{U_1(t)}{2}p_1, \tag{8}$$

$$a_3=\frac{U_1^2(t)}{8}p_1^2+\frac{U_1(t)}{4}\left(p_2-\frac{p_1^2}{2}\right)+\frac{U_2(t)}{8}p_1^2, \tag{9}$$

$$\begin{aligned}a_4=\frac{U_1(t)}{6}p_1a_3+\frac{a_2}{3}\left[\frac{U_1(t)}{2}\left(p_2-\frac{p_1^2}{2}\right)+\frac{U_2(t)}{4}p_1^2\right]\\+\frac{U_1(t)}{6}\left(p_3-p_1p_2+\frac{p_1^3}{4}\right)+\frac{U_2(t)}{6}p_1\left(p_2-\frac{p_1^2}{2}\right)+\frac{U_3(t)}{24}p_1^3.\end{aligned} \tag{10}$$

From the equality (8) and Lemma 1.1 first inequality of the theorem is clear.

From the equality (9) for a_3 we can write the following expression

$$a_3=\frac{U_1(t)}{4}p_2+\frac{U_1(t)\left[U_1(t)-1\right]+U_2(t)}{8}p_1^2;\ \text{that is,}\ a_3=\frac{t}{2}p_2+\frac{8t^2-2t-1}{8}p_1^2,\ t\in\left(\frac{1}{2},1\right).$$

Since the coefficients of p_2 and p_1^2 are positive for all $t\in\left(\frac{1}{2},1\right)$, from the last equality the inequality for $|a_3|$ is valid on using triangle inequality and the inequalities $|p_n|\le 2,\ n=2,3$.

From the equality (10) for a_4 we can write the following expression

$$\begin{aligned}a_4=\frac{U_1(t)}{6}p_1a_3+\frac{U_1^2(t)+2U_2(t)}{12}p_1\left(p_2-\frac{p_1^2}{2}\right)\\+\frac{U_1(t)}{6}\left[p_3-p_1p_2+\frac{U_1(t)+U_1(t)U_2(t)+U_3(t)}{4U_1(t)}p_1^3\right].\end{aligned}$$

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So that $a_4 = \frac{t}{3} p_1 a_3 + \frac{6t^2 - 1}{6} p_1 \left(p_2 - \frac{p_1^2}{2} \right) + \frac{t}{3} \left(p_3 - p_1 p_2 + \frac{4t^2 - 1}{2} p_1^3 \right)$, $t \in \left(\frac{1}{2}, 1 \right)$.

The last equality for a_4, we can write $a_4 = \frac{t}{3} p_1 a_3 + \frac{6t^2 - 1}{6} p_1 \left(p_2 - \frac{\nu}{2} p_1^2 \right) + \frac{t}{3} \left(p_3 - 2B p_1 p_2 + D p_1^3 \right)$, $t \in \left(\frac{1}{2}, 1 \right)$,

where $\nu = 1$, $B = \frac{1}{2}$ and $D = \frac{4t^2 - 1}{2}$.

We now use Lemma 1.2 and Lemma 1.3. Since $\nu \in [0,2]$, $B \in [0,1]$, and $B(2B-1) \le D \le B$ when $t \in \left(\frac{1}{2}, \frac{\sqrt{2}}{2} \right]$, applying triangle inequality, inequality obtained for $|a_3|$, Lemma 1.2 and Lemma 1.3, and again using the inequality $|p_1| \le 2$, we obtain $|a_4| \le \frac{1}{3} \left(8t^3 + 12t^2 + t - 2 \right)$ on the interval $t \in \left(\frac{1}{2}, \frac{\sqrt{2}}{2} \right]$.

It is easy see that $D \ge B$ for $t \in \left[\frac{\sqrt{2}}{2}, 1 \right)$. In this case expression for a_4, we can write

$$a_4 = \frac{t}{3} p_1 a_3 + \frac{6t^2 - 1}{6} p_1 \left(p_2 - \frac{\nu}{2} p_1^2 \right) + \frac{t}{3} \left(p_3 - 2B p_1 p_2 + B p_1^3 \right) + \frac{t}{3} \left(2t^2 - 1 \right) p_1^3 .$$

Then, the second inequality for $|a_4|$ is valid on the interval $t \in \left[\frac{\sqrt{2}}{2}, 1 \right)$ on using triangle inequality, inequality obtained for $|a_3|$, Lemma1.2, Lemma 1.3 (with $B = D$), and the inequalities $|p_n| \le 2$, $n = 1, 3$.

Thus, the proof of Theorem 2.1 is completed.

Theorem 2.2:

Let the function $f(z)$ given by (1) be in the class $S^* C(G; \beta, t)$, $\beta \ge 0$, $t \in \left(\frac{1}{2}, 1 \right)$. Then, $|a_2| \le \frac{2t}{1+\beta}$,

$$|a_3| \le \frac{8t^2 - 1}{2(1+2\beta)} \quad \text{and} \quad |a_4| \le \frac{1}{3(1+3\beta)} \begin{cases} 8t^3 + 12t^2 + t - 2 & \text{if } t \in \left(\frac{1}{2}, \frac{\sqrt{2}}{2} \right], \\ 24t^3 + 12t^2 - 7t - 2 & \text{if } t \in \left[\frac{\sqrt{2}}{2}, 1 \right). \end{cases}$$

Proof:

Let $f \in S^* C(G; \beta, t)$, $\beta \ge 0$, $t \in \left(\frac{1}{2}, 1 \right)$; that is,

$$\frac{z f'(z) + \beta z^2 f''(z)}{\beta z f'(z) + (1-\beta) f(z)} \prec \quad , \ z \in U \cdot \tag{11}$$

Define the function $F : U \to \square$ by

$$F(z) = \beta z f'(z) + (1-\beta) f(z), \ z \in U,$$

the condition (11) we can write as follows

$$\frac{zF'(z)}{F(z)} \prec G(t,z), z \in U.$$

It is clear that

$$F(z) = z + \sum_{n=2}^{\infty} A_n z^n , \ z \in U ,$$

where $A_n = (1+(n-1)\beta)a_n$, $n = 2,3,4,...$. So that, the function $F(z)$ belong to the class $S^*C(G;t)$, $t \in \left(\frac{1}{2},1\right)$.

Using Theorem 2.1 for $|A_n|$, $n = 2,3,4$ we can write

$$|A_2| \le 2t, , \ |A_3| \le \frac{8t^2-1}{2} , \ |A_4| \le \frac{1}{3}\begin{cases} 8t^3+12t^2+t-2 & \text{if } t \in \left(\frac{1}{2},\frac{\sqrt{2}}{2}\right], \\ 24t^3+12t^2-7t-2 & \text{if } t \in \left[\frac{\sqrt{2}}{2},1\right). \end{cases}$$

Applying equalities $A_n = (1+(n-1)\beta)a_n$ gives the inequalities for $|a_n|$, $n = 2,3,4$.

With this, the proof of Theorem 2.2 is completed.

The following theorem is direct result of the Theorem 2.1.

Theorem 2.3:

Let the function $f(z)$ given by (1) be in the class $C(G;t)$, $t \in \left(\frac{1}{2},1\right)$. Then,

$$|a_2| \le t, \ |a_3| \le \frac{8t^2-1}{6} , \ |a_4| \le \frac{1}{12}\begin{cases} 8t^3+12t^2+t-2 & \text{if } t \in \left(\frac{1}{2},\frac{\sqrt{2}}{2}\right], \\ 24t^3+12t^2-7t-2 & \text{if } t \in \left[\frac{\sqrt{2}}{2},1\right). \end{cases}$$

III. Conclusion

In this work, making the second kind Chebyshev polynomials, was introduced and investigated new subclasses of analytic functions on the open unit disk in the complex plane. Here, was given upper bound estimates for the initial three coefficients of the function belonging to these classes.

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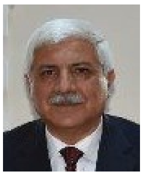

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