Fluid Turbulence Road to Quantum Cantorian Spacetime Via Nested Solitonic Golden Eddies Chaos Implied by Batchelor's Law - A Possible Rationale for the Mysterious Fine Tuning of the Cosmos*

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Abstract – We start from a nested golden mean logarithmic Eddy's model for fluid turbulence and Batchelor's law and then move to quantum-Cantorian spacetime physics. In the second section, a physico mathematical comparison is undertaken between the theory of massive gravity and the fractal Cantorian spacetime theory. We conclude that at a minimum the two theories are complimentary. However E-infinity theory may be simpler in several number theoretical and physical respects. Our first important conclusion is that quantum physics as well as classical physics are just different sides of the same nested eddies of the fluid turbulence coin. In this sense the nested (fractal) eddies paradigm is a fundamental model unifying classical and non-classical physics via nonlinear dynamics, solitonic spiral chaos and the golden mean number system. The final and most compelling conclusion of the present work is the strong possibility that the presented theory may be the only rational explanation for the major problem of the fine tuning of our cosmos. In fact the golden mean number system of the present theory converts fine tuning to a naturalness argument and establishes an analogy between quantum wave collapse and liquefaction.

Keywords – Golden Mean Spirals, Nested Eddies Model of Fluid Turbulence, Batchelor's Law, De Rham Massive Gravity, Ord-Nottale-El Naschie Fractal Cantorian Spacetime, Unification of All Fundamental Forces, Golden Mean Number System, Quantum Gravity, Graviton with Non-Zero Mass, Solitonic Spiral Chaos of Ji-Huan He, Thomas Kapitaniak and M.S. El Naschie, Possible Resolution of the Fine Tuning of the Cosmos, Naturalness Requirement, Liquefaction.

I. INTRODUCTION

The present work is an ambitious undertaking addressing almost the entire field of classical and quantum physics [1-85]. The aim is three fold. The first is to introduce a major new mental picture unifying classical and quantum mechanics via a theory of fluid turbulence and El Naschie-He-Kapitaniak solitonic spiral chaos [61-70]. The second is utilizing this theory to qualify a resolution for a comparative study of the theory of massive gravity [26-28] and E-infinity theory [8-25]. The ideas leading to our first aim could maybe be traced back to pencil drawings by Leonardo da Vinci dating to the beginning of the 16th century [62-69] up to the modern time research by the outstanding Australian applied mathematician, George Batchelor's Law of Turbulence [69], which was quite recently proven mathematically [68]. The third aim, and from the theory of knowledge point of view, the most import aim is the obvious possibility that the present model and mathematics represent possibly the only rational way to explain cosmic fine tuning of our universe [83]. We stress that attentive study of equations (1)-(9) of the present work clearly shows how the fine tuning paradox is converted here to a naturalness argument and in addition we realize an intimate analogy between liquefaction and quantum wave collapse [84-85].

*) Dedicated to the memory of Prof. G. Batchelor as well as Prof. F. Hoyle from the legendary Dept. of Applied Mathematics & Physics, Cambridge, U.K.

Coming from a superficially entirely different direction, E-infinity Cantorian spacetime theory was developed as a further extension of the Ord-Nottale pioneering theory of fractal spacetime [5-10]. Initially the said E-infinity theory was presented in two different but ultimately equivalent pictures. The first was basically a topological geometrical picture of a hierarchal conglomerate of infinitely many elementary random cantor sets of the Mauldin-Williams type [8-12]. The second picture was as a collection of infinitely many golden mean oscillators joined sequentially and in parallel [9]. However and as we have just found out, there is indeed a third picture which was tacitly employed by intensive modern turbulence research [61-72], namely what we may call nested eddies picture [65-68]. This new picture will be one of our three main concerns in the present work as we mentioned earlier on because it essentially employs not only a neat exact description of what is arguably the most difficult problem in classical mechanics, if not in all of physics, but it is also a remarkable path for unification of classical and quantum mechanics [42-54] via the fluid mechanics theory of turbulence [61-69] and this is all apart of using our present model and mathematics to give a complete and rational explanation for the fine tuning of our universe and our standard model of elementary particles [83].

The Paper is Organized in the Following Way:

Subsequent to the introduction we outline the problematics of the theory of massive gravity and compare it to our platonic E-infinity theory [8], [10-11], [67]. The third paragraph of this paper is then concerned with giving the minimal background information needed to tackle our problem. This section is followed by Section 4 outlining Heterotic strings [37], the Gavi-weak theory [38] and how it is applied in determining the pure dark energy density of the universe [34-41]. Subsequently Sections 5 and 6 are addressing the nested eddies model and a deep philosophical look at deterministic chaos respectively from the perspective of a remarkable giant of world literature, Leo Tolstoy [59]. Section 7 is an important discussion of mainly important findings of the present paper while Section 8 is a conclusion summing up the wisdom gained in a crisp and concise manner and commenting briefly on the possibility that cosmic fine tuning of our universe [83] may be easily and rationally explained using the golden mean number system of the present theory [1-83].

II. MASSIVE GRAVITY AND E-INFINITY

Einstein's relativity was conceived before the inception of the quantum relativity theory [1-25]. Consequently Einstein did not ponder the existence or non-existence of massive gravity. The theory of massive gravity which proposed a non-zero mass to the hypothetical spin 2 quantum particle, i.e. the graviton is a profound, radical theory which deserves careful consideration [25-30]. On the other hand in E-infinity Cantorian spacetime theory one can explain dark matter energy without involving anything more than classical topology and classical and chaotic mechanics [30-58]. However the massive gravity theory has roots going back in time as a fleeting look at the literature, particularly the powerful publication of Prof. Claudia de Rham [26-28] and her school of thoughts clearly reveal [28]. None the less, coming from a completely different direction, there is the said other way of looking at the same thing but through a radically different spacetime setting [5-25], [35-60], namely that of fractal geometry and the E-infinity theory of fractal Cantorian spacetime which is essentially the same as the fluid turbulence model outlined above [8-25]. The remarkable and in fact more than radical theoretical findings of this E-infinity theory is that the standard model as it is already contains all what it needs

and is complete when seen through fractal eyes using fractal logic which implies fractal counting [29-30]. The 14 particles counted using our ordinary integer counting include not only the Higgs boson but also the graviton. It is not only any graviton but a pre-quantum graviton with a fractal weight equal to $k = \phi^3(1 - \phi^3)$ to the power of two, i.e. the square of 'tHooft's renormalon k^2 and ϕ^3 is the inverse of the scale invariant Einstein fractal spacetime dimension $4 + \phi^3$ where $\phi = (\sqrt{5} - 1)/2$ is the golden mean [10-20]. This is what we christened fractal graviton while the corresponding fractal Higgs is given by $k_o = (\phi^5)(1 - \phi^5)$ where ϕ^5 is the famous Hardy's probability of quantum entanglement [31]. Incredible as it may sound, counted using fractal logic, the fourteen are only $\sqrt{\bar{a}_0} = 11.7082$, i.e. almost 12 [29], [60]. This situation eliminates the need to talk about this quasi graviton particle as massive or not. Seen in this way it is really good news not to contemplate the massiveness of the graviton since mass is by its very nature not a deep concept [32-34] and the relation between mass and energy is also open to questions that constitute a large part of the achievements of E-infinity Cantorian spacetime theory in elucidating the deep meaning of $E = mc^2$ of special relativity [29-34]. In the present work we will reconsider all the preceding questions in depth and we will draw on some of the fundamental concepts of Gross' Heterotic superstring theory [35-37] as well as the Gravi weak theory [38-40] to show how we can uphold the dark matter theory and arrive at accelerated cosmic expansion while keeping maximal simplicity without the need to invoke the massiveness of the graviton [26-28] and loose the consistency of the present framework of the standard model of high energy particle physics [34-37]. On balance it may be possible that a theory of massive graviton without the need for dark matter energy and a theory without a massive graviton but with the Heterotic superstring structure with ordinary energy, dark matter energy and pure dark energy are two different sides of the same theoretical coin [33-40]. We always strive for understanding although if we are really honest with ourselves, we must admit we do not know exactly what we mean with this word and that we do not fully understand the structure and functioning of our own brain [9]. At the end analysis only painstaking refined experiments would be able to discriminate between the two theories although it is only fair to admit that the present author, as probably one expects, tends to prefer the idea of keeping the dark matter energy theory in place and follow the path chartered by E-infinity Cantorian spacetime which we will explain in some detail next [5-60]. In the rest of the present paper we argue for the unity of physics by employing non-classical Cantorian fractal geometry and the associated golden mean number theory.

III. MINIMALISTIC BACKGROUND FOR E-INFINITY DARK ENERGY SET THEORETICAL MATHEMATICS

In a nutshell E-infinity Cantorian spacetime theory postulates that the zero set and the empty set are identical to the pre-quantum particle [10] and the pre-quantum wave respectively [10], [12], [13]. In both cases they are described with the bi-dimension $D(0) \equiv (0, \phi)$ and $D(-1) \equiv (-1, \phi^2)$ respectively where zero and minus one are the topological dimension while ϕ and ϕ^2 are the corresponding Hausdorff dimension respectively. This follows straightforwardly from A. Connes' dimensional function [10-13], [77-81]

$$D = a + b\,\phi \tag{1}$$

where $a, b \in Z$ and $\phi = (\sqrt{5} - 1)/2$ is describing Penrose fractal tiling universe which is modelling the holographic boundary of our cosmos according to our E-infinity Cantorian spacetime theory [8-14]. After reasoning that our universe is Kaluza-Klein like and possesses five topological dimensions, it is easily conclude-

-d and computed that the ordinary energy density of our universe is given by [8-19], [60]

$$\gamma(0) = \phi^5/2$$
$$\simeq 4.5\%$$
(2)

while the dark energy density is given by [8-14], [17-25], [60]

$$\gamma(D) = 5\phi^2/2$$
$$\simeq 95.5\%$$
(3)

On the other hand this dark energy, which cannot be measured directly consists of two parts. The first is dark matter energy amounting to 22.2% of the total maximal energy [13-14], [17-25] and the rest of about 73.3% is pure dark energy with negative (repulsive) pressure [17-25] to which the accelerated cosmic expansion, which is not to be confused with the ordinary Hubbled expansion is attributed. This accelerated expansion was solved quantitatively but not completely using D. Gross et al via the ingenious theory of Heterotic superstring theory [35-37]. It is the purpose of the present work to complete and refine the Heterotic derived accelerated expansion of the universe to show how it constitutes a rather simple explanation without the need to attribute a mass to the hypothetical graviton of an extended standard model of high energy particle physics as required by the equally brilliant theory of massive gravity of Prof. Claudia de Rham [26-28].

To understand the above strategy we need one more additional information gained from E-infinity theory, namely that from a number theoretical viewpoint, the 12 messenger particles of the standard model, namely $|SU(3)\,SU(2)\,U(1)| = 12$ are in fact 14 messenger particles within the fuzzy fractal counting, i.e. dimensional number counting weight of $\sqrt{\bar{\alpha}_0} = 11.7082$ when applying the fractal logic of counting in a Cantorian-fractal spacetime where $\bar{\alpha}_0 = 137 + k_0$ is the inverse electromagnetic coupling constant, $k_o = \phi^5(1 - \phi^5)$ and ϕ^5 is Hardy's quantum probability of two particles quantum entanglement [29-31]. In other words, 11.70803398 already includes a pre-graviton described by k^2 where $k = \phi^3(1 - \phi^3)$ is 'tHooft's renormalon as explained above and in numerous previous publications [16-25].

IV. HETEROTIC STRINGS PLUS GRAVI-WEAK LEADS TO FRACTAL PURE DARK ENERGY ACCELERATED EXPANSION VIA E-INFINITY FRACTAL CANTORIAN SPACETIME THEORY

Let us go first to the rudimentary integer form of applying Heterotic string theory to the issue of ordinary and dark energy density of the universe [24],[35-37]. To do this we recall the basic ingenious idea devised by D. Gross et al to join the old bosonic string theory with its 26 dimensions and the ten dimensional superstring theory into one theory via the deceptively trivial equation [24], [29], [35], [37], [60]

$$26 - 16 = 10$$
(4)

where an additional 16 bosonic dimensions are 'running' into an opposite direction to the rest. In what follows we will reason that the negative pressure responsible for the accelerated expansion of the universe is linked to the pure dark energy and these 16 extra bosonic dimension, as well as a subtle link between these ideas and the theory of Gravi-weak [38-40]. Now following previous expositions, the total maximal energy may be written using Gross et al Heterotic superstring theory slicing as [24], [29], [35-37], [60]

$$E = (22/22)\,mc^2$$

$$= \left(\frac{1 + 5 + 16}{22}\right)mc^2$$

$$= \left(\frac{1}{22}\right)mc^2 + \left(\frac{5}{22}\right)mc^2 + \left(\frac{16}{22}\right)mc^2$$

$$= (\gamma(O))mc^2 + (\gamma(DM))mc^2 + (\gamma(PD))mc^2 \tag{5}$$

$$= E(\text{Einstein})$$

$$= mc^2$$

Here m is the mass and c is the speed of light while $(\gamma(0))$, $(\gamma(DM))$ and $(\gamma(PD))$ are the ordinary, the dark matter and the pure dark energy densities respectively [24], [29], [35], [37]. In the above it was of course tacitly implied that we are dealing with modulus of γ irrespective of the sign so that our 100 core space dimensions as given by the fundamental equation [29-31], [35-37]

$$\sum_{i=1}^{4} \bar{\alpha}_i = \bar{\alpha}_1 + \bar{\alpha}_2 + \bar{\alpha}_3 + \bar{\alpha}_4 \tag{6}$$

$$= 60 + 30 + 9 + 1$$

$$= 100$$

are effectively subdivided into [10-19], [17-25]

$$\sum_{i=1}^{4} \bar{\alpha}_i = 4 + 22 + 74 \tag{7}$$

$$= (\gamma(0)) + \gamma(DM) + \gamma(PD)$$

In other words, we have assigned four dimensions for ordinary energy, 26 – 4 = 22 compactified bosonic dimensions for dark matter energy and the remaining 74 dimensions for the pure dark energy sector [34-37]. However from another viewpoint things are not additive in view of the 16 dimensions running in the opposite direction as devised by Gross et al's ingenious insight. Consequently we really have an auxiliary equation saying de facto that [13-14], [16-25].

$$|22| + |74| = 96\% \tag{8}$$

in accordance with Dvoretzky measure concentration theorem [13-25]. This situation also leads to including the minus sign of the negative 16 extra dimensions which are the source of 74 percent of pure dark energy as per the value of 16/22 expressed in a percentage of the 100% total of equations (7) and (8). Consequently it is obvious that we are justified in writing [41-44]

$$22 - 74 = -52 \tag{9}$$

This result is now consistent with $|F_4| = \dim F_4 = 52$ of the exceptional Lie groups that we invoke in the unification of the weak force with gravity, which is well known as the Gravi-weak unification [26-28], [41-44]. The weak force on the other hand is responsible for the radioactive decay that is balanced by the strong force in all stable forms of matter that we encounter in nature. In other words, the negative sign of Gross et al's 16 extra bosons is directly linked to the $|F_4| = 52$ and the repulsive force of the weak interaction [39-44] and hence the negative pressure of pure dark energy, which we and others consider responsible for the accelerated cosmic expansion [20-25]. This subtle mathematical explanation seems to us as compelling and simple as any but at a

minimum, it is equivalent and as a convincing an argument as assigning mass to the hypothetical graviton in the massive gravity theory [38-40]. At present this is an attractive explanation when remembering that we have not experimentally found the graviton with or without mass until this moment.

Finally for additional coverage of subtle intricate points of the above, the reader may be advised to consult some of the very recent publications on E-infinity and the platonic quantum set Cantorian spacetime theory [45-65], [74-76].

V. THE NESTED EDDIES MODEL OF FLUID TURBULENCE AND ITS RELATION TO QUANTUM-CANTORIAN SPACETIME OF E-INFINITY THEORY

Let us start this part by first citing what D. Bohm and B. Hiley said in their classical 'The Undivided Universe' [73]: "The principle current difficulty in the attempt to make theoretical physics coherent is the notion of a mathematical point in space-time without extension or duration". Clearly only a 'fractal', without going into detailed analysis of our conclusion at this early stage, we allege that a fractal-Cantorian point is the only possibility for the above requirement [8], [11], [73]. However any object however complex can reach the stage of a 'self similar' Cantor quasi "point" as it is trivially a resolution dependent issue. Thus it is natural to ask a logical question and inquire about the optimal self similar object which can produce after an almost infinite iteration a Cantor-like mathematical point. Again without going into a lengthy recounting of the meticulous reasoning, we take our conclusion at its face value that it is the logarithmic spiral [71-72] which constitutes the optimal choice for our self similar Cantor point in a fractal-Cantorian logarithmic spirals spacetime [61-62], [71-72]. It may be helpful as a plausible simple hint for understanding this choice to know that the growth factor of the logarithmic spiral is the inverse of the golden mean $1/\emptyset = 1 + \emptyset = 1.618033989$ and that the resulting Cantor set is randomized in the same way that E-infinity Mauldin-Williams Cantor sets [ ] are random and over and above of that, possess the golden mean $\emptyset = (\sqrt{5} - 1)/2$ as a Hausdorff dimension [8-9], [11]. This situation is represented in several of the figures in the references cited here which reflects the most important point mentioned earlier on [61-65], [71-72].

VI. FROM TOLSTOY VIA DETERMINISTIC CHAOS TO FINE TUNING OF OUR COSMOS

Writing about history is in general a difficult and intricate undertaking as is well known from literary works such as War and Peace by Leo Tolstoy [59]. However writing about history of science, let alone the inception of our own scientific ideas and their source and when it first came to mind, is more than a slippery slope with countless pitfalls which could be more misleading than helpful.

In the present paper we emphasized one direction from where our ideas came, namely the work of Ord, Nottale [6-10], gross [35] and in no minor measure, the theory of continuous pointless spacetime geometry of von Neumann and its extension and perfection to the theory of non-commutative geometry of Connes [50], [77-81]. By doing this we now feel that we may have underplayed the pre-eminent direction from which we realized the central role that the golden mean and the golden mean number system came to play in shaping the Cantorian E-infinity theory and the transfinite set theory based quantum mechanics. At that time, all these realizations came from the very fashionable and vigorous modern theory of chaos, fractals and nonlinear dynamics with R. Thom VAK and Kolmogorov-Arnold-Moser KAM theorem and the golden mean at its centre. In fact for a man initially trained as an engineer who went on to become a professor of engineering before

studying physics or pure mathematics, it is very doubtful that he would have taken number theory and the golden mean as well as the associated Cantor sets seriously if it were not for the impressive application which came from the pioneers of deterministic chaos, such as Roulle, Feigenbaum, Procaccia, Mandelbrot, and Rossler [15] to mention only a few of many dedicated and courageous scientist and engineers who had the courage to break with orthodoxy and discover a nova terra connecting determinism with indeterminism and coining what on the surface must appear as a totally contradictory name for a new science, i.e. deterministic chaos which was what the applied mathematician J. York proposed [15]. Thus these critical after thoughts must be followed by some additional references documenting some of the most important early papers which carried the seeds for the fully-fledged expansion and development of the present Author's E-infinity theory [10-20]. In this respect one must mention the role that the founding editor of a completely unorthodox and unusual Springer Journal played in teaching the E-infinity infant how to walk and we mean here the late Prof. Alan McKay and his journal Speculation in Science and Technology [55], [58]. Speaking of Leo Tolstoy [59], it was indeed Alan McKay who drew my attention first to the unexpected relevance of the great Leo Tolstoy to my work so with the kind permission of the patient reader, the writer and without much more ado would like to add the following references as additional an additional source [55-59]. It is in more than one respect important reading to the subject of the present paper. I must in particular stress the paper of Allen D. Allen, which A. McKay drew my attention to [58]. This paper, which was published in his remarkable speculation journal, was to my mind one of the first to draw strong attention to the role of fractals in quantum mechanics [55-59]. Returning to the great Leo Tolstoy [59], the Author admits that he strongly believes that he combined the two opposites, i.e. reality and fiction in the same way the heroes of deterministic chaos [15] reconciled chance and disorder with determinism in unheard of ways [56]. We hope this example of Tolstoy demonstrates clearly that fundamental ideas in science may come how ever indirectly from non-scientists but sensitive, intelligent, alert people from every corner of activity and particularly art, literature and philosophy [80-82]. In this sense we ask the reader to evaluate our proposal that the present theory seems to be the only plausible and rational possibility to explain the hotly debated and controversial subject of the fine tuning of our cosmos as well as our standard model of high energy elementary particles [10-20], [83].

VII. Discussion

Looking at the subject of the present work as objectively as humanly possible, we do not have any experimentally conclusive watertight proof that the theory of massive gravity is more real or convincing than the E-infinity Cantorian spacetime theory of dark matter and dark energy or do we? What is simpler and more natural? It seems fair to say that one is permitted to ask oneself what could be better from the following two alternatives:

(a) To assume that dark energy is real and that it is the energy of the empty set quantum wave while ordinary energy is the energy of the zero set quantum particle or

(b) To assume that the hitherto hypothetical massless graviton, i.e. the messenger particle of the gravitational field, against our initial supposition, does have a mass after all.

In this respect it is hard for the present author to be really, totally objective so our tentative most possible objective conclusion is that the situation is almost equal or almost identical with the scale may be tilting slightly towards (a), i.e. our E-infinity resolution of the mystery of dark energy which we do not in all events think is a

real mystery when we apply our physico-number theoretical machinery of transfinite set theory based quantum mechanics to it. In addition to the above and taking into account our new third golden nested eddies picture, which is based on the fluid turbulence theory from Leonardo da Vinci to George Batchelor, we are confident that E-infinity Cantorian spacetime and in particular the picture of golden mean solitonic fractal spiral chaos is a true ultimate theory of unification of physics and beyond. If what we just said turns out to be right, and we think it will, then we are seriously more than inclined to believe that the golden mean number system of E-infinity and the present golden spiral model may be the only rational explanation for the fine tuning of our universe and the standard model of high energy physics. After all this very serious discussion maybe the author is permitted to end on a light hearted note and admit that he finds it hard to resist the unforgettable quote of the marvellous fiction of Douglas Adams 'The Hitchhiker's Guide to the Galaxy' where it was concluded that the ultimate answer to every life and everything in the universe is 42. Well we almost agree but we insist that 42 would not do and that the real answer is, i.e. the golden mean number.

There are still two very important points which we need to discuss and focus upon here a little. The first point is with regard to the fine tuning quasi paradox and by the same token, the naturalness requirement [84]. Clearly fine tuning and naturalness are on opposite sides for obvious reasons. However even a fleeting glance at our golden mean number system mathematics involved in equations (1) – (9) of Section 4 reveal clearly that fine tuning is a logical consequence of using the golden mean number system of E-infinity Cantorian space. In other words, our theory automatically converts the fine tuning paradox into a fulfilment of the naturalness requirement. The second point is the remarkable and surprising analogy between the fluid mechanical and earthquake phenomenon of liquefaction [85] on the mundane side and the quantum wave collapse on the somewhat esoteric side. Nothing could attest more to the profound role of hydrodynamical turbulence in erecting a bridge between classical and quantum realities.

VIII. Conclusion

In what may be labelled a tour de force, the present work tackled a great deal of fundamental and difficult problems in quantum physics and comes to some startling conclusions which we may summarise in the following:

1. We introduced a new mental picture for visualizing and implementing E-infinity Cantorian spacetime besides two well known pictures, namely (a) the golden mean random Cantor sets and (b) the golden mean oscillators. This third picture is realized via the golden mean logarithmic spirals that connect our theory with fluid turbulence.

2. We found that E-infinity Cantorian spacetime theory may be in some respects simpler or at least equivalent to the theory of massive gravity.

3. We unravelled a deep and quite instructive analogy between liquefaction and quantum wave collapse and we conjectured that fluid turbulence is neither a fundamentally classical nor fundamentally quantum phenomena but rather a transition regime or a bridge between the two sides of our reality.

4. It is shown that the seemingly paradoxical problem of fine tuning regarding the equations of cosmology as well as the high energy particles standard model is resolved automatically by the golden mean number system of E-infinity theory. In other words the golden mean number system converts automatically the fine

tuning paradox to a logical fulfilment of the requirement of naturalness.

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AUTHOR'S PROFILE

Professor M.S. El Naschie was born in Cairo, Egypt on 10th October 1943. He received his elementary education in Egypt. He then moved to Germany where he received his college education and then his undergraduate education at the Technical University of Hannover where he earned his (Dipl-Ing) diploma, equivalent to a Master's degree in Engineering plus being a professional chartered engineer. After that he moved to the UK where he enlisted as a post graduate student in the stability research group of the late Lord Henry Chilver and obtained his Ph.D. degree in structural mechanics under the supervision of Professor J.M.T. Thompson, FRS. After his promotions up to the rank of full professor, he held various positions in the UK, Saudi Arabia and USA and was a visiting professor, senior scholar or adjunct professor in Surrey University, UK, Cornell, USA, Cambridge University, UK and Cairo University, Egypt. In 2012 he ran for the Presidency of Egypt but withdrew at the final stage and returned to academia and his beloved scientific research. He is presently a distinguished Professor at the Dept. of Physics, Faculty of Science of the University of Alexandria, Egypt. Professor El Naschie is well known for his research in structural stability in engineering as well as for his work on high energy physics and more recently for his work in cosmology and elucidation of the secret of dark energy and dark matter as well as for proposing a dark energy Casimir nanoreactor and a fuelless interstellar spaceship. He is the creator of E-infinity theory, which is a physical theory based on random Cantor sets and can be applied to micro, macro and mesoscopic systems. Professor El Naschie is the single or joint author of about one thousand publications in engineering, physics, mathematics, cosmology and political science. His current h-index is 82 and his i-10 index is 791 and total citations are 36427 according to Google Scholar Citation.