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Some Results on Soft *BCK/BCI*-Algebras

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Abstract **– The concept of Soft BCK/BCI-algebra and Soft BCK/BCI-subalgebra were introduced by Jun with some properties discussed. In this work, we extend the work of Jun by defining the basic algebraic operations such as restricted intersection, extended intersection, restricted union, Cartesian product, direct sum, OR-product on two Soft BCK/BCI-algebra and Soft BCK/BCI-subalgebra and family of Soft BCK/BCI-algebra and Soft BCK/BCI-subalgebra. We also present the algebraic properties of the operations and establish some basic results.**

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I. Introduction

The certainty of information has always been a major challenge in recent times. It is very difficult to obtain the expected degree of precision in available information. In an attempt to deal with the problem of imprecision in information, various models and theories were postulated, developed and employed by mathematician but none of them seems to proffer solution to the problem of imprecision and uncertainty. It was concluded that the fundamental cause of uncertainty lies in the set theory based on classical logic.

Molodtsov [1] in 1999 invented or discovered soft set theory as a new general mathematical tool to solve the problem of uncertainty in information. Maji *et al.* [2] showed the significance of soft set theory by applying it in decision making problems.

Aktas and Cagman [3] applied the notion of soft group theory on soft set. Jun [4], Jun and Park [5] have explored BCK/BCI-algebras and their applications in soft sense. However, Feng [6] investigated soft semi-rings and soft ideals. Moreover, Shabir and Irfan [7] restricted binary operation and investigated soft ideals over a semi group.

Recently, the algebraic study of soft set theory has received incredible attention by researchers. Aktas and Cagman [3] extended soft set theory to soft group. However, Acar et al. [8] introduced the basic idea of soft ring, which is a parameterized family of subrings and ideals of a ring. Atagun and Sezgin [9] presented soft subring and soft ideal, soft subfield over a field and soft sub-module over a left R-module. Celik *et al*. [10] described a novel concept of soft rings, soft ideals and introduced some new operations in soft set theory. The notion of soft modules and its properties are defined in [11].

In this paper, we extend the work of Jun by studying the basic algebraic properties such as restricted intersection, extended intersection, restricted union, Cartesian product, direct sum, OR-product on two Soft BCK/BCI-algebra and Soft BCK/BCI-subalgebra and family Soft BCK/BCI-algebra and Soft BCK/BCI-subalgebra. We also established some basic results on the various algebraic operations.

II. PRELIMINARIES

2.1 *Soft Set*

We first recall some basic notions in soft set theory. Let U be an initial universe set, E be a set of parameters or attributes with respect to U, $P(U)$ be the power set of U and $A \subseteq E$.

Definition 2.1.1 [1].

A pair(F, A) is called a **soft set** over U, where F is a mapping given by $F: A \rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $x \in A$, $F(x)$ may be considered as the set of x-elements or as the set of x-approximate elements of the soft set (F, A). The soft set (F, A) can be represented as a set of ordered pairs as follows: $(F, A) = \{(x, F(x)), x \in A, F(x) \in P(U)\}$

Definition 2.1.2 [2].

Let (F, A) and (G, B) be two soft sets over U. Then

(i) (F, A) is said to be a **soft subset** of (G, B), denoted by$(F, A) \tilde{\subseteq} (G, B)$, if $A \subseteq B$ and $F(x) \subseteq G(x), \forall x \in A$

(ii) (F, A) and (G, B) are said to be **soft equal**, denoted by $(F, A) = (G, B)$, if $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\subseteq} (F, A)$.

Definition 2.1.3 [12].

Let (F, A) be a soft set over U. Then the support of (F, A) written **supp** (F, A) is defined as **supp** $(F, A) = \{x \in A: F(x) \neq \emptyset\}$.

(i) (F, A) is called a **non-null** soft set if supp $(F, A) \neq \emptyset$.

(ii) (F, A) is called a **relative null** soft set denoted by$\emptyset_A$ if $F(x) = \emptyset, \forall x \in A$.

(iii) (F, A) is called a **relative whole** soft set, denoted by U_A if $F(x) = U, \forall x \in A$.

Definition 2.1.4. [13].

Let (F, A) be a soft set over U. If $F(x) \neq \emptyset$ for all $x \in A$, then (F, A) is called a non-empty soft set.

Definition 2.1.5 [12].

Let (F, A) and (G, B) be two soft sets over U. Then the **union** of (F, A) and (G, B), denoted by $(F, A) \tilde{\cup} (G, B)$ is a soft set defined as $(F, A) \tilde{\cup} (G, B) = (H, C), where\ C = A \cup B\ and\ \forall x \in C$,

$$H(x) = \begin{cases} F(x), & if\ x \in A - B \\ G(x), & if x \in B - A \\ F(x) \cup G(x), & if\ x \in A \cap B \end{cases}$$

Definition 2.1.6 [13].

Let (F, A) and (G, B) be two soft sets over U. Then the **restricted union** of (F, A) and (G, B), denoted by $(\Gamma, A) \tilde{\cup}_R (G, B)$ is a soft set defined as;$(F, A) \tilde{\cup}_R (G, B) = (H, C)$, where $C = A \cap B \neq \emptyset$ and $\forall$ $x \in C$, $H(x) = F(x) \cup G(x)$.

Definition 2.1.7 [12].

Let (F, A) and (G, B) be two soft sets over U. Then the **extended intersection** of (F, A) and (G, B), denoted by $(F, A)\ \tilde{\cap}_E\ (G, B)$, is a soft set defined as $(F, A)\ \tilde{\cap}_E\ (G, B) = (H, C)$ where $C = A \cup B$ and $\forall\ x \in C$,

$$H(x) = \begin{cases} F(x), & if\ x \in A - B \\ G(x), & if x \in B - A \\ F(x) \cap G(x), & if\ x \in A \cap B \end{cases}$$

Definition 2.1.8 [12].

Let (F, A) and (G, B) be two soft sets over U. Then the **restricted intersection** of (F, A) and (G, B) denoted by $(F, A) \Cap (G, B)$, is a soft set defined as $(F, A) \Cap (G, B) = (H, C)$ where $C = A \cap B$ and $\forall\ x \in C, H(x) = F(x) \cap G(x)$.

Definition 2.1.9 [2].

Let (F, A) and (G, B) be two soft sets over U. Then the **AND-product** or **AND-intersection** of (F, A) and (G, B) denoted by $(F, A)\tilde{\wedge}(G, B)$ is a soft set defined as $(F, A)\tilde{\wedge}(G, B) = (H, C),\ where\ C = A \times B\ and\ \forall\ (x, y) \in A \times B, H(x, y) = F(x) \cap G(y)$.

Definition 2.1.10 [2].

Let (F, A) and (G, B) be two soft sets over U. Then the **OR-product** or **OR-union** of (F, A) and (G, B), denoted by $(F, A)\tilde{\vee}(G, B)$ is a soft set defined as $(F, A)\tilde{\vee}(G, B) = (H, C), where\ C = A \times B\ and\ \forall\ (x, y) \in A \times B, H(x, y) = F(x) \cup G(y)$.

Definition 2.1.11. [2]

Let (F, A) and (G, B) be two soft sets over the universes U_1 and U_2 respectively. Then the **Cartesian product** of (F, A) and (G, B), denoted by $(F, A)\ \tilde{\times}\ (G, B)$is a soft define as: $(F, A)\ \tilde{\times}\ (G, B) = (H, C)$, where $C = A \times B$ and $\forall\ (x, y) \in A \times B$

$H(x, y) = F(x) \times G(y)$.

Definition 2.1.12. [16]

Let (F, A) and (G, B) be two nonempty soft sets over U. The **sum**$(F, A)\tilde{\mp}(G, B)$ is define as the soft set$(H, C) = (F, A)\tilde{\mp}(G, B)$, where $C = A \times B$ and$H(x, y) = F(x) + G(y), \forall\ (x, y) \in C$.

Definition 2.1.13. [6]

Let $(F_i, A_i)_{i \in I}$ be a nonempty family of soft sets over a common universe U. The union of these soft sets is defined to be the soft set (G, B) such that $B = \cup_{i \in I} A_i$ and for all $x \in B$, $G(x) = \cup_{i \in I(x)} F_i(x)\ where\ I(x) = \{i \in I: x \in A_i\}$. In this case we write $\tilde{\cup}_{i \in I}\ (F_i,\ A_i) = (G, B)$.

Definition 2.1.14. [6]

Let $(F_i, A_i)_{i \in I}$ be a nonempty family of soft sets over a common universe U. The AND-soft set $\tilde{\wedge}_{i \in I}(F_i,\ A_i)$ of these soft sets is defined to be the soft set (H, B) such that $B = \prod_{i \in I} A_i$ and $H(x) = \cap_{i \in I} F_i(x_i)\ for\ all\ x = (x_i)_{i \in I} \in B$.

Definition 2.1.15. [15]

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Let $(F_i, A_i)_{i\in I}$ be a nonempty family of soft sets over a common universe set U. The OR-soft set $\widetilde{\vee}_{i\in I}(F_i,\ A_i)$ of these soft sets is defined to be the soft set (H, B) such that $B = \prod_{i\in I} A_i$ and $H(x) = \cup_{i\in I} F_i(x_i)$ for all $x = (x_i)_{i\in I} \in B$.

Note that if $A_i = A$ and $F_i = F$ for all $i \in I$, then $\widetilde{\wedge}_{i\in I}(F_i,\ A_i)$ (res. $\widetilde{\vee}_{i\in I}(F_i,\ A_i)$) is denoted by $\widetilde{\wedge}_{i\in I}(F, A)$ (res. $\widetilde{\vee}_{i\in I}(F, A)$). In this case, $\prod_{i\in I} A_i = \prod_{i\in I} A$ means the direct power A^I.

Definition 2.1.16. [15]

The restricted union of a nonempty family of soft sets $(F_i, A_i)_{i\in\Delta}$ over a common universe U is defined as the soft set $(H, B) = \widetilde{\cup}_{R\ i\in\Delta} (\Gamma_i,\ A_i)$, where $B = \cap_{i\in\Delta}\ A_i \neq \emptyset$ and $H(x) = \cup_{i\in\Delta} F_i(x)$ for all $x \in B$.

Definition 2.1.17. [14]

The extended intersection of a nonempty family of soft sets $(F_i,\ A_i)_{i\in\Delta}$ over a common universe set U is defined as the soft set $(H, B) = \widetilde{\cap}_{E\ i\in\Delta} (F_i, A_i)$ such that $B = \cup_{i\in\Delta} A_i$ and $H(x) = \cap_{i\in\Delta} F_i(x)$ where $\Delta(x) = \{i \in \Delta : x \in A_i\}$ for all $x \in B$.

Definition 2.1.18. [14]

Let $(F_i, A_i)_{i\in\Delta}$ be a nonempty family of soft sets over a common universe set U. The restricted intersection of these soft sets is defined to be the soft set (G, B) such that $B = \cap_{i\in\Delta} A_i \neq \emptyset$ and for all $x \in B, G(x) = \cap_{i\in\Delta} F_i(x)$. In this case we write $⋒_{i\in\Delta} (F_i,\ A_i) = (G, B)$.

Definition 2.1.19. [14]

Let $(F_i, A_i)_{i\in\Delta}$ be a nonempty family of soft sets over $U_i, i \in \Delta$. The Cartesian product of $(F_i, A_i)_{i\in\Delta}$ over U_i is defined as the soft set $(H, B) = \widetilde{\prod}_{i\in\Delta} (F_i, A_i)\ where\ B = \prod_{i\in\Delta} A_i$ and $H(x) = \prod_{i\in\Delta} F_i(x_i)\ for\ all\ x = (x_i)_{i\in\Delta} \in B$. It is worth noting that if $A_i = A$ and $F_i = F$ for all $i \in \Delta$, then $\widetilde{\prod}_{i\in\Delta} (F_i, A_i)$ is denoted by $\prod_{i\in\Delta}(F, A)$. In this case $\prod_{i\in\Delta}(A_i) = \prod_{i\in\Delta} A$ means the direct power A^Δ.

Definition 2.1.20. [14]

Let $(F_i, A_i)_{i\in\Delta}$ be a nonempty family of soft sets over $U_i, i \in \Delta$. The sum of $(F_i, A_i)_{i\in\Delta}$ over U_i is defined as the soft set $(H, B) = \widetilde{\Sigma}_{i\in\Delta} (F_i, A_i)\ where\ B = \prod_{i\in\Delta} A_i$ and $H(x) = \Sigma_{i\in\Delta} F_i(x_i)\ for\ all\ x = (x_i)_{i\in\Delta} \in B$.

2.2. *BCK/BCI-algebras*

BCK and *BCI* algebras are algebraic structures introduced by Iseki *et al.* in 1966, that describe fragments of the propositional calculus involving implication known as *BCK* and *BCI* logics.

BCI-algebra

Definition 2.2.1. [4]

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a BCI-algebra if, for any $x, y, z \in X$, satisfies the following conditions:

$BCI - 1.\ \big((x * y) * (x * z)\big) * (z * y) = 0$

$BCI - 2.\ \big(x * (x * y)\big) * y = 0$

$BCI-3.\ x*x=0$

$BCI-4.\ x*y=0 \wedge y*x=0 \ \rightarrow x=y$

$BCI-5.\ x*0=0 \rightarrow x=0$

BCK-algebra

Definition 2.2.2.

A BCI-algebra $(X;*,0)$ of type $(2,0)$ is called a BCK-algebra if it satisfies the following condition: $BCK-1.$ $0*x=0, \forall\, x \in X$.

Any BCK-algebra X satisfies the following axioms for any $x, y, z \in X$;

(a_1) $x*0=x$,

(a_2) $x \leq y \ \Rightarrow x*z \leq y*z, z*y \leq z*x$,

(a_3) $(x*y)*z=(x*z)*y$,

(a_4) $(x*z)*(y*z) \leq x*y$.

Where $x \leq y$ if and only if $x*y=0$.

BCK-algebra is commutative if it satisfies the identity $x*(x*y)=y*(y*x)$. In this regard, $x*(x*y)=x\wedge y, \forall x, y \in X$, the greatest lower bound of x and y under the partial order $\leq$. The BCK-algebra is bounded if it has a largest element. Denoting this element by 1, one obtain $1*\big((1*x)\wedge(1*y)\big)=x\vee y$, the least upper bound of x and y. In this case, X is a distributive lattice with bounds 0 and 1. A BCK-algebra is positive implicative if it satisfies the identity $(x*y)*z=(x*z)*(y*z)$. This is equivalent to the identity $x*y=(x*y)*y$. X is called implicative if it satisfies the identity $x*(y*x)=x$. Every implicative BCK-algebra is commutative and positive implicative and a bounded BCK-algebra is Boolean algebra.

Definition 2.2.3.

A nonempty subset S of a BCK/BCI-algebra X is called a BCK/BCI-subalgebra of X if $x*y \in S$ for all $x, y \in S$.

III. Soft *BCK/BCI*- algebras

Let X be a BCK/BCI-algebra, A be a nonempty set and R be an arbitrary binary relation between an element of A and an element of X, that is R is a subset of $A \times X$ unless otherwise indicated. A set valued function $F: A \rightarrow P(X)$ defined by $F(x)=\{y \in X: xRy\}$ for all $x \in A$.

The pair (F, A) is then a soft set over X. For any element x of a BCK/BCI-algebra X, we define the order of x, denoted by $O(x)$, as $O(x)=\min\{n \in \mathbb{N}: 0*x^n=0\}$.

Definition 3.1. [4]

Let (F, A) be a soft set over X. Then (F, A) is called a soft BCK/BCI-algebra over X if $F(x)$ is a BCK/BCI-subalgebra of X for all $x \in A$.

The definition can be illustrated with the following example.

Example 3.1.Let $X = \{0, a, b, c, d\}$ be a BCK-algebra with the following Caley table [4].

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	a	a
b	b	b	0	b	b
c	c	c	c	0	c
d	d	d	d	d	0

Let (F, A) be a soft set over X, where $A = X$ and $F: A \rightarrow P(X)$ is a set-valued function defined by

$F(x) = \{y \in X: xRy \leftrightarrow y \in x^{-1}I\}$ for all $x \in A$, where $I = \{0, a\}$ and$x^{-1}I = \{y \in X: x\wedge y \in I\}$. Then $F(0) = X$, $F(a) = \{0, 0, b, c, d\}$, $F(b) = \{0, a, c, d\}$, $F(c) = \{0, a, b, d\}$, $F(d) = \{0, a, b, c\}$ are BCK-subalgebras of X. Therefore, (F, A) is a soft BCK-algebra over X.

Example 3.2. Consider a BCI-algebra $X = \{0, a, b, c\}$ with the following Caley table [4].

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let $A = X$ and let $F: A \rightarrow P(X)$ be a set-valued function defined by $F(x) = \{y \in X: xRy \leftrightarrow y = x^n, n \in \mathbb{N}\}$ for all $x \in A$, where $x^n = x * x * \dots * x$ in which x appears $n - times$. Then $F(0) = \{0\}$, $F(a) = \{0, a\}$, $F(b) = \{0, b\}$, $F(c) = \{0, c\}$ which are BCI-subalgebras of X. Hence, (F, A) is a soft BCI-algebra.

Suppose, we define a set-valued function $H: A \rightarrow P(X)$ by $H(x) = \{y \in X: xRy \leftrightarrow O(x) = o(y)\}$ for all $x \in A$, then $H(0) = \{0\}$ is a BCI-subalgebra of X, but $H(a) = H(b) = H(c) = \{a, b, c\}$ are not subalgebras of X. This shows that there exists a set-valued function $H: A \rightarrow P(X)$ such that, the soft set (H, A) is not a soft BCI-algebra over X.

Example 3.3. Let $X = \{0, a, b, c, d, e, f, g\}$ and consider the following Caley table [4].

*	0	a	b	c	d	E	f	g
0	0	0	0	0	d	D	d	d
A	a	0	0	0	e	D	d	d
B	b	b	0	0	f	F	d	d
C	c	b	a	0	g	F	e	d
D	d	d	d	d	0	0	0	0
E	e	d	d	d	a	0	0	0
f	f	f	d	d	b	B	0	0
g	g	f	e	d	c	B	a	0

Then $(X; *,\ 0)$ is a BCI-algebra. Let (F, A) be a soft set over X, where $A = X$ and $F: A \rightarrow P(X)$ is a set-valued function defined by $F(x) = \{0\} \cup \{y \in X: xRy \Leftrightarrow O(x) = O(y)\}$ for all $x \in A$. Then $F(0) = F(a) = F(c) = \{0, a, b, c\}$ is a BCI-subalgebra of X, but $F(d) = F(e) = F(f) = F(g) = \{0, d, e, f, g\}$ is not a BCI-subalgebra of X. Hence, (F, A) is not a soft BCI-algebra over X, but if we take $B = \{a, b, c\} \subset X$ and defined a set-valued function $G: B \rightarrow P(X)$ by $G(x) = \{y \in X: xRy \Leftrightarrow O(x) = O(y)\}$ for all $x \in B$. Then $(G, B)\ is\ a\ soft\ BCI-algebra\ over\ X, since\ G(a) = G(b) = G(c) = \{0, a, b, c\}$ is a BCI-subalgebra of X.

Let A be a fuzzy BCK/BCI-subalgebra of X with membership μ_A. Let us consider the family of α −level sets for the function μ_A given by $F(\alpha) = \{x \in X: \mu_A(x) \geq \alpha\}$, $\alpha \in [0,1]$.

Then, $F(\alpha)$ is a BCK/BCI-subalgebra of X. If we know the family F, we can find the functions $\mu_A(x)$ by means of the following formula: $\mu_A(x) = \sup\{\alpha \in [0,1]: x \in F(\alpha)\}$.

Therefore, every fuzzy BCK/BCI-subalgebra A may be considered as the soft BCK/BCI-algebra $(F, [0, 1])$.

Theorem 3.1.

Let (F, A) be a soft set over X. If B is a subset of A, then (F_B, B) is a soft BCK/BCI-algebra over X.

Proof:

Let X be a BCK/BCI-algebra and let (F, A) be a soft set defined over X. We need to show that (F, A) is a soft BCK/BCI-algebra over$X$. It implies that every BCK/BCI-subalgebra of X. Therefore, (F, A) is a soft BCK/BCI-algebra over X.

However, there are many situation in which every subsets of F is not a subfield. In that case, there must exist $B \subset A$ such that for each $a \in B$, $F(a) \in X$ is a subfield of X. Since every field X is a subfield of itself. Therefore, (F, B) is a subfield of X.

Hence, every BCK/BCI-algebra can be considered as a soft BCK/BCI-algebra.

The following example shows that there exists a soft set (F, A) over X such that

(i) (F, A) is not a soft BCI-algebra over X.

(ii) There exists a subset B of A such that (F_B, B) is a soft BCI-algebra over X.

Example 3.4.

Let (F, A) be a soft set over X given in the example **above.** It is important to note that (F, A) is not a soft BCI-algebra over X. However, if we take $B = \{a, b, c\} \subset A$, then (F_B, B) is a soft BCI-algebra over X.

Proposition 3.1.

Let (F, A) and (G, B) be soft BCK/BCI-algebra over X. Then

(i) If it is non-null, then the restricted intersection of $(F, A) \Cap (G, B)$ is a soft BCK/BCI-algebra over X.

(ii) If it is non-null, then the soft set $(F, A)\ \tilde{\cap}_E\ (G, B)$ is a soft BCK/BCI-algebra over X.

(iii) If $F(x)$ and $G(x)$ are ordered by inclusion relation for all $x \in supp\big((F, A)\ \tilde{\cup}_R\ (G, B)\big), then (F, A)\ \tilde{\cup}_R\ (G, B)$ is a soft BCK/BCI-algebra over X, whenever, it is non-null.

(iv) If it is non-null, then the soft set $(F, A)\widetilde{\wedge}(G, B)$ is a soft BCK/BCI-algebra over X.

(v) If A and B are disjoint, then $(F, A) \widetilde{\cup} (G, B)$ is a soft BCK/BCI-algebra over X, and if it is non-null.

(vi) If it is non-null, then the soft set $(F, A)\widetilde{\vee}(G, B) = (N, A \times B)$ is a soft BCK/BCI-algebra over $X$and if $F(x)$ and $G(y)$ are ordered by inclusion relation for all $(x, y) \in supp((N, A \times B)$.

(vii) If it is non-null, then the soft set $(F, A)\widetilde{\mp}(G, B)$ is a soft BCK/BCI-algebra over X.

Proof:

(i) Let $(F, A) \Cap (G, B) = (K, C)$ where $K(x) = F(x) \cap G(x)$ for all $x \in C = A \cap B \neq \emptyset$. Note that $K: \to P(X)$ is a mapping and therefore by hypothesis (K, C) is a non-null soft set over X. If $x \in supp(K, C)$, then $K(x) = F(x) \cap G(x) \neq \emptyset$. It follows that $F(x) \neq \emptyset$ and $G(x) \neq \emptyset$ are both BCK/BCI-subalgebras over$X$. Hence, $K(x)$ is a BCK/BCI-subalgebra over$X$, for all $x \in supp(K, C)$.Thus, (K, C) is a softBCK/BCI-algebra over X.

(ii) Let $(F, A) \widetilde{\cap}_E (G, B) = (M, A \cup B)$, where $M(x) = \begin{cases} F(x), & if\ x \in A - B \\ G(x), & if x \in B - A \\ F(x) \cap G(x), & if\ x \in A \cap B \end{cases}$

For all $x \in A \cup B$. Then by the hypothesis $(M, A \cup B)$ is a non-null soft set over X.

Let $x \in supp(M, A \cup B)$. If $x \in A - B$, then $\emptyset \neq M(x) = F(x)$. If $x \in B - A$, then $\emptyset \neq M(x) = G(x)$, and if $x \in A \cap B$, then $M(x) = F(x) \cap G(x) \neq \emptyset$. Since, $F(x) \neq \emptyset$ and $G(x) \neq \emptyset$, are both BCK/BCI-subalgebras over X then $M(x)$ is a BCK/BCI-subalgebra for all $x \in supp\ (M, A \cup B)$.

Therefore, $(F, A) \widetilde{\cap}_E (G, B) = (M, A \cup B)$ is a softBCK/BCI-algebra over X.

(iii) Let $(F, A) \widetilde{\cup}_R (G, B) = (R, A \cap B)$ where $R(x) = F(x) \cup G(x)$ for all $x \in A \cap B \neq \emptyset$. Then by hypothesis $(R, A \cap B)$ is a non-null soft set over X. If $x \in supp(R, A \cap B)$, $R(x) = F(x) \cup G(x) \neq \emptyset$. Since, $F(x)$ and $G(x)$ are ordered by inclusion relation for all $x \in supp(R, A \cap B), F(x) \cup G(x) = F(x)$ or $F(x) \cup G(x) = G(x)$. Since, $F(x) \neq \emptyset$ and $G(x) \neq \emptyset$ are both BCK/BCI-subalgebras over $X, R(x)$ is a BCK/BCI-subalgebra over X for all $x \in supp\ (R, A \cap B)$. Therefore, $(R, A \cap B)$ is a soft BCK/BCI-algebra over X.

(iv) Let $(F, A)\widetilde{\wedge}(G, B) = (Q, A \times B)$, where $Q(x, y) = F(x) \cap G(y)$, for all $(x, y) \in A \times B$. Then by hypothesis, $(Q, A \times B)$ is non-null soft set over X. If $(x, y) \in supp(Q, A \times B)$, then $Q(x, y) = F(x) \cap G(y) \neq \emptyset$. It follows that $F(x) \neq \emptyset$ and $G(y) \neq \emptyset$ are both BCK/BCI-subalgebras over X. Hence, $Q(x, y)$ is a BCK/BCI-algebra over X for all $(x, y) \in supp(Q, A \times B)$. Therefore,$(F, A)\widetilde{\wedge}(G, B)$ is a soft BCK/BCI-algebra over X.

(v) Let $(F, A) \widetilde{\cup} (G, B) = (V, A \cup B)$ where $V(x) = \begin{cases} F(x), & if\ x \in A - B, \\ G(x), & if x \in B - A, \\ F(x) \cup G(x), & if\ x \in A \cap B, \end{cases}$

For all $x \in A \cup B$, and $A \cap B = \emptyset$, it follows that either $x \in A - B$ or $x \in B - A$, forall $x \in A \cup B$.

If $x \in A - B$, then $V(x) = F(x)$ is a BCK/BCI-subalgebra over$X$ and if $x \in B - A$, then $V(x) = G(x)$ is a BCK/BCI-subalgebra over X. Therefore, $(F, A) \widetilde{\cup} (G, B)$ is a soft BCK/BCI-algebra over X.

(vi) Let $(F, A)\widetilde{\vee}(G, B) = (N, A \times B)$, where $N(x, y) = F(x) \cup G(y)$, for all $(x, y) \in A \times B$. Then by hypothe-

-sis, $(N, A \times B)$ is a non-null soft set over F. If $(x, y) \in supp(N, A \times B)$, then $N(x, y) = F(x) \cup G(y) \neq \emptyset$. Since $F(x)$ and $G(y)$ are ordered by inclusion relation for all $(x, y) \in supp(N, A \times B)$, $F(x) \cup G(y) = F(x)$ or $F(x) \cup G(y) = G(y)$. Since, $F(x) \neq \emptyset$ and $G(y) \neq \emptyset$ are both BCK/BCI-subalgebra over$X$ for all $(x, y) \in supp(N, A \times B)$. Therefore, $(F, A)\tilde{\vee}(G, B)$ is a soft BCK/BCI-algebra over X.

(vi) $(F, A)\tilde{\mp}(G, B) = (H, A \times B)$, where $H(x, y) = F(x) + G(y)$, for all $(x, y) \in A \times B$. Then by the hypothesis, $(H, A \times B)$ is a non-null soft set over X. Suppose $(x, y) \in supp(H, A \times B)$, then $H(x, y) = F(x) + G(y) \neq \emptyset$. It means that $F(x) \neq \emptyset$ and $G(y) \neq \emptyset$ are both BCK/BCI-subalgebra of X. Hence, $H(x, y)$ is a BCK/BCI-subalgebra of X for all $(x, y) \in supp(H, A \times B)$. Therefore, $(F, A)\tilde{\mp}(G, B)$ is a soft BCK/BCI-algebra over X.

Proposition 3.2.

Let (F, A) and (G, B) be two soft BCK/BCI-algebras over M and N respectively. If it is non-null, then the product $(F, A) \times (G, B)$ is a soft BCK/BCI-algebra over $M \times N$.

Proof:

By definition 2.1.11, we can write $(F, A) \times (G, B) = (J, A \times B)$, where $J(x, y) = F(x) \times G(y)$, for all $(x, y) \in A \times B$. Then by hypothesis, $(J, A \times B)$ is non-null soft set over $M \times N$. If $(x, y) \in supp\ (J, A \times B)$, then $J(x, y) = F(x) \times G(y) \neq \emptyset$. Since, $F(x) \neq \emptyset$ is a BCK/BCI-subalgebra over M and $G(y) \neq \emptyset$ is a BCK/BCI-subalgebra over N, it implies that $J(x, y)$ is a BCK/BCI-subalgebra of $M \times N$ for all $(x, y) \in supp(J, A \times B)$. Therefore, $(F, A) \times (G, B)$ is a soft BCK/BCI-algebra over$M \times N$.

Proposition 3.3.

Let $(F_i,\ A_i)_{i \in I}$ be a nonempty family of soft BCK/BCI-algebras over X. Then the following holds:

(i) $\tilde{\wedge}_{\boldsymbol{i \in I}}(F_i,\ A_i)$ is a soft BCK/BCI-algebra over X, if it is non-null.

(ii) $\Cap_{\boldsymbol{i \in I}}\ (F_i,\ A_i)$ is a soft BCK/BCI-algebra over X, if it is non-null.

(iii) $\tilde{\cap}_{\boldsymbol{E\ i \in I}}\ (F_i,\ A_i)$ is a soft BCK/BCI-algebra over X, if it is non-null.

(iv) If $\{A_i : i \in I\}$ are pairwise disjoint, that is, $i \neq j$ implies $A_i \cap A_j = \emptyset$, then $\tilde{\cup}_{i \in I}\ (F_i,\ A_i)$ is a soft BCK/BCI-algebra over X.

(v) Let $F_i(x_i) \subseteq F_j(x_j)$ or $F_j(x_j) \subseteq F_i(x_i)$, for all $i, j \in I$ and $x_i \in A_i$, then $\tilde{\cup}_{R\ i \in I}\ (F_i,\ A_i)$ is a soft BCK/BCI-algebra over X, whenever it is non-null.

(vi) Let $F_i(x_i) \subseteq F_j(x_j)$ or $F_j(x_j) \subseteq F_i(x_i)$, for all $i, j \in I$ and $x_i \in A_i$, then $\tilde{\vee}_{\ i \in I}(F_i,\ A_i)$ is a soft BCK/BCI-algebra over X, whenever it is non-null.

(vii) $\tilde{\Sigma}_{i \in I}(F_i,\ A_i)$ is a softBCK/BCI-algebra over X, whenever it is non-null.

Proof:

(i) By definition 2.1.14, we can write $\tilde{\wedge}_{\boldsymbol{i \in I}}(F_i,\ A_i) = (H, C)$, where $C = \prod_{i \in I} A_i$, for all $x = (x_i) \in C$, we have $H(x) = \cap_{i \in I} F_i(x_i)$. By the hypothesis (H, C) is a non-null soft set over X. If $x = (x_i)_{i \in I} \in supp(H, C)$. Then, $H(x) = \cap_{i \in I} F_i(x_i) \neq \emptyset$, so we have $F_i(x_i) \neq \emptyset$ for all $i \in I$, since $(\Gamma_i,\ A_i)$ is a family of soft

BCK/BCI-algebras over X for all $i \in I$, we have $\cap_{i\in I} F_i(x_i)$ are BCK/BCI-subalgebras of X, for all $x = (x_i)_{i\in I} \in C$. Therefore, $\tilde{\wedge}_{i\in I}(F_i, A_i)$ is a softBCK/BCI-algebra over X.

(ii) Let $(F_i, A_i)_{i\in I}$ be a nonempty family of soft BCK/BCI-algebras over X. By definition 2.1.18, we can write $⋒_{i\in I} (F_i, A_i) = (G, B)$, where $B = \cap_{i\in\Delta} A_i \neq \emptyset$ and for all $x = (x_i)_{i\in\Delta} \in B$, $G(x) = \cap_{i\in\Delta} F_i(x)$.

Suppose that (G, B) is a non-null soft set over X. If $x \in supp(G, B)$, then $G(x) = \cap_{i\in\Delta} F_i(x) \neq \emptyset$, so we have $F_i(x) \neq \emptyset$ for all $i \in \Delta$, since(F_i, A_i) is a family of soft BCK/BCI-algebras over X for all $i \in \Delta$.

It follows that, $\cap_{i\in\Delta} F_i(x_i)$ are BCK/BCI-subalgebras of X, for all $x = (x_i)_{i\in I} \in B$. Hence, $⋒_{i\in I} (F_i, A_i)$ is a soft BCK/BCI-algebra over X.

(iii) Suppose that $(F_i, A_i)_{i\in I}$ be a nonempty family of soft BCK/BCI-algebras over X. By definition 2.1.17, we can write$\tilde{\cap}_{E\, i\in I} (F_i, A_i) = (M, C)$, where $C =\cup_{i\in\Delta} A_i$ and $M(x) = \cap_{i\in\Delta} F_i(x)$ where $\Delta(x) = \{i \in \Delta : x \in A_i\}$ for all $x \in C = \cup_{i\in\Delta} A_i$. Suppose that (M, C) is a non-null soft set over X. If$x \in supp(M, C)$, then $M(x) = \cap_{i\in\Delta} F_i(x) \neq \emptyset$, so, we have $F_i(x) \neq \emptyset$ for all $i \in \Delta$. Since (F_i, A_i) is a family of soft BCK/BCI-algebras over X for all $i \in \Delta$. It follows that, $\cap_{i\in\Delta} F_i(x)$ are BCK/BCI-subalgebras of X for all $x \in supp(M, C)$. Therefore, $\tilde{\cap}_{E\, i\in I} (F_i, A_i)$ is a softBCK/BCI-algebra over X.

(iv) Let $(F_i, A_i)_{i\in I}$ be a nonempty family of soft BCK/BCI-algebras over X. By definition 2.1.13, we can write$\tilde{\cup}_{i\in I} (F_i, A_i) = (G, B)$, where $B = \cup_{i\in I} A_i$ and for all $x = (x_i)_{i\in I} \in B$, $G(x) = \cup_{i\in I} F_i(x)$ where $I(x) = \{i \in I : x \in A_i\}$. Then by the hypothesis, (G, B) is a non-null soft set over X. If $x \in supp(G, B)$, then $G(x) = \cup_{i\in I} F_i(x) \neq \emptyset$. Since, $\{A_i : i \in I\}$ are pairwise disjoint, that is, $i \neq j$ implies $A_i \cap A_j = \emptyset$,so we have $F_i(x) \neq \emptyset$ for all $i \in I(x)$. Since (F_i, A_i) is a family of softBCK/BCI-algebras over X for all $i \in I(x)$. Therefore, $\tilde{\cup}_{i\in I} (F_i, A_i)$ is a softBCK/BCI-algebra over X.

(v) Let $(F_i, A_i)_{i\in I}$ be a nonempty family of soft BCK/BCI -algebras over X. By definition 2.1.16, we can write$\tilde{\cup}_{R\, i\in I} (F_i, A_i) = (H, B)$, where $B =\cap_{i\in I} A_i \neq \emptyset$ and $H(x) = \cup_{i\in I} F_i(x)$ for all $x = (x_i)_{i\in I} \in B$. By the hypothesis (H, B) is a non-null soft set over X. If $x \in supp(H, B)$, then $H(x) = \cup_{i\in I} F_i(x) \neq \emptyset$ for all $x \in B$. Since, $F_i(x_i) \subseteq F_j(x_j)$or $F_j(x_j) \subseteq F_i(x_i)$, for all $i, j \in I$and $x_i \in A_i$, so we have $F_i(x) \neq \emptyset$ for all $i \in I$. Since (F_i, A_i) is a family of soft BCK/BCI-algebras over X for all $i \in I$. It follows that $\cup_{i\in I} F_i(x)$ are BCK/BCI-subalgebras of X for all $x \in B$. Therefore, $\tilde{\cup}_{R\, i\in I} (F_i, A_i)$ is a soft BCK/BCI-algebra over X.

(vi) Assume that $(F_i, A_i)_{i\in I}$ be a nonempty family of soft BCK/BCI-algebras over X. By definition 2.1.15, we can write $\tilde{\vee}_{i\in I}(F_i, A_i) = (M, C)$, where $C = \prod_{i\in I} A_i \neq \emptyset$ and $M(x) = \cup_{i\in I} F_i(x_i)$ for all $x = (x_i)_{i\in I} \in C$. By the hypothesis, (M, C) is a non-null soft set over X. If $x \in supp(M, C)$, then $M(x) = \cup_{i\in I} F_i(x_i) \neq \emptyset$ for all $x = (x_i)_{i\in I} \in C$. Since, $F_i(x_i) \subseteq F_j(x_j)$or $F_j(x_j) \subseteq F_i(x_i)$, for all $i, j \in I$and $x_i \in A_i$,so we have $F_i(x_i) \neq \emptyset$ for all $i \in I$. Since (F_i, A_i) is a family of softBCK/BCI-algebras over X for all $i \in I$, it follows that $\cup_{i\in I} F_i(x_i)$ are BCK/BCI-subalgebras of X for all $x \in C$. Therefore, $\tilde{\vee}_{i\in I}(F_i, A_i)$ is a soft BCK/BCI-algebra over X.

(vii) By definition 2.1.20, we can write $\tilde{\Sigma}_{i\in I}(F_i, A_i) = (G, B)$, where $B = \prod_{i\in I} A_i$ and $G(x) = \sum_{i\in I} F_i(x_i)$ for all $x = (x_i)_{i\in I} \in B$. By the hypothesis (G, B) is a non-null soft set over X. If $x \in supp(G, B)$, then $G(x) = \sum_{i\in I} F_i(x_i) \neq \emptyset$ for all $x \in B$. So, we have $F_i(x_i) \neq \emptyset$ for all $i \in I$.Since (F_i, A_i) is a family of soft BCK/BCI-algebras over X for all $i \in I$, it follows that $\sum_{i\in I} F_i(x_i)$ are BCK/BCI-subalgebras of X for all x

$\in B$. Therefore, $\tilde{\Sigma}_{i\in I}(F_i,\ A_i)$ is a softBCK/BCI-algebra over X.

Proposition 3.4.

Let $(F_i,\ A_i)_{i\in I}$ be a nonempty family of soft BCK/BCI-algebra over X_i. Then $\tilde{\prod}_{i\in I}(F_i, A_i)$ is a soft BCK/BCI-algebra over$\prod_{i\in I} F_i$.

Proof:

By definition 2.1.19, We can write $(H,B) = \tilde{\prod}_{i\in I}(F_i, A_i)$, where $B = \prod_{i\in I} A_i$ and $H(x) = \prod_{i\in I} F_i(x_i)$ for all $x = (x_i)_{i\in I} \in B$. Let $x = (x_i)_{i\in I} \in supp(H,B)$. Then, $H(x) = \prod_{i\in I} F_i(x_i) \neq \emptyset$, so we have $F_i(x_i) \neq \emptyset$ for all $i \in I$. Since (F_i, A_i) is a family of softBCK/BCI-algebras over X_i , for all $i \in I$ we have $\prod_{i\in I} F_i(x_i)$ is aBCK/BCI-subalgebra of $\prod_{i\in I} X_i$, for all $x = (x_i)_{i\in I} \in B$. That is, the Cartesian product $\tilde{\prod}_{i\in I}(F_i, A_i)$ is a soft BCK/BCI-algebra over $\prod_{i\in I} X_i$.

Definition 3.2[15]

Let (F,A) and (G,B) be two soft BCK/BCI-algebras over X. Then (F,A) is a soft BCK/BCI-subalgebra of (G,B) written $(F,A) \tilde{<}_F (G,B)$ if the following conditions hold:

(i) $A \subseteq B$,

(ii) $F(x) <_X G(x)$, which means $F(x)$ is aBCK/BCI-subalgebra of $G(x)$for all $x \in supp(F,A)$.

(The notation $\tilde{<}_X$ refers to soft BCK/BCI-subalgebra).

Proposition 3.5.

Let (F,A),(G,A) and (H,B) be soft BCK/BCI-algebras over X. Then the following hold:

(i) If $F(x) \subseteq G(x)$ for all $x \in A$, then (F,A) is a soft BCK/BCI-subalgebra of (G,A).

(ii) $(F,A) \Cap (H,B)$ is a soft BCK/BCI-subalgebra of both (F,A) *and* (H,B) , if it is non- null.

(iii) $(F,A) \cap_E (G,A)$ is a soft BCK/BCI-subalgebra of both (F,A) *and* (G,A) if it is non-null.

Proof:

(i) Since$F(x) \subseteq G(x)$ for all $x \in A$ and $A \cap A = A \subseteq A$ it is obvious that$F(x)$ is a BCK/BCI-subalgebra of $G(x)$, denoted by $F(x) <_X G(x)$. Therefore, $(F,\ A)$ is a soft BCK/BCI-subalgebra of (G,A). (the notation $<_X$ refers to BCK/BCI-subalgebra).

(ii) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, the first condition of definition 3.5 is satisfied. Let $(F,A) \Cap (H,B) = (K,C)$, where $C = A \cap B$ and $K(x) = F(x) \cap H(x)$ for all $x \in C$. Since $K(x) = F(x) \cap H(x) \subseteq F(x)$ and $K(x) = \Gamma(x) \cap H(x) \subseteq H(x)$ for all $x \in C$. Therefore, $(\Gamma,A) \Cap (H,B)$ is a soft BCK/BCI-subalgebra of both (F,A) *and* (H,B)(see definition 3.2).

(iii) Since$A \cap A = A \subseteq A. Let\ (F,A) \cap_E (G,A) = (Q,A)$where $Q(x) = F(x) \cap G(x)$ for all $x \in A$. Since $Q(x) = F(x) \cap G(x) \subseteq F(x)$ and $Q(x) = F(x) \cap G(x) \subseteq G(x)$ for all $x \in A$. Therefore, $(F,A) \cap_E (G,A)$is a soft BCK/BCI-subalgebra of both (F,A) *and* (G,A) (see definition 3.2).

Proposition 3. 6.

Let (F, A) be a soft BCK/BCI-algebra over X and $(F_i, A_i)_{i\in I}$ be a nonempty family of soft BCK/BCI-subalgebras of (F, A). Then the following results hold:

(i) $\widetilde{\wedge}_{i\in I}(F_i,\ A_i)$ is a soft BCK/BCI-subalgebra of (F, A), if it is non-null.

(ii) $⋒_{i\in I}\ (F_i,\ A_i)$ is a soft BCK/BCI-subalgebra of (F, A), if it is non-null.

(iii) $\widetilde{\cap}_{E\ i\in I}\ (F_i,\ A_i)$ is a soft BCK/BCI-subalgebra of (F, A), if it is non-null.

(iv) If $\{A_i : i \in I\}$ are pairwise disjoint, that is, $i \neq j$ implies $A_i \cap A_j = \emptyset$, then $\widetilde{\cup}_{i\in I}\ (F_i,\ A_i)$ is a soft BCK/BCI-subalgebraof (F, A).

(v) Let $F_i(x_i) \subseteq F_j(x_j)$ or $F_j(x_j) \subseteq F_i(x_i)$, for all $i, j \in I$ and $x_i \in A_i$, then $\widetilde{\cup}_{R\ i\in I}\ (F_i,\ A_i)$ is a soft BCK/BCI-subalgebra of (F, A), whenever it is non-null.

(vi) Let $F_i(x_i) \subseteq F_j(x_j)$ or $F_j(x_j) \subseteq F_i(x_i)$, for all $i, j \in I$ and $x_i \in A_i$, then $\widetilde{\vee}_{i\in I}(F_i,\ A_i)$ is a soft BCK/BCI-subalgebra of(F, A), whenever it is non-null.

(vii) $\tilde{\Sigma}_{i\in I}(F_i,\ A_i)$ is a soft BCK/BCI-subalgebra of (F, A), whenever it is non-null.

(viii) $\widetilde{\prod}_{i\in I}\ (F_i, A_i)$ is a soft BCK/BCI-subalgebra of (F, A), whenever it is non-null.

Proof:

(i) Let $\widetilde{\wedge}_{i\in I}(F_i,\ A_i) = (K, C)$, where $C = \prod_{i\in I} A_i$ and $K(x) = \cap_{i\in I}\ F_i(x_i)$. for all $x = (x_i)_{i\in I} \in C$. Suppose that (K, C) is a non-null soft set over X. If $x \in supp(K, C)$, then $K(x) = \cap_{i\in I}\ F_i(x) \neq \emptyset$. Thus, we have$F_i(x) \neq \emptyset$ for all $i \in I$.Since $(F_i,\ A_i)$ is a family of soft BCK/BCI-algebras over X for all $i \in I$. It follows that, $C = \prod_{i\in I} A_i \neq \emptyset$, it means that C is nonempty and$C \subseteq A$, meaningevery $x \in C$ implies $x \in A$. Also,$K(x) = \cap_{i\in I}\ F_i(x_i) <_X F(x)$, which means $K(x)$ is a soft BCK/BCI-subalgebra of $F(x)$. Therefore,$\widetilde{\wedge}_{i\in I}(F_i,\ A_i)$ is a soft BCK/BCI-subalgebra of (F, A).

(ii) Let $⋒_{i\in I}\ (F_i,\ A_i) = (H, C)$, where $C = \cap_{i\in\Delta}\ A_i \neq \emptyset$ for all $x = (x_i)_{i\in\Delta} \in C$, $H(x) = \cap_{i\in\Delta}\ F_i(x)$. By the hypothesis (H, C) is a non-null soft set over X. If $x \in supp(H, C)$, then $H(x) = \cap_{i\in\Delta}\ F_i(x) \neq \emptyset$, so we have $F_i(x) \neq \emptyset$ for all $i \in \Delta$. Since $(F_i,\ A_i)$ is a family of soft BCK/BCI-algebras over X for all $i \in \Delta$. It follows that, $C = \cap_{i\in\Delta}\ A_i \neq \emptyset$, it implies that $C$is nonempty and $C \subseteq A$, means every $x \in C$ implies $x \in A$. Also,$H(x) = \cap_{i\in\Delta}\ F_i(x) <_X F(x)$, which means $H(x)$ is a BCK/BCI-subalgebra of $F(x)$ for all $x \in supp(H, C)$. Therefore, $⋒_{i\in I}\ (F_i,\ A_i)$ is a soft BCK/BCI-subalgebra of (F, A).

(iii) Let$\widetilde{\cap}_{E\ i\in I}\ (F_i,\ A_i) = (G, B)$, where $B = \cup_{i\in\Delta}\ A_i$ and$G(x) = \cap_{i\in\Delta}\ F_i(x)$ where $\Delta(x) = \{i \in \Delta : x \in A_i\}$ for all $x \in B = \cup_{i\in\Delta}\ A_i$. By the hypothesis (G, B) is a non-null soft set over X. If $x \in supp(G, B)$, then $G(x) = \cap_{i\in\Delta}\ F_i(x) \neq \emptyset$. Since, $\{A_i : i \in I\}$ are pairwise disjoint, that is, $i \neq j$ implies $A_i \cap A_j = \emptyset$, so we have $F_i(x) \neq \emptyset$ for all $i \in \Delta$, since $(F_i,\ A_i)$ is a family of soft BCK/BCI-algebras over X for all $i \in \Delta$. It follows that, $B = \cup_{i\in\Delta}\ A_i \neq \emptyset$, it means B is nonempty and $B \subseteq A$, it means $x \in B \Rightarrow x \in A$ for all $x \in B$. Also, $G(x) = \cap_{i\in\Delta}\ F_i(x) <_X F(x)$, which means $G(x)$ is a BCK/BCI-subalgebra of $F(x)$ for all $x \in supp(G, B)$. Therefore, $\widetilde{\cap}_{E\ i\in I}\ (F_i,\ A_i)$ is a softBCK/BCI-subalgebra of (F, A).

(iv) Let $\widetilde{\cup}_{i\in I}\ (F_i,\ A_i) = (K, C)$, where $C = \cup_{i\in I}\ A_i$ and for all $x \in C$, $K(x) = \cup_{i\in I(x)}\ F_i(x)$ where $I(x) = \{i \in I : x \in A_i\}$. By the hypothesis, (K, C) is non-null soft set over X. If $x \in supp(K, C)$, then $K(x) =$

$\cup_{i\in I(x)} F_i(x) \neq \emptyset$, so we have $F_i(x) \neq \emptyset$ for all $i \in I(x)$. It follows that, $C = \cup_{i\in I} A_i \neq \emptyset$. It means C is nonempty and $C \subseteq A$, this means $x \in B \Rightarrow x \in A$ for all $x \in C$. Also, $K(x) = \cup_{i\in I(x)} F_i(x) <_X F(x)$, which means $K(x)$ is a BCK/BCI-subalgebra of $F(x)$ for all $x \in supp\ (K, C)$. Therefore, $\tilde{\cup}_{i\in I} (F_i, A_i)$ is a soft BCK/BCI-subalgebra of (F, A).

(v) Let $\tilde{\cup}_{R\, i\in I} (F_i, A_i) = (H, C)$, where $C = \cap_{i\in I} A_i$ and $H(x) = \cup_{i\in I(x)} F_i(x)$ for all $x \in C$. Then by the hypothesis(H, C) is a non-null soft set over X. If $x \in supp(H, C)$, then $H(x) = \cup_{i\in I(x)} F_i(x) \neq \emptyset$. Since, $F_i(x_i) \subseteq F_j(x_j)$ or $F_j(x_j) \subseteq F_i(x_i)$, for all $i, j \in I$ and $x_i \in A_i$, so we have $F_i(x) \neq \emptyset$ for all $i \in I$. It follows that $C = \cap_{i\in I} A_i \neq \emptyset$, this means C is nonempty and $C \subseteq A$, it means that$x \in C \Rightarrow x \in A$ for all $x \in C$. Also, $H(x) = \cup_{i\in I(x)} F_i(x) <_X F(x)$, it means that $H(x)$ is aBCK/BCI-subalgebra of $F(x)$ for all $x \in supp(H, C)$.

Therefore, $\tilde{\cup}_{R\, i\in I} (F_i, A_i)$ is a soft BCK/BCI-subalgebra of(F, A).

(vi) Let $\tilde{\vee}_{i\in I}(F_i, A_i)$= (M, D), where $D = \prod_{i\in I} A_i$ and $M(x) = \cup_{i\in I} F_i(x_i)$ for all$x = (x_i) \in D$. Then by the hypothesis (M, D) is a non-null soft set over X. If $x \in supp(M, D)$, then $M(x) = \cup_{i\in I} F_i(x_i) \neq \emptyset$. Since, $F_i(x_i) \subseteq F_j(x_j)$ or $F_j(x_j) \subseteq F_i(x_i)$, for all $i, j \in I$ and $x_i \in A_i$, so we have $F_i(x_i) \neq \emptyset$ for all $i \in I$. Since (F_i, A_i) is a family of soft BCK/BCI-algebras over X for all $i \in I$. It follows that, $D = \prod_{i\in I} A_i \neq \emptyset$, it means D is nonempty and $D \subseteq A$, it means that for every $x \in D \Rightarrow x \in A$. Also, $M(x) = \cup_{i\in I} F_i(x_i) <_X F(x)$, which means that$M(x)$ is a BCK/BCI-subalgebra of $F(x)$ for all $x \in supp(M, D)$.

Therefore, $\tilde{\vee}_{i\in I}(\Gamma_i, A_i)$ is a soft BCK/BCI-subalgebra of (F, A).

(vii) Let $\tilde{\Sigma}_{i\in I}(F_i, A_i) = (W, C)$, where $C = \prod_{i\in I} A_i$ and $W(x) = \sum_{i\in I} F_i(x_i)$ for all $x = (x_i)_{i\in I} \in C$. Then by the hypothesis (W, C) is a non-null soft set over X. If$x \in supp(W, C)$, then $W(x) = \sum_{i\in I} F_i(x_i) \neq \emptyset$ for all $i \in I$. Since (F_i, A_i) is a family of soft BCK/BCI-algebras over X for all $i \in I$. It follows that, $C = \prod_{i\in I} A_i \neq \emptyset$. It means C is nonempty and$C \subseteq A$. It means $x \in C \Rightarrow x \in A$ for all $x \in C$. Also, $W(x) = \sum_{i\in I} F_i(x_i) <_X F(x)$, which means that $W(x)$ is a BCK/BCI-subalgebra of$F(x)$ for all $x \in supp(W, C)$.

Therefore, $\tilde{\Sigma}_{i\in I}(F_i, A_i)$ is a soft BCK/BCI-subalgebra of (F, A).

(viii) Let $\tilde{\prod}_{i\in I} (F_i, A_i) = (H, B)$, where $B = \prod_{i\in\Delta} A_i$ and $H(x) = \prod_{i\in\Delta} F_i(x_i)$ for all $x = (x_i)_{i\in\Delta} \in B$. Then by the hypothesis (H, B) is a non-null soft set over X. If $x \in supp(H, B)$, then $H(x) = \prod_{i\in\Delta} F_i(x_i) \neq \emptyset$, so we have $F_i(x_i) \neq \emptyset$ for all $i \in \Delta$. Since (Γ_i, A_i) is a family of soft BCK/BCI-algebras over X for all $i \in \Delta$. It follows that, $B = \prod_{i\in\Delta} A_i \neq \emptyset$, it means that B is nonempty and $B \subseteq A$. Also, $H(x) = \prod_{i\in\Delta} F_i(x_i) <_X F(x)$, which means $H(x)$ is a BCK/BCI -subalgebra of$F(x)$ for all $x \in supp\ (H, B)$.

Therefore, $\tilde{\prod}_{i\in I} (F_i, A_i)$ is a soft BCK/BCI-subalgebra of (F, A).

Proposition 3.7

Let (F, A) be a soft BCK/BCI-algebra over X and $(F_i, A_i)_{i\in I}$ be a nonempty family of soft BCK/BCI-subalgebra of (F, A). Then $\Cap_{i\in I} (F_i, A_i)$ is a soft BCK/BCI-subalgebra of (F_i, A_i) for each $i \in I$, if it is non-null.

Proof:

Let $\Cap_{i\in I}(F_i, A_i) = (H, C)$, where $C = \cap_{i\in I} A_i \neq \emptyset$ and $H(x) = \cap_{i\in I} F_i(x)$ for all $x \in C$. The parameter set of the soft set $\Cap_{i\in I}(F_i, A_i)$, that is $\cap_{i\in I} A_i$ is a subset of the parameter set of the soft set $(F_i, A_i)_{i\in I}$ for all $i \in I$. Then by the hypothesis (H, C) is a non-null soft set over X. If $x \in supp(H, C)$, then $H(x) = \cap_{i\in I} F_i(x) \neq \emptyset$. Therefore, $F_i(x) \neq \emptyset$ are BCK/BCI-subalgebras of X for all $i \in I$. Thus, $H(x) = \cap_{i\in I} F_i(x)$ is a BCK/BCI-subalgebra of X. Moreover, since $\cap_{i\in I} F_i(x) \subseteq F_i(x)$, for all $i \in I$ and for all $x \in \cap_{i\in I} A_i$. Hence, $\cap_{i\in I} F_i(x) <_X F_i(x)$, for all $x \in \cap_{i\in I} A_i$. Therefore, $\Cap_{i\in I}(F_i, A_i)$ is a soft BCK/BCI-subalgebra of (F_i, A_i) for each $i \in I$,

Definition 3.3 [15]

Let (F, A) be a soft BCK/BCI-algebras over X. Then,

(i) (F, A) is called trivial soft BCK/BCI-algebras over X if $F(x) = \{0_X\}$ $(the\ zero\ element\ of\ X)$ for all $x \in supp(F, A)$.

(ii) (F, A) is said to be whole soft BCK/BCI-algebras over X if $F(x) = X$ for all $x \in supp(F, A)$.

Proposition 3.8.

Let (F, A) and (G, B) be two softBCK/BCI-algebras over X. Then,

(i) If (F, A) and (G, B) are trivial soft BCK/BCI-algebras over X, then $(F, A) \Cap (G, B)$ is a trivial softBCK/BCI-algebra over X.

(ii) If (F, A) and (G, B) are whole softBCK/BCI-algebras over X, then $(F, A) \Cap (G, B)$ is a whole soft BCK/BCI-algebra over X.

(iii) If (F, A) is a trivial soft BCK/BCI-algebra over X and (G, B) is a whole soft BCK/BCI-algebra over X, then $(F, A) \Cap (G, B)$ is a trivial soft BCK/BCI-algebra over X.

(iv) If (F, A) and (G, B) are trivial soft BCK/BCI-algebras over X, then $(F, A) + (G, B)$ is a trivial soft BCK/BCI-algebra over X.

(v) If (F, A) is a trivial soft BCK/BCI-algebra over X and (G, B) is a whole softBCK/BCI-algebra over X, then $(F, A) + (G, B)$ is a whole soft BCK/BCI-algebra over X.

Proof:

(i) Assume that, $(F, A) \Cap (G, B) = (H, C)$, where $C = A \cap B$ and for all $x \in C$, $H(x) = F(x) \cap G(x)$. Since (F, A) and (G, B) are trivial soft BCK/BCI-algebras over X, it implies that $F(x) = \{0_x\}$ for all $x \in supp(F, A)$ and $G(x) = \{0_x\}$ for all $x \in supp(G, B)$. Thus, $H(x) = F(x) \cap G(x) = \{0_x\} \cap \{0_x\} = \{0_x\}$ is trivial, $\forall x \in supp(H, A \cap B)$. Therefore, $(F, A) \Cap (G, B)$ is a trivial softBCK/BCI-algebra over X.($\{0_x\}$refers to set of zero elements of the BCK/BCI-algebra$X$).

(ii) Suppose, $(F, A) \Cap (G, B) = (K, C)$, where $C = A \cap B$, for all $x \in C$, $K(x) = F(x) \cap G(x)$. Since (F, A) and (G, B) are whole soft BCK/BCI-algebras over X, it follows that $F(x) = X$, for all $x \in supp\ (F, A)$ and $G(x) = X$, for all $x \in supp\ (G, B)$. This implies that $K(x) = F(x) \cap G(x) = X \cap X = X$, for all $\forall x \in supp\ (K, A \cap B)$. Therefore, $(F, A) \Cap (G, B)$ is a whole soft BCK/BCI-algebra over X.

(iii) Let $(F, A) \Cap (G, B) = (T, N)$, where $N = A \cap B$, for all $x \in N$, $T(x) = F(x) \cap G(x)$. Since (F, A) is a tri-

-vial softBCK/BCI-algebra over X, it implies that $F(x) = \{0_x\}, \forall x \in supp(F, A)$ and (G, B) is a whole softBCK/BCI-algebra over X, then $G(x) = X$, for all $\forall\, x \in supp(G, B)$. It means that $T(x) = F(x) \cap G(x) = \{0_x\} \cap F = \{0_x\}$. Therefore, $(F, A) \Cap (G, B)$ is a trivial softBCK/BCI-algebra over X.

(iv) Let $(F, A) + (G, B) = (W, A \times B)$, where $W(x, y) = F(x) + G(y)$, for all $(x, y) \in A \times B$. Let $(W, A \times B)$be a non-null soft set over X. If $(x, y) \in supp(W, A \times B)$, then $W(x, y) = F(x) + G(y) \neq \emptyset$. It means that, $F(x) = \{0_x\}$ is a trivial soft BCK/BCI-algebra over X, for all $x \in supp(F, A)$ and $G(y) = \{0_x\}$ is a trivial soft BCK/BCI-algebra over X, then $W(x, y) = F(x) + G(y) = \{0_x\} + \{0_x\} = \{0_x\}$, for all $x \in supp(W, A \times B)$. Therefore, $(F, A) + (G, B)$ is a trivial soft BCK/BCI-algebra over X.

(v) Let $(F, A) + (G, B) = (T,\ A \times B)$, where $T(x, y) = F(x) + G(y)$, for all $(x, y) \in A \times B$. Let, $(T,\ A \times B)$be a non-null soft set over X. If $(x, y) \in supp(T, A \times B)$, then $T(x, y) = F(x) + G(y) \neq \emptyset$. Since, (F, A) is a trivial soft BCK/BCI-algebra over X, it implies that for all $x \in supp(F, A)$, $F(x) = \{0_x\}$ similarly (G, B) is whole soft BCK/BCI-algebra, it means for all $x \in supp(G, B)$, and $G(y) = X$.

It implies that, $T(x, y) = F(x) + G(y) = \{0_x\} + X = X$ is a whole soft BCK/BCI-algebra over X. Therefore, $(F, A) + (G, B)$ is a whole soft BCK/BCI-algebra over X.

IV. Conclusion

In this paper, we studied Soft BCK/BCI-algebras introduced by Y.B. Jun and further defined some basic algebraic operations such as restricted intersection ($\Cap$), restricted union ($\tilde{\cup}_R$), OR-product ($\tilde{\vee}$), direct sum ($\tilde{+}$), extended intersection ($\tilde{\cap}_E$) and Cartesian product ($\tilde{\times}$) on two and family of soft BCK/BCI-algebras. We also established some basic results on the algebraic properties of the operations.

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