International Journal of Applied Science and Mathematics
Volume 6, Issue 4, ISSN (Online): 2394-2894

One-Dimensional Unsteady Heat Transfer Model Based on Partial Differential Equations

Tianzhen Wang and Shujiang Ding*
School of Mathematics and Statistics, Shandong University of Technology, Zibo, China.
*Corresponding author email id: 1395698201@qq.com

Date of publication (dd/mm/yyyy): 16/07/2019

***Abstract* – Thermal protective clothing is widely used at present. It is very important to study the heat transfer model of thermal protective clothing. This paper mainly studies the heat transfer model of thermal protective clothing. By using advanced heat transfer knowledge, we get the partial differential equation of heat conduction. Then, we use the inverse Laplace transformation and iterative method to get the temperature distribution function. By comparing the simulated data with the actual data, the rationality of the model could be seen.**

***Keywords* – Thermal Protective Clothing, Partial Differential Equation, Iteration Method, Temperature Distribution.**

I. Introduction

With the rapid development of industry, high-temperature working environment is everywhere. The appearance of thermal protective clothing avoids the harm of heat source to human body. The heat transfer model of thermal protective clothing is a hot topic. Some scholars put forward their opinions on the heat transfer model of thermal protective clothing from different perspectives. Gibson [1] first put forward the heat and mass transfer model of single-layer porous media under high temperature, but he did not consider the influence of thermal radiation in the fabric layer. In order to improve the model, Torvi [2] established a heat transfer model of heat protective clothing shell material, which fully considered the influence of radiation. Based on Gibson and Torvi models, Chitrphiromsri [3] developed a heat and moisture coupling model for protective clothing with porous media. Professional thermal protective clothing is made of three layers of material, which are recorded as layers I, II and III. The layer I is in contact with the external environment. The air between the clothes and the human body is the layer IV. This paper established a one-dimensional unsteady heat conduction model under specified conditions, which is the surface temperature is controlled at 37ºC, the ambient temperature is 75ºC.

II. Modeling of Heat Conduction

In order to simplify the problem, we think the human body is a cylinder. Supposing the radius of the base of the cylinder is 100mm and the height of the cylinder is 150cm. Then the cylinder and the thermal protective clothing (each layer is still approximately a cylinder) are drawn by MATLAB.

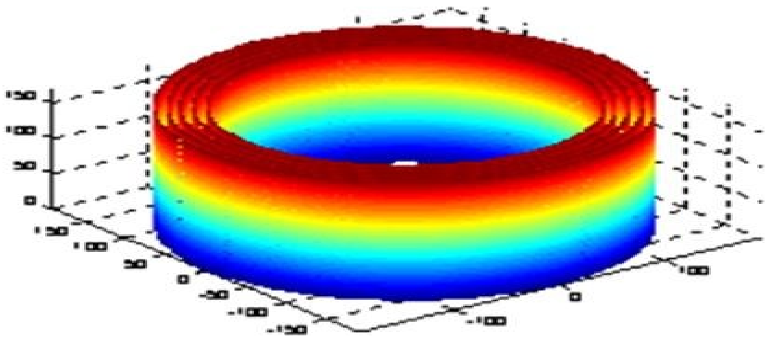

Fig. 1. The cylinder model

Heat conduction from the outside to the inside. According to the Fourier law of heat conduction, in the process of heat conduction, the unit time through a given section of the heat conduction, is proportional to the vertical direction of the section of the rate of change in temperature and area, and the direction of heat transfer is opposite to the direction of temperature rise [4].

Firstly, take layer I for analysis. The internal temperature of the human body is T_0, the external temperature of layer I is T_1, and the internal temperature of layer I is T_2, followed by T_3, T_4 and T_5. The change of the inner temperature T_1 is caused by the temperature difference between the ambient temperature and the inner side of the layer I.

We consider the heat conduction differential equation of the heat transfer model [5]

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}$$

Where $a = \frac{\lambda}{\rho c_p}$, ρ is the density of material, Cp is the specific heat.

The initial condition is

$$T(x, t_0) = T_0 = 37$$

$$T(x \to +\infty, t) = T_0 \quad \text{(The temperature at infinity is still } T_0\text{)}$$

The boundary condition is

$$T(x = 0, t \geq t_0) = T_1 = 75$$

For this energy equation, we use the Laplace transformation to solve.

2.1 *Definition*

The Laplace transformation of $T(x,t)$ is

$$\Theta = \Theta(x,s): L(T(x,t)) = \Theta(x,s)(x,t)$$

By the linearization and differentiation of the Laplace transformation, we get

$$s\Theta(x,s) - T(x,t_0) = a\frac{d^2\Theta(x,s)}{dx^2}$$

Then we get the general solution

$$\Theta(x,s) = A(s)\mathrm{e}^{-mx} + B(s)e^{mx} + \Theta_0(s)$$

Where $\mathrm{m} = \sqrt{s/a}$.

By the initial condition and the boundary condition we get

$$T(x,t_0) = T_0 \Rightarrow \Theta_0(s) = \frac{T_0}{s}$$

$$\forall t, x \to \infty, T(\infty,t) = T_0 \Rightarrow \forall s, B(s) = 0$$

Take x = 0 into Laplace transformation we obtain $A(s) = \frac{T_1 - T_0}{s}$

Then we have

$$\Theta(x,s) = \frac{T_1 - T_0}{s} e^{-x\sqrt{s/a}} + \frac{T_0}{s}$$

2.2 *Definition*

The error function of x is

$$erf(x) = \frac{2}{\pi} \int_0^{\infty} e^{-\varepsilon^2} d\zeta$$

and the complementary error function is

$$erfc(x) = 1 - erf(x) = \frac{2}{\pi} \int_x^{\infty} e^{-\varepsilon^2} d\zeta$$

The temperature distribution function is the inverse Laplace transformation.

$$T(x,t) = L^{-1}(\Theta(x,s)) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st}(x,s) d_s$$

So we get the temperature distribution function

$$\frac{T(x,t) - T_0}{T_1 - T_0} = erfc\left(\frac{x}{2\sqrt{a(t-t_0)}}\right)$$

$$T(x,t) = (T_1 - T_0)\ erfc\left(\frac{x}{2\sqrt{a(t-t_0)}}\right) + T_0$$

By using cyclic iteration method, the function of T2, T3, T4 and T5 are obtained

$$T(x_1,t) = (T_1 - T_0)\ erfc\left(\frac{x_1}{2\sqrt{a_1(t-t_0)}}\right) + T_0 = T_2,$$

$$T(x_2,t) = (T_2 - T_0)\ erfc\left(\frac{x_2}{2\sqrt{a_2(t-t_0)}}\right) + T_0 = T_3$$

$$T(x_3,t) = (T_3 - T_0)\ erfc\left(\frac{x_3}{2\sqrt{a_3(t-t_0)}}\right) + T_0 = T_4$$

$$T(x_4,t) = (T_4 - T_0)\ erfc\left(\frac{x_4}{2\sqrt{a_4(t-t_0)}}\right) + T_0 = T_5$$

III. Model Solution

The parameter value of clothing material as follows.

Table 1. The parameter value of clothing material

Layer	Density (kg/m3)	Specific Heat (J/(kg·°C))
I	300	1377
II	862	2100

Layer	Density (kg/m3)	Specific Heat (J/(kg·ºC))
III	74.2	1726
IV	1.18	1005

We use the parameter values to figure out $a_i(i=1,2,3,4)$, and then we plot the temperature change at the different layers. The result has shown in Fig.2.

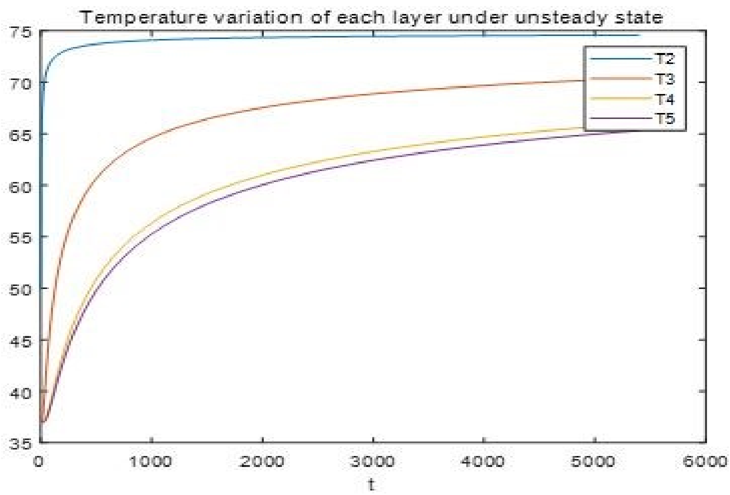

Fig. 2. Temperature variation of each layer under unsteady state

IV. Conclusion

This paper mainly discusses the heat transfer model of thermal protective clothing. The whole process is regarded as a one-dimensional unsteady heat transfer model. According to the partial differential equation of heat conduction, initial conditions and boundary conditions, the model is solved by iteration method. This paper does not take into account heat radiation and heat-moisture transfer, which are inevitable in the use of thermal protective clothing, which will be our research direction.

References

[1] Linzhen Lu, Heat Transfer Model and Parameter Optimization Decision of Multilayer Thermal Protective Clothing [D], Zhejiang University of Science and Technology, 2017.
[2] TORVIDA Heat Transfer in Thin Fibrous Materials under High Heat Flux Conditions [D]. Edmonton: University of Alberta, 1997.
[3] Bin Pan. Mathematical Modeling of Heat Transfer in Thermal Protective Clothing and Inverse Problem of Parameter. Determination [D]. Zhejiang University of Science and Technology, 2017.
[4] Shiming Yang, Wenquan Tao. Heat Transfer [M]. Higher Education Press, 2006.
[5] Anna-maria bianchi, Yves foettner, Jacarina aday, heat transfer [M], liaoning: Dalian University of Technology Press, 2008.6.

Authors Profile'

Tianzhen Wang,
Shandong in China, was born in 1998 and studying at the third grade of Shandong University of Technology now. She has published dissertations in the International Journal of Trend in Research and Development. Ms. Wang won the second prize in the 9th Shandong College Students Mathematical Contest and the 11th "Certification Cup" Mathematical Modeling Network Challenge and winning the third prize of the Shandong province in the 2018 Chinese College Students Mathematical Modeling Contest.

Shujiang Ding (Corresponding author**)**