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Product Market Equilibrium Modeling Based on the Implicit Function Theorem and its Practice in Teaching

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Abstract **–Following the analysis of the necessity of new modeling method, this paper provides a new mathematical equilibrium model for the product market equilibrium based on the implicit function theorem, which modified the linear assumption into non-linear, and built a link between the multipliers theory and equilibrium model in Macroeconomics. Compared with literature, our modeling method can simultaneously analyze the existence of equilibrium solution and the multipliers followed.**

Keywords **– Implicit Function Theorem, Product Market Equilibrium, Abstract Function, AMS Classification Numbers: 15A69, 97B10.**

I. INTRODUCTION

Product market equilibrium model is one of the important teaching contents in Principles of Macroeconomics. Furthermore, it is the basis of the well-known IS-LM model [1]. It is an important teaching work to help students understand the equilibrium conditions and master the analysis of variation from equilibrium solutions. However, in real process of teaching, we found that more students didn't know hypothesis of linear function, the link of all the multipliers, the relationship between economic equilibrium and multipliers, and so on. Perhaps this is due to lack of discussion of those questions in the textbooks, such as [1]. So the reform of teaching design for modeling product market equilibrium must be carried out.

The necessity of a new modeling method for product market equilibrium analysis is as follows.

1.1. *Assumption of Linear Function for the Consumption Function Need to Be Modified*

In traditional teaching practice and textbook of Macroeconomics Principle, the gross consumption and gross income are usually assumed to be linear functions [1]. Under these assumptions, if gross income increases by Y units, gross consumption will increase by Y*D units with the fixed proportion of D. But in fact, the growth of the consumption from low income group tends to be bigger than from high one. If most of the increase of income comes from low income group, compared with from high one, the consumption will grow at a higher proportion, not a fixed ratio.

Therefore, an assumption of linear function can't reflect characteristic of reality. It is necessary to modify it into an assumption of abstract function [2]. The discussion followed with it will be more logical and practical.

1.2. *Need to Build a Link between the Multipliers Model and Market Equilibrium Model*

In the teaching practice of elementary Macroeconomics, a model of multipliers is built to analyze the significant policy parameters effects on macroeconomic trend. Those parameters include government purchase multiplier, private investment multiplier, tax multiplier, and so on.

But these multiplier models in most macroeconomics textbooks are separated from the product market equilibrium theory, which causes the students to be lack of a global thinking and the model thinking.

1.3. *Economic Equilibrium Analysis is Worth to be Further Highlighted*

Economic equilibrium analysis is one of the core methods in economics. It is better for the students to learn economic equilibrium analysis earlier. Cultivating the insight of equilibrium would be beneficial to learn the advanced economics.

Following the analysis of the necessity of new modeling method, this paper provides a new mathematical equilibrium model for the product market equilibrium based on the implicit function theorem, which modified the linear assumption into non-linear, and built a link between the multipliers theory and equilibrium model in Macroeconomics. Compared with literature [3], our modeling method can simultaneously analyze the existence of equilibrium solution and the multipliers followed.

II. Teaching Design of Product Equilibrium Model Analysis Based on Implicit Function Theorem

In order to solve the problems above in teaching Macroeconomics, the equilibrium simultaneous equations are constructed on the basis of implicit function theorem. Differentials and derivatives from the equations will be derived. The linear algebra solving method based on Jacobian matrix [5] will be applied.

2.1. *Mathematical Modeling of Product Market Equilibrium*

Suppose the real income in product market is y, which measures the aggregate supply in product market. Aggregate demand will be separated into $c+i+g+x-m$ [1], c is the endogenous consumption function which depends on y and is described as $c=c\left(y-T+t_r\right)$, $c(\ )$ is subject to $0<c'<1$. $T=T(y)$ is the endogenous tax function, which is subject to $0<T'<1$. $m=m(y)$ is the endogenous import function, which is subject to $0<m'<1$. The exogenous parameters include g which is government purchase, t_r which is government transfer payment, i which is private investment, and x which is export.

When product market equilibrium appears, the simultaneous equations (1) hold true. For the multiplier theory needs to be established on product market equilibrium, the existence of equilibrium turns to be an important problem. Whether the product market equilibrium exists or not, is determined by whether (1) have solutions.

$$\begin{cases} y=c+i+g+x-m \\ c=c(y-T+t_r) \\ T=T(y) \\ m=m(y) \end{cases} \tag{1}$$

2.2. *Equilibrium Analysis on the Basis of Implicit Function Theorem*

In order to analyze the existence of solutions to simultaneous equations (1) [3], (1) is transformed into simultaneous equations (2). Jacobian matrix (3) is derived from (2) [6] [7].

$$\begin{cases} y-(c+i+g+x-m)=0 \\ c-c(y-T+t_r)=0 \\ T-T(y)=0 \\ m-m(y)=0 \end{cases} \tag{2}$$

$$|J|=\begin{vmatrix} 1 & -1 & 0 & -1 \\ -c' & 1 & c' & 0 \\ -T' & 0 & 1 & 0 \\ -m' & 0 & 0 & 1 \end{vmatrix}=1-c'+m'+c'T'>0 \tag{3}$$

According to the implicit function theorem, the simultaneous equations (2) have solutions for $|J|>0$. These solutions are the product market equilibrium expressed as the functions (4) with exogenous parameters of i, g, x, t_r, c', m'.

$$\begin{cases} y^* = F_1(i,g,x,t_r,c',m') \\ c^* = F_2(i,g,x,t_r,c',m') \\ T^* = F_3(i,g,x,t_r,c',m') \\ m^* = F_4(i,g,x,t_r,c',m') \end{cases} \tag{4}$$

The derivatives of those equilibrium variables with respect to private investment *i* are derived, in order to discuss the private investment's effect on equilibrium variables such as national income, consumption, tax, and import. With implicit function theorem, equation (5) is derived. The partial derivative of the equilibrium income with respect to investment $\frac{\partial y^*}{\partial i}$ will be given in (6) based on the Cramer Rule [5] from linear algebra. $\frac{\partial y^*}{\partial i}$ is the private investment multiplier in nature. Other multipliers can be gained with similar method above.

$$\begin{pmatrix} 1 & -1 & 0 & -1 \\ -c' & 1 & c' & 0 \\ -T' & 0 & 1 & 0 \\ -m' & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{\partial y^*}{\partial i} \\ \frac{\partial c^*}{\partial i} \\ \frac{\partial T^*}{\partial i} \\ \frac{\partial m^*}{\partial i} \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

$$\frac{\partial y^*}{\partial i}=\frac{1}{1-c'(1-T')+m'} \tag{6}$$

III. Conclusion

Firstly, compared with the assumption of linear functions, the abstract function will help to analyze the macroeconomic problems with both non-linear and linear models. The analysis would be more general and more logical.

Secondly, the macroeconomics theory will impress the learners greatly, with the help of combination of the multiplier model and market equilibrium model. This will cultivate a global insight of macroeconomic issues,

and construct the ability of solving problems with calculus and linear algebra.

Finally, the modified model expressed product market equilibrium as solutions to simultaneous equations. The existence of equilibrium solutions was discussed by implicit function theorem. So students would have the experience of applying the mathematical models to analyze economic issues.

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