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The Application of Derivative in Economics

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***Abstract* – This paper takes the relationship and function between derivative and economics as the starting point, discusses the application and significance of marginal analysis and elastic analysis in economic problems with examples of derivative in economics, and forms a general method of derivative to solve economic problems. This paper uses derivative to reflect the importance of mathematics in economic life.**

***Keywords* – Derivatives, Economics, Marginal Analysis, Marginal Cost, Marginal Revenue, Marginal Profit, Elastic Analysis, AMS Classification Numbers: 15A69, 97B10.**

I. Introduction

The emergence of the marginal utility school in the 1970s is considered to be the symbol of a comprehensive revolution in economics. This revolution is called marginal revolution. This revolution changed economics from the production, supply and cost emphasized by classical economics to the consumption, demand and utility concerned by modern economics.

Derivative plays a very important role in economic life. People usually use derivative in the analysis of marginal cost, marginal profit and marginal effect [1]. The marginal revolution contains two important contents, namely, the marginal utility theory of value and the extensive application of the marginal analysis method. The method of marginal analysis is actually a mathematical analysis method, that is, using calculus in mathematics to observe economic problems. The wide use of marginal analysis is a major change in economic research [2].

II. The Mathematical Definition of Derivative and its Economic Significance

2.1. *Mathematical Definition of Derivative*

Let the function $y=f(x)$ be defined in a neighborhood of a point x_0, when the argument gets Δx increment at the point x_0, the function y get Δy increment accordingly, as $\Delta x \to 0$, if the limit of the ratio of Δy to Δx exists, then the function is said to have a derivative at the point, which is written as $f'(x_0)$, that is $f'(x_0)=\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$.

In practice, it is necessary to discuss the change speed of various variables with different meanings. Mathematically, it is the change rate of function, and the concept of derivative is its accurate description.

2.2. *Economic Significance of Derivative*

In economics, the derivative of a function is called its marginal function, and the value at a specific point is called the marginal value of the function at that point, that is, the marginal refers to the degree to which the dependent variable changes with the change of the independent variable, that is, the amount that the dependent variable will change as the independent variable changes by one unit [3].

III. The Application of Derivative in Economics

3.1. *Derivative and Marginal Analysis*

Marginal refers to the amount of change of the dependent variable when the independent variable in economics changes by one unit. Marginal analysis is the analysis of the rate of change of economic variables.

a. *Marginal Cost*

In the process of economic calculation, cost minimization is generally expressed by marginal cost. Marginal cost is the change in total cost for each additional unit of goods produced under other conditions unchanged [4].

Generally speaking, the marginal cost decreases with the increase of production, which is due to the existence of scale effect. If the production cost of producing a bicycle is very high, the cost of producing each of the 10000 bicycles will be reduced. However, there is no lack of cases that the marginal cost increases with the increase of production, which is usually due to the existence of silent cost [5].

According to the concept of margin, if the product output is Q and its corresponding cost function $C(Q)$ is derivable, then its marginal cost can be defined as: $C'(Q)=\lim\limits_{\Delta Q\to 0}\frac{\Delta C}{\Delta Q}=\lim\limits_{\Delta Q\to 0}\frac{C(Q+\Delta Q)-C(Q)}{\Delta Q}$.

Therefore, the marginal cost is the derivative of the cost function with respect to output.

Example 1 When the output of toothbrush in a factory is Q, the cost function is $C(Q)=100+0.02Q^2$, and the marginal cost of producing 10 and 100 a products is calculated [6].

According to the above definition of marginal cost, the marginal cost function is $C'(Q)=0.04Q$, $C'(10)=0.4$, $C'(100)=4$.

Therefore, the marginal cost of producing 10 products is 0.4, while the marginal cost of producing 100 products is 4.

Let's calculate the average cost again: When $Q=10$, the average cost is $\frac{C(10)}{10}=10.2$, when $Q=100$, the average cost is $\frac{C(100)}{100}=3$.

Conclusion: when the output is 10, the output must be increased because the marginal cost is 0.4. For each additional product, the cost will only increase by 0.4, which is far lower than the average cost of 10.2. When the output is 100, it is not suitable to increase the output, because the marginal cost is 4, which is higher than the average cost of 3.

b. *Marginal Income*

Similar to the marginal cost, the marginal income is the derivative $R'(Q)$ of the income function $R(Q)$ with respect to output. Its economic meaning is the increased income ΔQ. The product is sold when the output is Q.

Example 2 When the output of computer host is Q, the revenue function is $R(Q)=1800+680Q-0.3Q^2$.

Calculate the marginal revenue when the production of main engine is 110 pieces.

According to the above introduction of marginal income, the marginal income function is $R'(Q)=680-0.6Q$.

So, $R'(110)=614$. Therefore, when the mouse output is 110, the marginal income is 614.

Let's calculate the average income again: When the output is 110 pieces, the average income is $\frac{R(110)}{110}\approx 663.36$.

Conclusion: when the output is 110 pieces, the income increased by one product is 614 yuan, while the average income per product is 663.36 yuan. O bviously, increasing the production cannot increase the average income.

c. *Marginal Profit*

Similar to marginal income, marginal profit is defined as the derivative of the total profit function $L(Q)$ of goods with respect to the quantity of products sold, that is $L'(Q)$. Meaning: when the sales quantity is Q, the profit increased by selling another piece. It should be noted that the marginal profit $L'(Q)<0$ is different from the profit $L(Q)<0$. $L'(Q)<0$ means that when the sales volume is Q, the profit from selling another product is less than the current average profit per product. However, the total profit from selling products increases, which is what we call a small profit in life. The average profit of each product decreases, the enterprise is still profitable, and the total profit is still increasing. Therefore, this situation is beneficial to the enterprise. However, $L(Q)<0$ means that the sales volume is Q and the profit is negative, which means that the enterprise is in a state of loss [6].

Example 3 The total cost of producing certain automobile parts in an automobile enterprise is a function of output, recorded as $C(Q)=1.2Q^2+5Q+180$, it is known that the sales unit price of the product is 605 yuan, to calculate the profit function and marginal profit of the product, and to calculate the marginal profit , when the sales volume is 190 pieces, 250 pieces and 280 pieces. Explain the economic significance of marginal profit at different sales volumes.

This question involves the output and sales volume. Assuming that the output is consistent with the sales volume, the total income of the product is $R(Q)=605Q$, we know the total cost is like this, $C(Q)=1.2Q^2+5Q+180$.

Then according to the profit function = revenue function - cost function, so $L(Q)=R(Q)-C(Q)=-1.2Q^2+600Q-180$.

The marginal profit is like this,

$L'(Q)=-2.4Q+600$,

$L'(190)=-2.4\times 190+600=144$,

$L'(250)=-2.4\times 250+600=0$,

$L'(280)=-2.4\times 280+600=-72$.

When the sales volume is 190 pieces, 250 pieces and 280 pieces, the marginal profits are 144, 0 and -72 yuan respectively.

Conclusion: when the sales volume is 190 pieces, the profit will increase by approximately 144 yuan when a-

-nother unit product is produced; When the sales volume is 250 pieces, the profit of another product will not increase; When the sales volume is 280 pieces, the profit will be reduced by 72 yuan if another product is produced.

3.2. *Derivative and Elasticity*

The concept of elasticity in economics was put forward by alfredmarshall. It refers to the attribute that a variable changes in a certain proportion relative to another variable [7]. The concept of elasticity can be applied to all causal variables. The variable as a cause is usually called an independent variable, and the quantity changed by its action is called a dependent variable. The relationship between independent variable and dependent variable is as follows: $y = f(x)$, then the x elasticity of y is as follows: $Ey/Ex = (\Delta y/y)/(\Delta x/x) = f'(x) \cdot x/y$.

a. *Price Elasticity of Demand*

Price Elasticity of Demand, in economics it is generally used to measure that the quantity of demand changes with the change of commodity price.

b. *Price Elasticity of Supply*

Price Elasticity of Supply indicates the extent to which a 1% change in price causes a change in supply. Like the price elasticity of demand, the supply price elasticity coefficient is also determined by the ratio of the percentage of supply change to the percentage of price change.

According to the supply theorem in economics, the supply quantity and price change in the same direction, that is, the greater the price change of the commodity, the production of the enterprise will change accordingly.

c. *Income Elasticity of Demand*

Income Elasticity of Demand refers to measuring the response of the demand for an article to the change in consumer income, that is, the change percentage of the demand for 1% of the change in consumer income.

d. *Cross-Price Elasticity of Demand*

Cross-Price Elasticity of Demand means that every 1% change in the price of some other goods will change the demand of the goods by a few percent. From this concept, we can deduce two important concepts in Economics: substitute and complementary [8].

The cross elasticity of demand reflects the sensitivity of consumers to changes in the demand for a certain commodity corresponding to changes in the prices of other commodities. Its elasticity coefficient is defined as the percentage of changes in the demand divided by the percentage of changes in the prices of other commodities. The cross elasticity coefficient can be greater than 0, equal to 0 or less than 0, which indicates that there is a substitution, uncorrelation or complementary relationship between the two commodities.

IV. Conclusion

Through the analysis of various applications of derivatives in economics and the explanation of corresponding application examples, this paper leads the readers to experience how to apply abstract mathematical theory to specific economic practice. Mathematics is closely related to economics, especially microeconomics. Using mathematical quantitative analysis, especially the establishment of mathematical models, to solve problems in the economic field has become an integral part of the whole theoretical system of economics [9]. Derivatives are widely used in economics. We have mastered the marginal and elastic problems of derivatives in microeconomics, In many business decisions, it can play a very important role if we can use advanced mathematical knowledge to analyze and solve the phenomena in economics and summarize them into the mathematical field [10].

It is very important for enterprises to conduct demand analysis. If the enterprise leaves the social demand and blind production, it will cause the huge wave cost of the source of capital enterprises such as if it leaves the accurate analysis of the need for price, it is impossible to reach the goal of maximizing profit [8]. Using mathematics as an analytical tool can provide objective and accurate data to decision-makers, and at the same time, in the process of analysis and inclusion, it can provide new ideas and perspectives for decision-makers. Therefore, we should use mathematical knowledge more fully in economic research in the future, so that economics will move towards quantification, precision, and accuracy [11].

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