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On the Relative Mix Transition Probabilities of three Machine Each of two Different Types in Repairman Problem with Batch Deterministic Repairs

Sunday Olanrewaju Agboola[1] and Titilola Obilade[2]
[1]Department of Mathematical Sciences, Nigerian Army University, Biu, Nigeria.
[2]Department of Mathematics, Obafemi Awolowo University, Ile-Ife, Nigeria.
*Corresponding author email id: agboolasunday70@gmail.com

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Abstract **– The relative mix in the distribution of each machine type and the link between the durability and maintenance of each type of machine is presented. Compared to existing machine repair problem, this study considered the repairman problem with multiple batch deterministic repairs of two different machine types A_1 and A_2. There are integer $k_1 = 3$ of type A_1 machines and integer $k_2 = 3$ of type A_2 machines in the system such that $k_1 + k_2 = k$. Each type $A_1(A_2)$ machine occasionally breaks down and moves into repair queue $Q_1(Q_2)$ at Poisson rate $\lambda_1(\lambda_2)$. The Repair station has a capacity to repair at most two machines in a session. Flow balance equations were obtained for each state of the system as defined by observing the system at repair time points. The resulting equations were solved for stationary probabilities for repair point and occupancy mode conditions using Gaussian elimination. Steady state repair completion probabilities $(p1, p2, \ldots, p49)$ and steady state occupancy probabilities $(qj : j = 1, 2, \cdots, 13)$ were obtained. Subsequently, the work are evaluated with adopted arrival rates $\lambda_1 = 0.25$, 0.50, 0.75, 1.00, $\cdots, 2.50$, $\lambda_2 = 2.00$, and specific deterministic repair times D_{st} for s and t units of type A_1 and A_2 machines respectively, using Minkwoski distance of the various runs and system configuration as a guide, it was established that the closer the system is to a balanced situation the less is the discrepancy in the relative mix of the state probability distribution. We concluded that the relative mix can be rather different from each other depending on the appropriate parameters for the model.**

Keywords **– Machine Repair Problem, Flow Balance Equations, Occupancy Mode Condition, Relative Mix.**

I. Introduction

The performance of any machining system plays a vital role in human life. This has pervaded every field of our lives in different activities ensuring our almost total dependence on machines. However, as the time passes, machines become prone to failure. The failure of machines may result to loss of production, money, goodwill and inconvenience, among others. If at any time a machine fails, it is sent to the repair facility for necessary repair. To avoid the aforementioned loss in machining system, the spare and appropriate repair facility should be incorporated. In view of such a design, the standby units play an important role - so that the machining system may keep working to provide the desired grade of service at all times. If a machine fails, an available standby unit replaces it and the failed machine is sent for immediate repair. A standby unit may be cold standby type which has zero failure rate. It may also be warm standby that has failure rate of non-zero. Today's machining systems are highly sophisticated and complex. They comprise a number of complicated parts. Failure of any part(s) or whole machine directly affects the service system being sought. Thus, a machining system can be out of order at any stage or can have different reasons or models for its failure. Various form of Matrix-analytic methods in queueing theory was introduced [1] while the the classical machine repair problem was discussed [2] this was extended to the Machine Repair Problem with Spares and N-Policy Vacation [3]. The algorithm proposed for busy period subcomponent analysis of bulk queue was presented [4, 5] and this was extended to Queue with Multiple Working Vacation [6]. A heuristic for the traveling repairman problem with profits was analysed [7], this was extended to Bilevel control of degraded machining system with warm standby setup and vacation [8] while the availability analysis of repairable system with warm standby, switching failure and reboot delay, also with an unreliable server and controlled arrival of failed machines was demostrated [9, 10, 11] while the Relative Mix Transition Probabilities in Repairman Problem of Two machines each of two different types with batch deterministic repair was introduced [12, 13]. However, in this study, this is extended to the relative mix transition probabilities of three machine each of two different types in repairman problem with batch deterministic repairs.

Notation

- λ_i Failure rate of type $A_i : i = 1,2$ machines.
- μ_i repair rate of type $A_i : i = 1,2$ machines.
- $A_i,\ i = 1,2$ Machine type A_1 and A_2.
- n_i The number of failed units of type $A_i : i = 1,2$ machines on the repair queue.
- P_n Probability that there are n failed units present in the system in steady state.
- k Total number of machines in the system.
- k_i Total number of type $A_i : i = 1,2$ machines in the system.
- Q_i Repair queue of type $A_i : i = 1,2$ machines in the system.
- D_{st} Constant deterministic time taking if there is s of type A_1 and t of type A_2 mahines in repair.

II. Methodology

The study area consisted of the relative mix transition probabilities of three machine each of two different types in repairman problem with batch deterministic repairs.

2.1. *Mathematical Model*

Consider a closed system of k machines that are either working or under repairs. Suppose there are integer $k_1 = 3$ of type A_1 machines and integer $k_2 = 3$ of type A_2 machines in the system such that $k_1 + k_2 = k$. Suppose each type A_1 machine occasionally breaks down and moves into repair queue Q_1 at Poisson rate λ_1. Suppose also that each type A2 machine occasionally breaks down and moves into repair queue Q_2 at Poisson rate λ_2. The Repair station has a capacity to repair at most two machines in a session. Subject to the availability of machine on the repair queues, the repair manager may pick the two for repairs from Q_1 or the two from Q2. It may also choose to pick one from either Q_1 and Q_2. The random number of machine chosen from Q_1 is given by the binomial distribution $B(2,\tau)$, where τ is the probability of randomly choosing a machine from Q_1. This is equivalent, from the point of view of choosing of Q_2, to the distribution $B(2,1-\tau)$, where $1-\tau$ is the probability of randomly choosing a machine from Q_2. We assume that repair takes a constant deterministic time D_{11} if there is one of each machine type in repair. We also assume that repair takes a constant deterministic time

D_{20} if the two machines are of type A1 and repair take a constant deterministic time D_{02} if the two machines are of type A_2. In the event that only one queue is occupied, two of the machine types are served at D_{20} or D_{02} and each available machine for repair in Q_1 or Q_2 is served at constant time D_{10} or D_{01} depending on which type is available or not available. Our main interest is in the distribution of the mix in number of each of machine type on the repair queues. Noting that the system is not a straight forward Markovian system, we examine the distribution mix at repair completion times.

2.2. *State Definition*

Suppose (n_1, n_2, i) denotes the state that there are integer n_1 type A_1 machines and integer n_2 type A_2 machines on the repair queue at the moment just after a repair completion of type i, $i = 0, 1, 2, 3, 4$ as discussed [6]. Let $p(n_1, n_2, i)$ denotes the probability that there are integer n_1 type A_1 machines and integer n_2 type A_2 machines on the repair queue at the moment just after a repair completion of type $i = 0, 1, 2, 3, 4$. Let $n = n_1 + n_2$. The symbol i takes the value of 0 if the repaired consists of one of the type A_1 machine and one of the type A_2 machines. It takes the value 1 if two of the type A_1 are repaired and the value 2 if two of the type A_2 are repaired. It takes the value 3 if only one of type A_1 is repaired and value 4 if only one of type A_2 is repaired. The diagram to represent the Repairman Problem with Multiple Batch Deterministic Repairs is shown in Fig. 1:

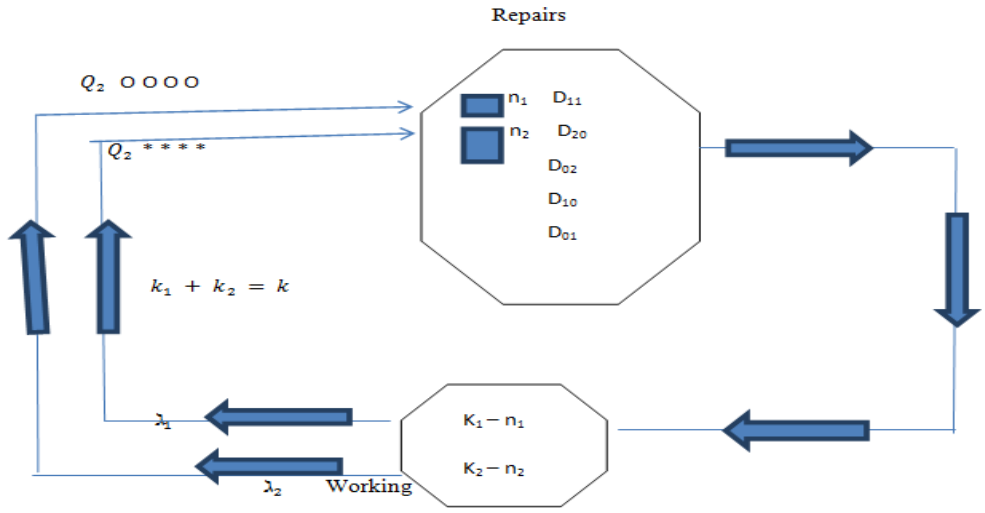

Fig. 1. Repairman Problem with Multiple batch Deterministic Repair.

2.3. *Enumerations of States*

We note as follows:

$0 \leq n1 \leq k1 - 1 \; and \; 0 \leq n2 \leq k2 - 1 \; if \; i = 0$

$0 \leq n1 \leq k1 - 2 \; and \; 0 \leq n2 \leq k2 \; if \; i = 1$

$0 \leq n1 \leq k1 \; and \; 0 \leq n2 \leq k2 - 2 \; if \; i = 2$

$0 \leq n1 \leq k1 - 1 \; and \; 0 \leq n2 \leq k2 \; if \; i = 3$

$0 \leq n1 \leq k1 \; and \; 0 \leq n2 \leq k2 - 1 \; if \; i = 4$

The total number of possible states is calculated as follows: $k_1k_2 + [(k_1 - 1)(k_2 + 1)] + [(k_1 + 1)(k_2 - 1)] + [(k_1)(k_2 + 1)] + [(k_1 + 1)(k_2)] = 5k_1k_2 + k_1 + k_2 - 2$.

Thus, if $k_1 = k_2 = 3$, the total number of possible states is 49. Table 1 gives the 49 possible states for the specific case of $k_1 = k_2 = 3$. The table also contains the pseudonyms.

Table 1. The Chronological Order of the States for $k_1 = k_2 = 3$.

State & Pseudonyms	**State & Pseudonyms**	**State & Pseudonyms**	**State & Pseudonyms**
0 0 0 (1)	1 0 0 (17)	2 0 0 (33)	3 0 2 (45)
0 0 1 (2)	1 0 1 (18)	2 0 2 (34)	3 0 4 (46)
0 0 2 (3)	1 0 2 (19)	2 0 3 (35)	3 1 2 (47)
0 0 3 (4)	1 0 3 (20)	2 0 4 (36)	3 1 4 (48)
0 0 4 (5)	1 0 4 (21)	2 1 0 (37)	3 2 4 (49)
0 1 0 (6)	1 1 0 (22)	2 1 2 (38)	
0 1 1 (7)	1 1 1 (23)	2 1 3 (39)	
0 1 2 (8)	1 1 2 (24)	2 1 4 (40)	
0 1 3 (9)	1 1 3 (25)	2 2 0 (41)	
0 1 4 (10)	1 1 4 (26)	2 2 3 (42)	
0 2 0 (11)	1 2 0 (27)	2 2 4 (43)	
0 2 1 (12)	1 2 1 (28)	2 3 3 (44)	
0 2 3 (13)	1 2 3 (29)		
0 2 4 (14)	1 2 4 (30)		
0 3 1 (15)	1 3 1 (31)		
0 3 3 (16)	1 3 3 (32)		

Table 2. Feasible transition points from Specific States ($G = n_1, n_2, i$) to Possible State ($H = n_1^x, n_2^x, j$) for case $k_1 = k_2 = 3$.

G/H	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
1				P	P				P	P			P	P		P				P	P				P
2				P	P				P	P			P	P		P				P	P				P
3				P	P				P	P			P	P		P				P	P				P
4				P	P				P	P			P	P		P				P	P				P
5				P	P				P	P			P	P		P				P	P				P
6					P					P				P							P				
7					P					P				P							P				
8					P					P				P							P				
9					P					P				P							P				
10					P					P				P							P				
11			P					P											P					P	
12			P					P											P					P	

G/H	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
13			P					P											P					P	
14			P					P											P					P	
15			p					P											P					p	
16								p																	
17				P					P				P			P				P					
18				P					P				P			P				P					
19				P					P				P			P				P					
20				P					P				P			P				P					
21				p					p				p			p				p					
22	P					P					P						P					P			
23	P					P					P						P					P			
24	P					P					P						P					P			
25	p					P					p						p					p			
26	p					p					p						p		P			P			
27						P					P								P			P		P	
28						P					P								P			P		P	
29						P					P								P			P		P	
30						p					p								p			p		P	
31											P													P	
32											p													p	
33		P					P					P			P			P					P		
34		P					P					P			P			P					P		
35		P					P					P			P			P					P		
36		p					P					P			P			p					P		
37							P					P			P								P		
38							P					P			P								P		
39							P					P			P								P		
40							p					P			P								P		
41												P			P										
42												P			P										
43												P			P										
44															P										

G/H	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
45																		p							
46																		P							
47																		p							
48																									
49																									

Table 2.

G/H	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
1	P			P						P	P			P	P		P	P	P				P	P
2	P			P						P	P			P	P		P	P	P				P	P
3	P			P						P	P			P	P		P	P	P				P	P
4	P			P						P	P			P	P		P	P	P				P	P
5	p			P						p	P			p	P		P	P	P				P	P
6	P										P				P			P					P	P
7	p										P				P			P					P	P
8	P										P				P			P					P	P
9	P										P				P			P					P	P
10	p										p				P			P					P	P
11									P				P							P		P		
12									P				P							P		P		
13									P				P							P		P		
14									P				P							p		P		
15													P									P		
16													p									p		
17				P			P							P			P		P					
18				P			P							P			P		P					
19				P			P							P			P		P					
20				P			P							P			P		P					
21				P			p							p			P		p					
22		P														P								
23		P														P								

G/H	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
24		P														P								
25		P														P								
26		P						p				P				P								
27												P	P			P				P		P		
28												P	P			P				P		P		
29												P	P			P				P		P		
30												p	P			P				p		P		
31		P											P			P						P		
32		p											p			p						p		
33			P			P																		
34			P			P																		
35			P			P																		
36			P			P																		
37			P			P		p				P				P								
38			P			P		P				P				P								
39			P			P		P				P				P								
40			P			P		p				P				P								
41			P			P						P	P			P				P		P		
42			P			P						P	P			P				P		P		
43			P			P						P	P			P				P		P		
44						P							p			p						p		
45			P			P																		
46			P			P																		
47			P			P	P					p				P								
48			P			P	p					p				p								
49			P			p						p				p				p		p		

2.4. *Sample Transit Problem*

We present three samples of transition probabilities from each state (n_1, n_2, i) into the feasible state (n_1^x, n_2^x, j) out of 49 as follows:

From state 0 1 0, we have:

$-\ -\ 003 := \frac{\lambda_1}{\lambda}\ exp[-\lambda_1 D_{10}]exp[-\lambda_2 D_{10}]$

$-\ -\ 004 := \frac{\lambda_2}{\lambda}\ exp[-\lambda_1 D_{01}]exp[-\lambda_2 D_{01}]$

$-\ -014 := \frac{\lambda_2}{\lambda}\ [1-\exp(-\lambda_2 D_{01})]exp[-\lambda_1 D_{01}]$

$-\ -024 := \frac{\lambda_2}{\lambda}\ exp[-\lambda_1 D_{01}][1-\exp(-\lambda_2 D_{01})(1+\lambda_2 D_{01})]$

$-\ -\ 104 := \frac{\lambda_2}{\lambda}\ [\lambda_1 D_{01} exp[-\lambda_1 D_{01}]exp[-\lambda_2 D_{01}]]$

$-\ -\ 114 := \frac{\lambda_2}{\lambda}\ [\lambda_1 D_{01} exp[-\lambda_1 D_{01}][1-exp[-\lambda_2 D_{01}]]]$

$-\ -\ 124 :=\ [\lambda_1 D_{01} exp[-\lambda_1 D_{01}][1-exp[-\lambda_2 D_{01}][1+\lambda_2 D_{01}]]]$

$-\ -\ 204 :=\ [1-exp[-\lambda_1 D_{01}][1+\lambda_1 D_{01}]]exp[-\lambda_2 D_{01}]$

$-\ -\ 214 :=\ [1-exp[-\lambda_1 D_{01}][1+\lambda_1 D_{01}]][1-exp[-\lambda_2 D_{01}]]$

$-\ -\ 224 :=\ [1-exp[-\lambda_1 D_{01}][1+\lambda_1 D_{01}]][1-exp[-\lambda_2 D_{01}][1+\lambda_2 D_{01}]]$

$-\ -\ 304 :=\ [1-exp[-\lambda_1 D_{01}][1+\lambda_1 D_{01}+1/2[\lambda_1 D_{01}]]][exp[-\lambda_2 D_{01}]$

$-\ -\ 314 :=\ [1-exp[-\lambda_1 D_{01}][1+\lambda_1 D_{01}+1/2[\lambda_1 D_{01}]^2]][1-exp[-\lambda_2 D_{01}]]$

$-\ -\ 324 :=\ [1-exp[-\lambda_1 D_{01}][1+\lambda_1 D_{01}+1/2[\lambda_1 D_{01}]^2]][1-exp[-\lambda_2 D_{01}][1+\lambda_2 D_{01}]$

The same for 0 1 1 – 0 1 4.

From state 0 2 0,we have :

$-\ -\ -0\ 0\ 2 = exp[-\lambda_1 D_{02}]$

$-\ -\ -0\ 1\ 2 = exp[-\lambda_1 D_{02}][1-exp[-\lambda_2 D_{02}]]$

$-\ -\ -1\ 0\ 2 = \lambda_1 D_{02} exp[-\lambda_1 D_{02}]$

$-\ -\ -1\ 1\ 2 = \lambda_1 D_{02} exp[-\lambda_1 D_{02}][1-exp[-\lambda_2 D_{02}]$

$-\ -\ -2\ 0\ 2 = [1-exp[-\lambda_1 D_{02}][1+\lambda_1 D_{02}]]$

$-\ -\ -2\ 1\ 2 = [1-exp[-\lambda_1 D_{02}][1+\lambda_1 D_{02}]]\ [1-exp[-\lambda_2 D_{02}]$

$-\ -\ -30\ 2 = [1-exp[-\lambda_1 D_{02}]\left[1+\lambda_1 D_{02}+\frac{1}{2}[\lambda_1 D_{02}]^2\right]$

$-\ -\ -31\ 2 = [1-exp[-\lambda_1 D_{02}]\left[1+\lambda_1 D_{02}+\frac{1}{2}[\lambda_1 D_{02}]^2\right]\ [1-exp[-\lambda_1 D_{02}]$

The same for 0 2 1 – 0 2 4

⋮

From state 0 3 1, we have:

$- - - 01\ 2 = exp[-\lambda_1 D_{02}]$

$- - - 11\ 2 = \lambda_1 D_{02} exp[-\lambda_1 D_{02}]$

$- - - 21\ 2 = [1 - exp[-\lambda_1 D_{02}][1 + \lambda_1 D_{02}]]$

$- - - 31\ 2 = [1 - exp[-\lambda_1 D_{02}]\left[1 + \lambda_1 D_{02} + \frac{1}{2}[\lambda_1 D_{02}]^2\right]]$

The same for the state 0 3 3

2.5. *System of Linear Equations for the Transition Probabilities for* $k_1 = k_2 = 3$

We assume the following notations: $a_{st} = exp[-\lambda_1 D_{st}]$

$b_{st} = exp[-\lambda_2 D_{st}]$ (1)

$(s, t) \in \{(1,1), \ (2,0), \ (1,0), \ (0,1) \ (0,2)\}$

The flow balance equations for the system are generally given by $pj = \sum_{i=1}^{49} q_{ij} p_i$, $j = 1,2, \ldots, 49$.

So that by notations (1.0) and from table 1 and table 2, the steady state probabilities are given as follows: λ_1

$p1 = a_{11} b_{11}(p22 + p23 + p24 + p25 + p26)$ (2)

$p2 = b_{20}(p33 + p34 + p35 + p36)$ (3)

$p3 = a_{02}(p11 + p12 + p13 + p14)$ (4)

$p4 = \frac{\lambda_1}{\lambda} a_{10} b_{10}(p1 + p2 + p3 + p4 + p5) + a_{10} b_{10}(p17 + p18 + p19 + p20 + p21)$

$p4 = a_{10} b_{10}[\frac{\lambda_1}{\lambda} (p1 + p2 + p3 + p4 + p5) + (p17 + p18 + p19 + p20 + p21)]$ (5)

$p5 = \frac{\lambda_2}{\lambda} a_{01} b_{01}(p1 + p2 + p3 + p4 + p5) + a_{22} b_{22}(p6 + p7 + p8 + p9 + p10)$

$p5 = a_{01} b_{01}[\frac{\lambda_2}{\lambda}(p1 + p2 + p3 + p4 + p5) + (p6 + p7 + p8 + p9 + p10)]$ (6)

$p6 = a_{11}(1 - b_{11})(p22 + p23 + p24 + p25 + p26) + 1\ 2 a_{11}(p27 + p28 + p29 + p30)$ (7)

$p7 = \lambda_2 D_{20} b_{20}(p33 + p34 + p35 + p36) + 1\ 2 b_{20}(p37 + p38 + p39 + p40)$ (8)

$\vdots$

$p48 = (1 - a_{02} - \lambda_1 D_{02} a_{02} - 1\ 2(\lambda_1 D_{02})2)(1 - b_{02})(p1 + p2 + p3 + p4 + p5) + (1 - a_{02} - \lambda_1 D_{02} a_{02} - 1\ 2(\lambda_1 D_{02})2)(1 - b_{02})(p6 + p7 + p8 + p9 + p10)$ (49)

$p49 = \frac{\lambda_2}{\lambda}(1 - a_{01} - \lambda_1 D_{01} a_{01} - 1\ 2(\lambda_1 D_{01})2)(1 - b_{01} - \lambda_2 D_{01} b_{01})(p1 + p2 + p3 + p4 + p5) + (1 - a01 - \lambda_1 D_{01} a_{01} - 1/\ 2(\lambda_1 D_{01})2)(1 - b_{01} - \lambda_2 D_{01} b_{01})(p6 + p7 + p8 + p9 + p10)$ (50)

$p1 + p2 + p3 + p4 + p5 + \cdots + p49 = 1$ (51)

Solution to Steady State Probabilities Linear Equations for $k_1 = k_2 = 3$

Let :

$$q1 = p1 + p2 + p3 + p4 + p5 \quad (52)$$

denote the probability of repair queue being empty following on completion of repair.

i.e. no unit under repairs(all are working) :

$$q2 = p6 + p7 + p8 + p9 + p10 \quad (53)$$

denote the probability of repair queue being solely ocupied by one unit of type A_2 immediately following the completion of a repair. i.e. only one unit of type A_2 machine is under repairs. The two other units of type A_2 and three units of type A_1 are working:

⋮

$$q11 = p41 + p42 + p43 \quad (62)$$

denote probability of repair queue being occupied by two units of each of type A_1 and type A_2 machines. The others (one of type A1 and one of type A2 are working) :

$$q12 = p45 + p46 + p47 + p48 \quad (63)$$

denote probability of repair queue being occupied by three units of type A_1 machines. i.e. three units of type A_2 machine are working,

$$q13 = p44 + p49 \quad (64)$$

denote the probability of repair queue being occupied by either two or three units of type A_1 and type A_2 machines. Summing Equations (2), ..., (6), we have:

$$(1 - (\frac{\lambda_1}{\lambda} a_{10}b_{10} + \frac{\lambda_2}{\lambda} a_{10}b_{10}))q1 - a_{10}b_{10}q2 - a_{02}q3 + a_{10}b_{10}q5 - a_{11}b_{11}q6 - b_{20}q9 = 0 \quad (65)$$

Similarly, Summing Equations (7), ..., (11), we have :

$$-(\frac{\lambda_1}{\lambda}\lambda_2 D_{10}a_{10}b_{10} + \frac{\lambda_2}{\lambda}(1 - b_{10})a_{01})q1 + (1 - a_{01}(1 - b_{10}))q2 - a_{02}(1 - b_{02})q3 - a_{02}q4 - a_{10}b_{10}\lambda_2 D_{10}q5 - a_{11}(1 - b_{11})q6 - \frac{a_{11}}{2} q7 - \lambda_1 D_{20}b_{20}q9 - \frac{b_{20}}{2} q10 = 0 \quad (66)$$

⋮

Similarly, Summing Equations (42),...,(44), we have:

$$\frac{1}{\lambda}\lambda_1(1 - a_{10} - \lambda_1 D_{10}a_{10})(1 - b_{10} - \lambda_2 D_{10}b_{10}) + \lambda_2(1 - a_{02} - \lambda_1 D_{02}a_{02})(1 - b_{02} - \lambda_2 D_{02}b_{02})q1 + (1 - a_{02} - \lambda_1 D_{02}a_{02})(1 - b_{02} - \lambda_2 D_{02}a_{02})q2 + [(1 - a_{10} - \lambda_1 D_{10}a_{10})(1 - b_{10} - \lambda_2 D_{10}b_{10} - \frac{1}{2}(\lambda_2 D_{10})^2 b_{10})]q5 + (1 - a_{11} - \lambda_1 D_{11}a_{11})(1 - b_{11} - \lambda_2 D_{11}b_{11})q6 + \frac{1}{2}(1 - a_{11} - \lambda_1 D_{11}a_{11} + \frac{1}{2}(\lambda_1 D_{11})^2 a_{02})(1 - b_{11})q7 + 1\,2(1 - a_{11} - \lambda_1 D_{11}a_{11})q8 - q11 \quad (77)$$

Similarly, Summing Equations (46),...,(48), we have:

$$[(1 - a_{01} - \lambda_1 D_{01}a_{01} - \frac{1}{2}(\lambda_1 D_{01})^2 a_{01})(1 - b_{01}) + \lambda_1 \lambda \frac{1}{3}(1 - a_{02})(1 - a_{10} - \lambda_1 D_{10}a_{10})(1 - b_{10} - \lambda_2 D_{10}b_{10} - \frac{1}{2}(\lambda_2 D_{10})^2 b_{10})]q1 + (1 - a_{01} - \lambda_1 D_{01}a_{01} - \frac{1}{2}(\lambda_1 D_{01})^2 a_{01})q2 + (1 - a_{02} - \lambda_1 D_{02}a_{02} - \frac{1}{2}(\lambda_1 D_{02})^2 a_{02})q3 + (1 - a_{02} - \lambda_1 D_{02}a_{02} - \frac{1}{2}(\lambda_1 D_{02})^2 a_{02})(1 - b2)q4 + [(1 - a_{11} - \lambda_1 D_{11}a_{11})(1 -$$

$b_{11} - \lambda_2 D_{10} b_{10} - \frac{1}{2}(\lambda_2 D_{02})^2 b_{10})]q5 + 1\,2(1 - a_{02} - \lambda_1 D_{02} a_{02} - \frac{1}{2}(\lambda_1 D_{02})2\, a_{02})q7 + \frac{1}{3}(1 - a_{02} - \lambda_1 D_{02} a_{02})q8 + \frac{D_{02}}{4}\, q11 - q12 + 1/3$ (78)

III. Results

This section analyse the Steady State Probabilities, $p_j,\ j = 1,2,\ldots,49,$ at Repair completion and Steady State Occupancy Probabilities, $q_j,\ j = 1,2,\ldots,13,$ Corresponding to Breaking Down Shares $\tau_i,\ i = 1,2$ and Minkowski distance of a balanced system for Various Runs with different adopted arrival rates $\lambda_1 = 0.25,\ 0.50,\ 0.75, 1.00,\cdots,2.50,$ where $\lambda_2 = 2.00,$ and specific deterministic repair times D_{st}, $(s, t) \in \{(1, 1), (2, 0), (1, 0), (0, 1), (0, 2)\}$.

3.1. *Numerical Illustration*

Equation (65) to (78) are amenable to solution by gausian elimination method and other methods for solving a system of linear equation. Adopting some realistic values (Table 3) for the parameter as relevant to the application discussed [2]. It is fairly straight forward to obtain the results given on Table 4 and Table 5 for the steady states probability p_j and occupancy probabilities q_j respectively. A closed look at Table 3 identify Run 8 has a balanced breakdown rates for the two machines.

Table 3. Parameters values a dopted for various run.

Runs	λ_1	λ_2	D_{11}	D_{20}	D_{02}	D_{10}	D_{01}
1	0.25	2	11/60	9/60	10/60	5/60	6/60
2	0.50	2	11/60	9/60	10/60	5/60	6/60
3	0.75	2	11/60	9/60	10/60	5/60	6/60
4	1.00	2	11/60	9/60	10/60	5/60	6/60
5	1.25	2	11/60	9/60	10/60	5/60	6/60
6	1.50	2	11/60	9/60	10/60	5/60	6/60
7	1.75	2	11/60	9/60	10/60	5/60	6/60
8	2.00	2	11/60	9/60	10/60	5/60	6/60
9	2.25	2	11/60	9/60	10/60	5/60	6/60
10	2.50	2	11/60	9/60	10/60	5/60	6/60

Table 4: Steady state probabilities, $p_j,\ j = 1,2,\ldots,49,$ at repair completion for various runs.

p_j**/Run**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
p_1	0.0638	0.0530	0.0495	0.0275	0.0174	0.0173	0.0134	0.0125	0.0113	0.0101
p_2	0.0682	0.0858	0.0910	0.01014	0.1024	0.1025	0.1027	0.1030	0.1041	0.1053
p_3	0.1068	0.1074	0.1173	0.1653	0.16603	0.1661	0.1674	0.1675	0.1677	0.1678
p_4	0.0550	0.0688	0.0722	0.0335	0.0912	0.0913	0.0925	0.0928	0.0930	0.0935
p_5	0.1223	0.0959	0.0764	0.0255	0.0224	0.0223	0.0222	0.0216	0.0199	0.0106

p_j/Run	1	2	3	4	5	6	7	8	9	10
p_6	0.0411	0.0394	0.0225	0.0136	0.0096	0.0094	0.0092	0.0091	0.0088	0.0085
p_7	0.0430	0.0544	0.0652	0.0712	0.0735	0.0737	0.0740	0.0743	0.0748	0.0751
p_8	0.0304	0.0307	0.0317	0.0321	0.0332	0.0338	0.0034	0.0342	0.0344	0.0347
p_9	0.0108	0.0115	0.0192	0.0206	0.0222	0.0224	0.0226	0.0228	0.0230	0.0231
p_{10}	0.0320	0.0212	0.0169	0.0149	0.0121	0.0099	0.0083	0.0069	0.0041	0.0018
p_{11}	0.0073	0.0054	0.0045	0.00418	0.0039	0.00373	0.00371	0.00369	0.00369	0.00365
p_{12}	0.0078	0.0143	0.0236	0.0279	0.0292	0.0294	0.0295	0.0297	0.0300	0.0301
p_{13}	0.0005	0.0010	0.0018	0.0029	0.0030	0.0043	0.0082	0.0083	0.0111	0.0147
p_{14}	0.0026	0.0021	0.0016	0.0010	0.0009	0.0008	0.0007	0.0005	0.0004	0.0003
p_{15}	0.0015	0.0019	0.0031	0.0051	0.0062	0.0063	0.0065	0.0067	0.0068	0.0070
p_{16}	0.00003	0.0001	0.00002	0.00005	0.00005	0.00015	0.00072	0.0010	0.0011	0.00112
p_{17}	0.0151	0.0328	0.0354	0.0366	0.0393	0.0394	0.0396	0.0398	0.0399	0.0401
p_{18}	0.1244	0.1104	0.0951	0.0868	0.0784	0.0783	0.0780	0.0776	0.0777	0.0775
p_{19}	0.0076	0.0087	0.0140	0.0172	0.0178	0.0179	0.0181	0.0183	0.0184	0.0119
p_{20}	0.0007	0.0029	0.0074	0.0159	0.0169	0.0275	0.0370	0.0376	0.0404	0.0409
p_{21}	0.0031	0.0048	0.0091	0.0176	0.0183	0.0184	0.0185	0.0186	0.0188	0.0189
p_{22}	0.0027	0.0028	0.0033	0.0034	0.0038	0.00384	0.00386	0.00371	0.00388	0.00389
p_{23}	0.0970	0.0857	0.0554	0.0365	0.0338	0.0275	0.0138	0.0102	0.0039	0.0023
p_{24}	0.0038	0.0085	0.0078	0.0063	0.0065	0.0063	0.00424	0.00415	0.0041	0.00404
p_{25}	0.00012	0.0005	0.0012	0.0027	0.0032	0.0034	0.0036	0.0037	0.0038	0.0040
p_{26}	0.0007	0.0011	0.0020	0.0039	0.0040	0.00419	0.00432	0.00457	0.00464	0.00482
p_{27}	0.0029	0.0030	0.0033	0.0094	0.0095	0.00963	0.00968	0.00974	0.00981	0.00988
p_{28}	0.0792	0.0793	0.0793	0.0935	0.0941	0.09424	0.09428	0.09426	0.1117	0.1120
p_{29}	0.00001	0.00004	0.0001	0.0002	0.0004	0.00042	0.00053	0.00059	0.00064	0.00071
p_{30}	0.00002	0.00001	0.00001	0.000006	0.000005	$3.8x10^{-5}$	$1.2x10^{-5}$	$1.0x10^{-5}$	$6x10^{-6}$	$4.5x10^{-6}$
p_{31}	0.0159	0.0135	0.0092	0.0006	0.0004	$1.6x10^{-4}$	$1.6x10^{-4}$	$4.9x10^{-5}$	$3.2x10^{-6}$	$1.0x10^{-6}$
p_{32}	$4.2x10^{-7}$	$1.6x10^{-4}$	$4.0x10^{-4}$	$8.8x10^{-4}$	$2.3x10^{-4}$	$4.3x10^{-7}$	$5.7x10^{-4}$	$6.0x10^{-4}$	$6.2x10^{-4}$	$6.2x10^{-4}$
p_{33}	0.0372	0.0393	0.0593	0.060	0.0607	0.0607	0.0609	0.0611	0.0612	0.0140
p_{34}	0.0001	0.0007	0.0010	0.0022	0.0023	0.0024	0.0025	0.0026	0.0027	0.0028
p_{35}	$7.8x10^{-4}$	$6.0x10^{-4}$	0.0002	0.0007	0.0008	0.0017	0.00176	0.0021	0.0042	0.0064

p_j|Run	1	2	3	4	5	6	7	8	9	10
p_{36}	$3.9\text{x}10^{-5}$	$1.2\text{x}10^{-4}$	$3.5\text{x}10^{-4}$	$9.1\text{x}10^{-4}$	$9.3\text{x}10^{-4}$	$9.6\text{x}10^{-4}$	0.0011	0.0012	0.0014	0.0015
p_{37}	$2.7\text{x}10^{-4}$	0.0012	0.0030	0.0063	0.0068	0.0083	0.0084	0.00844	0.00858	0.00863
p_{38}	$3.5\text{x}10^{-4}$	$6.5\text{x}10^{-4}$	$8.2\text{x}10^{-4}$	$9.1\text{x}10^{-4}$	0.0013	0.00052	0.0011	0.0023	0.0040	0.0061
p_{39}	**$1.4\text{x}10^{-7}$	**$9.1\text{x}10^{-7}$	**$1.0\text{x}10^{-5}$	**$3.6\text{x}10^{-5}$	**$5.6\text{x}10^{-5}$	**$7.2\text{x}10^{-5}$	**$7.7\text{x}10^{-5}$	**$8.4\text{x}10^{-5}$	**$9.2\text{x}10^{-5}$	**$1.1\text{x}10^{-4}$
p_{40}	$8.5\text{x}10^{-7}$	$2.7\text{x}10^{-5}$	$7.8\text{x}10^{-5}$	$2.0\text{x}10^{-5}$	$4.3\text{x}10^{-4}$	$4.4\text{x}10^{-4}$	$4.5\text{x}10^{-4}$	$4.6\text{x}10^{-4}$	$4.7\text{x}10^{-4}$	$4.74\text{x}10^{-4}$
p_{41}	$1.4\text{x}10^{-5}$	$4.8\text{x}10^{-5}$	$7.7\text{x}10^{-5}$	$8.5\text{x}10^{-5}$	$3.5\text{x}10^{-4}$	$3.56\text{x}10^{-4}$	$3.57\text{x}10^{-4}$	$3.7\text{x}10^{-4}$	$3.8\text{x}10^{-4}$	$4.0\text{x}10^{-4}$
p_{42}	$3.76\text{x}10^{-7}$	$3.17\text{x}10^{-7}$	$1.2\text{x}10^{-4}$	$3.4\text{x}10^{-4}$	$8,6\text{x}10^{-4}$	$9.8\text{x}10^{-7}$	$1.0\text{x}10^{-5}$	$1.2\text{x}10^{-5}$	$2.1\text{x}10^{-5}$	$4.9\text{x}10^{-5}$
p_{43}	$8.24\text{x}10^{-7}$	$2.61\text{x}10^{-7}$	$7.5\text{x}10^{-4}$	$1.9\text{x}10^{-5}$	$4.3\text{x}10^{-5}$	$8.2\text{x}10^{-7}$	$2.6\text{x}10^{-7}$	$7.5\text{x}10^{-4}$	$1.9\text{x}10^{-5}$	$4.3\text{x}10^{-5}$
p_{44}	$6.2\text{x}10^{-7}$	$4.8\text{x}10^{-7}$	$1.8\text{x}10^{-7}$	$5.3\text{x}10^{-7}$	$1.4\text{x}10^{-4}$	$3.4\text{x}10^{-7}$	$5.2\text{x}10^{-7}$	$7.7\text{x}10^{-7}$	$9.4\text{x}10^{-7}$	$1.5\text{x}10^{-4}$
p_{45}	$1.2\text{x}10^{-5}$	$2.2\text{x}10^{-5}$	$7.8\text{x}10^{-5}$	$1.2\text{x}10^{-4}$	$5.9\text{x}10^{-4}$	$5.7\text{x}10^{-4}$	$7.4\text{x}10^{-4}$	$8.2\text{x}10^{-4}$	$8.3\text{x}10^{-4}$	$9.1\text{x}10^{-4}$
p_{46}	$2.2\text{x}10^{-7}$	$2.5\text{x}10^{-7}$	$2.0\text{x}10^{-6}$	$1.2\text{x}10^{-5}$	$6.0\text{x}10^{-5}$	$5.7\text{x}10^{-4}$	$7.4\text{x}10^{-4}$	$8.4\text{x}10^{-5}$	$9.2\text{x}10^{-5}$	$9.1\text{x}10^{-5}$
p_{47}	*0.5627	*0.2993	*0.1606	*0.1681	*0.3982	*$7.2\text{x}10^{-5}$	*$8.2\text{x}10^{-5}$	*$8.8\text{x}10^{-5}$	*$9.2\text{x}10^{-5}$	*$1.0\text{x}10^{-4}$
p_{48}	$1.5\text{x}10^{-7}$	$5.7\text{x}10^{-7}$	$2.6\text{x}10^{-4}$	$8.7\text{x}10^{-4}$	$2.2\text{x}10^{-5}$	$6.8\text{x}10^{-5}$	$7.3\text{x}10^{-5}$	$8.1\text{x}10^{-5}$	$8.0\text{x}10^{-4}$	$9.7\text{x}10^{-5}$
p_{49}	$1.3\text{x}10^{-4}$	$4.5\text{x}10^{-4}$	$1.9\text{x}10^{-7}$	$6.4\text{x}10^{-7}$	$1.2\text{x}10^{-4}$	$6.8\text{x}10^{-5}$	$3.5\text{x}10^{-5}$	$4.6\text{x}10^{-5}$	$5.7\text{x}10^{-5}$	$7.8\text{x}10^{-5}$

Table 5. Steady State Occupancy Probabilities, q_j, $j = 1,2,\ldots,13$, corresponding to breaking down shares τ_i, $i = 1,2$ and Minkowski distance of a balanced system as discussed in Agboola, 2016.

Occupancy Probabilities at Repair stations													
Run	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}	q_{12}	q_{13}
1	0.4162	0.1573	0.0182	0.0015	0.1510	0.1082	0.0817	0.0159	0.0374	0.0006	0.000015	$1.9\text{x}10^{-5}$	$6.0\text{x}10^{-4}$
2	0.4110	0.1572	0.0228	0.0019	0.1597	0.0985	0.0823	0.0135	0.0401	0.0018	$5.1\text{x}10^{-5}$	$4.8\text{x}10^{-5}$	$9.4\text{x}10^{-8}$
3	0.4063	0.1553	0.0314	0.0031	0.1611	0.0802	0.0827	0.0092	0.0608	0.0039	$7.8\text{x}10^{-5}$	$1.3\text{x}10^{-4}$	$3.8\text{x}10^{-7}$
4	0.4031	0.1524	0.0359	0.0051	0.1740	0.0530	0.1032	0.0006	0.0638	0.0074	0.0001	0.0002	$1.2\text{x}10^{-6}$
5	0.3995	0.1506	0.0369	0.0063	0.1767	0.0514	0.1040	0.0005	0.0639	0.0085	0.0004	0.0007	$2.6\text{x}10^{-6}$
6	0.3993	0.1481	0.0382	0.0065	0.1821	0.0452	0.1043	0.0002	0.6530	0.0094	0.0004	0.0008	$2.4\text{x}10^{-6}$
7	0.3982	0.1481	0.0421	0.0072	0.1912	0.0298	0.1045	0.0001	0.0663	0.0100	0.0004	0.0010	$2.5\text{x}10^{-6}$
8	0.3974	0.1473	0.0422	0.0080	0.1921	0.0265	0.1047	0.0001	0.0671	0.0131	0.0005	0.0011	$2.8\text{x}10^{-6}$
9	0.3959	0.1451	0.0452	0.0079	0.1952	0.0194	0.1050	0.0001	0.0696	0.0131	0.0005	0.0011	$3.53\text{x}10^{-6}$
10	0.3873	0.1432	0.0488	0.0081	0.1961	0.0191	0.1053	0.0001	0.0721	0.0153	0.0005	0.0012	$4.85\text{x}10^{-6}$
M.D	3.0365	0.5269	0.6160	0.9221	0.8015	0.4464	0.0000	0.9374	0.3718	0.8946	0.9748	0.9714	0.9756

IV. Discussion

We have presented the steady states repair completion probabilities p_j, $j = 1,2,\ldots,49$ and Occupancy repair completion probabilities, $q_j, j = 1,2,\ldots,13$. Indicated in one (*) asterisk in Table 4 are the most frequent states for the runs. Also indicated in two (**) asterisks are the less frequent states. The adopted parameters are justified for efficiency of service by the fact that the most frequent states are those for which the system is idle while the less frequent states are those for which the system has a high load for repairs. The last row in Table 5 gives the Minkowski distance of the distribution of the occupancy probabilities at repair station respectively, for each run compared to run 8 where there is some balance in the breakdown rate $(\lambda_1 = \lambda_2)$. It would appear that the closer the system is to a balanced situation, the less the discrepancy in the mix for the state probability distribution. We concluded that the relative mix can be rather different from each other depending on the appropriate parameter for the model.

V. Conclusion

This study considered the repairman problem with multiple batch deterministic repairs of two different machine types A_1 and A_2. There are integer $k_1 = 3$ of type A_1 machines and integer $k_2 = 3$ of type A_2 machines in the system such that $k_1 + k_2 = k$. Each type $A_1(A_2)$ machine occasionally breaks down and moves into repair queue $Q_1(Q_2)$ at Poisson rate $\lambda_1(\lambda_2)$. The Repair station has a capacity to repair at most two machines in a session. Flow balance equations were obtained for each state of the system as defined by observing the system at repair time points. The resulting equations were solved for stationary probabilities for repair point and occupancy mode conditions using Gaussian elimination. Steady state repair completion probabilities $(p1, p2, \dots, p49)$ and steady state occupancy probabilities $(qj : j = 1,2,\cdots,13)$ were obtained. Subsequently, the work is evaluated with adopted arrival rates $\lambda_1 = 0.25, 0.50, 0.75, 1.00,\cdots,2.50$, $\lambda_2 = 2.00$, and specific deterministic repair times D_{st}, $(s,t) \in \{(1,1), (2,0), (1,0), (0,1)\ (0,2)\}$ for s and t units of type A_1 and A_2 machines respectively. Using Minkowski distance of the various runs and system configuration as a guide, it was established that the closer the system is to a balanced situation the less is the discrepancy in the relative mix of the state probability distribution. We concluded that the relative mix can be rather different from each other depending on the appropriate parameter for the model.

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AUTHOR'S PROFILE

Sunday Olanrewaju Agboola, is an Associate Professor of Applied Mathematics/Statistics at Nigerian Army University Biu. He attended The Polytechnic Ibadan Nigeria for National Diploma in Statistics between 1995 to 1997 and proceeded to Ladoke Akintola University of Technology Ogbomoso Nigeria in 1997 where he graduated in the year 2002 with B. Tech. (Hons) Pure and Applied Mathematics. In 2004, he commenced his 1st Second Degree (Master's Degree) in Mathematics at the prestigious University of Ibadan Nigeria and graduated in 2007. He proceeded to Obafemi Awolowo University Ile – Ife between 2008 to 2016 for his 2nd Master's Degree and Ph.D in Statistics. He has taught Post graduates and Undergraduates in Mathematics and Statistics courses at several Universities among which includes: Fede--ral University Lokoja, Bells University of Technology Ota, KolaDaisi University Ibadan, Nigeria and Nigerian Army University Biu. He researches in the areas of Operations Research, Computational Mathematics, Mathematical Modelling, Numerical Analysis, Probability Theories and Stochastic Processes, and he is currently the Acting Dean Faculty of Natural and Applied Sciences. Titilola Obilade is a Professor of Mathematical Statistics at Obafemi Awolowo University, ILE-IFE. He holds BSc Maths (Ife), MSc Statistics and Operations Research as well as PhD from Trinity college, Dublin.