International Journal of Applied Science and Mathematics
Volume 2, Issue 3, ISSN (Online): 2394-2894

On the Estimation of Longevity Using Parametric Models

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Abstract — **The Cox PH model is the most common method of analyzing prognostic factors in clinical data. The model allows estimate and make inference about the parameters without assuming any distribution for the survival time. However, when the proportional hazards assumption is not tenable, these models will not be suitable. In this study, we introduced derived longevity model (DLM) from non-parametric using parametric models, in which specific probability distribution is assumed for the survival times. This study centered on DLMs using parametric based survival function models and discussions of findings from combined data of universities academic retirees from two premier universities in Western Nigeria. Estimated mean of post retirement years for universities academic retirees from DLMs using Exponential, Weibull and Gompertz models without the effect of explanatory variables are 22.66, 22.35 and 22.08 years. However, DLMs with the effect of explanatory variables are 22.78, 22.82 and 22.48 years respectively. We recommend DLM using Weibull models for reliability, maintainability and safety (RMS) work as parametric model.**

Keywords — **DLM, Exponential, Gompertz, Retirees, Variance, Weibull.**

I. Introduction

Parametric models are family of distributions that can be described using a finite number of parameters. These conventional models assume an underline probability density function which gives rise to a parametric to estimate association with the distribution. Parametric models are mostly used to describe the risk of failure at any point in time called hazard function. According to (4), the hazard function also known as the failure rate, hazard rate, or force of mortality is the ratio of the probability density function to the survival function. Failure rate is the frequency with which an engineered system or component fails, expresses failures per hour. It is often denoted by the Greek letter λ (lambda) and is important in consistency engineering (Finkelstein, 2008).

According to (6), the associated difficulty with these methods is of establishing that the underlying assumption of the life time is correctly specified. Failure to do so will result in incorrect conclusions. Since life time is a time measure, hence continuous, the distribution involved needs be continuous. Properties of continuous distribution are well recognized and readily available such that their use in modeling, if the data does comply, makes them more eligible for use. Moreover, for parametric models all four functions characterizing lifetime data, namely probability density function, probability distribution, hazard or cumulative hazard rates and survivor functions, are of closed form for most of these distributions. This renders estimation of a single failure time event more feasible.

The parametric distributions include the Weibull, Exponential, Gompartz, Lognormal, e.t.c, most of which emanate from the generalized gamma (GG) family of distributions denoted as: $f(t)=\frac{\beta}{\Gamma(k)\theta}\left(\frac{t}{\theta}\right)^{k\beta-1}\ell^{-\left(\frac{t}{\theta}\right)^{\beta}}$, where $\theta>0$ is a scale parameter, $\beta>0$ and $k<0$ are shape parameters and $\Gamma(x)$ is the Gamma function of x, which is defined by: $\Gamma(x)=\int_0^{\alpha} S^{x-1}\ell^{-s}ds$. The problem of the conventional parametric models is that they are mostly used to describe the risk of failure at any point in time call hazard function, failure rate, hazard rate, or force of mortality which is the ratio of the probability density function to the survival function (4).

We developed longevity (DLM) based survival functions using parametric models from infusion of conventional life table and Kaplan Meier models. The DLMs were applied on universities academic retirees' data obtained from University of Ibadan (UI) and Obafemi Awolowo University (OAU) in Western Nigeria. The DLMs could be used in replacing the conventional parametric models. The DLMs can be used for computing both the expected years of life remaining at a given time, the proportion of a population which will survive beyond a certain time and the rate they will die or fail.

II. Survival Function Infused With Life Expectancy Models

The expected years of survival in life table model (1 and 8) is given as

$$e(x)=\frac{T(x)}{l(x)} \qquad (1)$$

Where $T(x)$ is the total life years lived by the cohort after age x

$$e(x)=\frac{\sum_{Lp}^{Fp} nL(x)}{l(x)} \qquad (2)$$

Where $l(x)$ is the number of survivors in the life table at exact age x out of an initial population call radix at age

0. $nL(x)$ is life years lived by the total population between ages x and x+n and where "n" is the age interval, Fp is the first point or first age and Lp is last point or last age in the observation.

Estimating variance of an estimated life expectancy given in equation 2 (3) under an assumption that grouped data in the age intervals are independent across intervals and total deaths (d_i) within interval i, can be modeled as having a binomial distribution with probability q_i but with unknown number of independent trials, Chiang developed Taylor-Linearization methods to estimate $Var\left[\hat{e}(x)\right]$, where a "hat" denotes an estimated quantity:

$$Var\left[\hat{e}(x)\right]=\sum_{i=1}^{n}\hat{p}_i^{\,2}\left[(1-a_i)n_i+\hat{e}_{i+i}\right]^2 Var(\hat{p}_i) \quad (3)$$

Where n_i is the length of interval i and a_i is the average fraction of n_i lived by those who die within the interval assumed to be 0.5 for every interval except the last, p_i is the probability of surviving to age x given survival to age $x+i$. $Var(\hat{p}_i)=\left[\frac{\hat{q}_i^2(1-\hat{q}_i)}{d_i}\right]$ and $q_i=1-p_i$

Thus, from Equation (2) we have:

$$nL(x)=L(x,n)=0.5n\left(l(x)+l(x+n)\right), \quad (5)$$

$$e(x)=\frac{\sum_{Lp}^{Fp}o.5n\left[l(x)+l(x+n)\right]}{l(x)} \quad (6)$$

$$=\frac{0.5n\left(\sum_{Lp}^{Fp}\left[l(x)+l(x+n)\right]\right)}{l(x)} \quad (7)$$

According to (2), we can express $l(x+n)$ as $l(x+n)=P(x)*l(x)$, where $p(x)$ is the probability of survival from age x to x+n.

If $p(x)=\frac{l(x+n)}{l(x)}$, then we have

$$\ell(x)=0.5n\sum_{Lp}^{Fp}(p(x)+1) \quad (8)$$

$$e_x=0.5n\sum_{Lp}^{Fp}p(x)+0.5n\sum_{Lp}^{Fp}1 \quad (9)$$

2.1 Kaplan Meier survival function model

The Kaplan Meier estimate of survival function is

$$\hat{s}(x)=\prod_{\le x}\hat{p}(x) \quad (10)$$

and its variance is

$$var(KM)=\hat{s}(t)^2\sum\frac{d_x}{l_x\left(l_x-d_x\right)} \quad (11)$$

The Kaplan Meier estimate of survival function could be written as:

$$\hat{s}(x)=\hat{p}(x)\times\hat{s}(x-1) \quad (12)$$

Where $p(0)=1$, and $\hat{p}(x)=\frac{\hat{s}(x)}{\hat{s}(x-1)}$, then we have

$$\sum_{Lp}^{Fp}\hat{p}(x)=\sum_{Lp}^{Fp}\frac{\hat{s}(x)}{\hat{s}(x-1)} \quad (13)$$

According to (2), the derived longevity model is

$$LG(x)=0.5n\sum_{Lp}^{Fp}\frac{\hat{s}(x)}{\hat{s}(x-1)}+0.5n\sum_{Lp}^{Fp}1 \quad (14)$$

$$L\hat{G}(x)=0.5n\left[\sum_{Lp}^{Fp}\left[\hat{s}(x)(\hat{s}(x-1)^{-1})\right]+I_{\ge x}\right] \quad (15)$$

Where $I_{\ge x}$ = summing 1 up to the age x from the highest age.

2.2 Longevity estimation (DLM) using Weibull model

Suppose that survival times are assumed to have a Weibull distribution with scale parameter λ shape parameter γ, so the survival and hazard function of a W (λ,γ) distribution are given respectively by $\hat{s}(x)=\exp(-\lambda x^{\gamma})$ and $h(x)=\lambda\gamma(x)^{\gamma-1}$ with λ,γ > 0. The hazard rate increases when γ > 1 and decreases when γ < 1 as time goes on. When γ = 1, the hazard rate remains constant, which is the special exponential case. Under the Weibull model, the hazard function of a particular respondent with covariates $(y_1,y_2,y_3,\ldots\ldots y_p)$ is given by:

$$h(x)/y=\lambda\gamma(x)^{\gamma-1}\exp(\beta_1y_1+\beta_2y_2+\ldots.+\beta_py_p)=\lambda\gamma(x)^{\gamma-1}\exp(\beta' y)$$

The effects of the explanatory variables in the model alter the scale parameter of the distribution, while the shape parameter remains constant. The corresponding survival function is given by:

$$\hat{s}(x)/y=\exp\left(-\lambda x^{\gamma}\right)\exp(\beta' y) \quad (16)$$

and

$$\hat{s}(x-1)/y=\exp\left(-\lambda(x-1)^{\gamma}\right)\exp(\beta' y) \quad (17)$$

Given equations (16) and (17), the DLM using Weibull distribution is

$$L\hat{G}(x)_{WB}=(0.5n)\sum_{Lp}^{Fp}\left[\left(\frac{\exp\left(-\lambda x^{\gamma}\right)\exp\left(\beta' y\right)}{\exp\left(-\lambda(x-1)^{\gamma}\exp\left(\beta' y\right)\right)}\right)+I_{\ge x}\right] \quad (18)$$

Therefore,

$$L\hat{G}(x)_{wB}=(0.5n)\sum_{Lp}^{Fp}\left[\exp\begin{pmatrix}-\lambda x^{\gamma}\exp\left(\beta' y\right)\\-\lambda(x-1)^{\gamma}\exp\beta' y\end{pmatrix}+I_{\ge x}\right] \quad (19)$$

$$L\hat{G}(x)_{wB} = (0.5n)\sum_{Lp}^{Fp}\left[\exp\left(\frac{-\lambda\exp\left(\beta' y\right)}{\left[(x)^{\gamma}-(x-1)^{\gamma}\right]}\right)+I_{\geq x}\right] \quad (20)$$

Therefore

$$L\hat{G}(x)_{wb} = 0.5n\left\{\sum_{Lp}^{Fp}\left(\frac{s_{(x)}}{s_{(x-1)}}\right)+I_{\geq x}\right\} \quad (21)$$

Where $s(x)=\exp\left(-\lambda x^{\gamma}\right)\exp\left(\beta' y\right)$ and

$s(x-1)=\exp\left(-\lambda(x-1)^{\gamma}\right)\exp\left(\beta' y\right)$

The variance of DLM from Weibull model can be estimated using ratio estimator as

$$\operatorname{var}\left(L\hat{G}(x)_{wb}\right)=(0.5n)^{2}\left\{\sum_{Lp}^{Fp}\left(\operatorname{var}\frac{\hat{s}(x)}{\hat{s}(x-1)}\right)\right\} \quad (22)$$

$$Var\left[L\hat{G}(x)_{wb}\right]=(0.5n)^{2}$$

$$\left[\sum_{Lp}^{Fp}\left(\begin{array}{l}\frac{\mu_{\hat{s}_{(x)}}^{2}}{\mu_{\hat{s}_{(x-1)}}^{4}}\sigma_{\hat{s}_{(x-1)}}^{2}+\frac{1}{\mu_{\hat{s}_{(x)}}^{2}}\sigma_{\hat{s}_{(x)}}^{2}\\ -2\rho\sigma_{\hat{s}_{(x)}}\sigma_{\hat{s}_{(x-1)}}\mu_{\hat{s}_{(x)}}\mu_{\hat{s}_{(x-1)}}^{-3}\end{array}\right)\right] \quad (23)$$

According to (2)

$\sigma_{\hat{s}(x)}=\sqrt{Var(\hat{s}(x))}$, $\sigma_{\hat{s}(x-1)}=\sqrt{Var(\hat{s}(x-1))}$,

$\rho=corr\left[\hat{s}(x),\hat{s}(x-1)\right]$, $\mu_{\hat{S}(x)}=\sum_{i}^{n}\hat{S}(x)n$ and

$\mu_{\hat{S}(x-1)}=\sum_{i}^{n}\hat{S}(x-1)n$.

2.3 Longevity estimation (DLM) using Exponential model

In literature, the Exponential model is a special case of the Weibull model when γ = 1. The hazard function under this model is assumed to be constant over time. The Survival function and Hazard function are written respectively as $\hat{s}(x)=\exp(-\lambda x)$ and $h(x)=\lambda$. Given the explanatory variables $y=y_1,y_2,\ldots\ldots\ldots,y_p$, the hazard and survival function becomes respectively as $h(x)/y=\lambda\exp(\beta' y)$ and

$$\hat{s}(x)/y=\exp(-\lambda x)\exp\left(\beta' y\right) \quad (24)$$

and

$$\hat{s}(x-1)/y=\exp\left(-\lambda(x-1)\exp\left(\beta' y\right)\right. \quad (25)$$

Therefore, DLM using Exponential model is

$$L\hat{G}(x)_{EXP}=0.5n\left\{\sum_{Lp}^{Fp}\exp\left(\frac{-\lambda\exp\left(\beta' y\right)}{\left[x-(x-1)\right]}\right)+I_{\geq x}\right\} \quad (26)$$

Hence,

$$L\hat{G}(x)_{EXP}=0.5n\left\{\sum_{Lp}^{Fp}\left(\frac{s(x)}{s(x-1)}\right)+I_{\geq x}\right\} \quad (27)$$

Where $s(x)=\exp(-\lambda x)\exp\left(\beta' y\right)$ and

$s(x-1)=\exp\left(-\lambda(x-1)\right)\exp\left(\beta' y\right)$.

The variance of DLM using Exponential model Can be estimated using ratio estimator as

$$\operatorname{var}\left(L\hat{G}(x)_{Exp}\right)=(0.5n)^{2}\left\{\sum_{Lp}^{Fp}\left(\operatorname{var}\frac{\hat{s}(x)}{\hat{s}(x-1)}\right)\right\} \quad (28)$$

$$Var\left[L\hat{G}(x)_{EXP}\right]=(0.5n)^{2}$$

$$\left[\sum_{Lp}^{Fp}\left(\begin{array}{l}\frac{\mu_{\hat{s}_{(x)}}^{2}}{\mu_{\hat{s}_{(x-1)}}^{4}}\sigma_{\hat{s}_{(x-1)}}^{2}+\frac{1}{\mu_{\hat{s}_{(x)}}^{2}}\sigma_{\hat{s}_{(x)}}^{2}\\ -2\rho\sigma_{\hat{s}_{(x)}}\sigma_{\hat{s}_{(x-1)}}\mu_{\hat{s}_{(x)}}\mu_{\hat{s}_{(x-1)}}^{-3}\end{array}\right)\right] \quad (29)$$

Where;

$\sigma_{\hat{s}(x)}=\sqrt{Var(\hat{s}(x))}$, $\sigma_{\hat{s}(x-1)}=\sqrt{Var(\hat{s}(x-1))}$,

$\rho=corr\left[\hat{s}(x),\hat{s}(x-1)\right]$, $\mu_{\hat{S}(x)}=\sum_{i}^{n}\hat{S}(x)n$

and $\mu_{\hat{S}(x-1)}=\sum_{i}^{n}\hat{S}(x-1)n$.

2.4 Longevity estimation (DLM) using Gompertz model

The survival and hazard function of the Gompertz distribution are given respectively by

$\hat{s}(x)=\exp\left(\frac{\lambda}{\theta}(1-\exp(\theta x))\right)$ and

$\hat{h}(x)=\lambda\exp(\theta x)$. For $0\leq x<\infty$ and $\lambda>0$. The parameter θ determines the shape of the hazard function and to be equals to zero. The survival time has an exponential distribution. The hazard function of a particular person is given by $h(x)/\underline{y}=\lambda\exp\left(\sum\hat{\beta}_i y_i\right)\exp(\theta x)$. Consequently, we may write the survival function as:

$$\hat{s}(x)/y=\exp\left(\frac{\lambda}{\varphi}\left[1-\exp(\varphi x)\right]\right)\left(\exp(\beta' y)\right) \quad (30)$$

$$\hat{s}(x-1)/y=\exp\left(\frac{\lambda}{\varphi}\left[1-\exp(\varphi(x-1)\right]\right)\left(\exp(\beta' y)\right) \quad (31)$$

By using equation (30) and (31), we have the longevity estimator for Gompertz model distribution as

$$L\hat{G}(x)_{GP}=0.5n\left\{\sum_{Lp}^{Fp}\left(\frac{\exp\left(\frac{\lambda}{\varphi}[1-\exp(\varphi x)]\right)\left(\exp(\beta' y)\right)}{\exp\left(\frac{\lambda}{\varphi}[1-\exp(\varphi x-1)]\right)\left(\exp(\beta' y)\right)}\right)+I_{\geq x}\right\} \quad (32)$$

$$L\hat{G}(x)_{GP}=0.5n\left\{\sum_{Lp}^{Fp}\exp\left(\begin{array}{l}\frac{\lambda}{\varphi}(1-\exp(\theta x))\\-\frac{\lambda}{\varphi}(1-\exp(\theta x-1))\end{array}\right)+I_{\geq x}\right\} \quad (33)$$

$$L\hat{G}(x)_{GP}=0.5n\left\{\sum_{Lp}^{Fp}\left(\exp\frac{\lambda}{\varphi}\left[\begin{array}{l}(1-\exp(\theta x))\\-(1-\exp(\theta x-1))\end{array}\right]\right)+I_{\geq x}\right\} \quad (34)$$

We derive the variance of $LG(x)_{GP}$ model by using equation (ratio estimator) as

$$\operatorname{var}\left(L\hat{G}(x)_{GP}\right)=(0.5n)^2\left\{\sum_{Lp}^{Fp}\left(\operatorname{var}\frac{\hat{s}(x)}{\hat{s}(x-1)}\right)\right\} \quad (35)$$

Where $s(x)=\exp\left(\frac{\lambda}{\varphi}[1-\exp(\varphi x)]\right)\left(\exp(\beta' y)\right)$

and

$$s(x-1)=\exp\left(\frac{\lambda}{\varphi}[1-\exp(\varphi(x-1))]\right)\left(\exp(\beta' y)\right).$$

We use ratio estimator technique since $\hat{s}(x)$ and $\hat{s}(x-1)$ are dependence, to give

$$Var\left[L\hat{G}(x)_{GP}\right]=(0.5n)^2\left[\sum_{Lp}^{Fp}\left(\begin{array}{l}\frac{\mu^2_{\hat{s}(x)}}{\mu^4_{\hat{s}(x-1)}}\sigma^2_{\hat{s}(x-1)}+\frac{1}{\mu^2_{\hat{s}(x)}}\sigma^2_{\hat{s}(x)}\\-2\rho\sigma_{\hat{s}(x)}\sigma_{\hat{s}(x-1)}\mu_{\hat{s}(x)}\mu^{-3}_{\hat{s}(x-1)}\end{array}\right)\right] \quad (36)$$

Where;

$\sigma_{\hat{s}(x)}=\sqrt{Var(\hat{s}(x))}$, $\sigma_{\hat{s}(x-1)}=\sqrt{Var(\hat{s}(x-1))}$, $\rho=corr\left[\hat{s}(x),\hat{s}(x-1)\right]$, $\mu_{\hat{S}(x)}=\sum_i^n\hat{S}(x)n$ and $\mu_{\hat{S}(x-1)}=\sum_i^n\hat{S}(x-1)n$.

III. Analysis, Results and Discussion

There are two stages in the sampling procedure adopted for the study. The first stage is purposive. This is the choice of conditions of retirement based on length of service (35 years) and compulsory retirement age (65). The second stage is convenient sampling. This is base on large population of retirees and long history of the establishment in Nigeria. The researcher and his trained research assistants administered copies of the questionnaire on the sampled retirees at the University of Ibadan. From 325 retirees sampled, records indicated that 107 academic retirees have died. Those who live (218) were given questionnaires in other to confirm information obtained from records and to provide supplementary information. A total of one hundred and ninety-seven (197) respondents were reachable, these represent 90.37% rate of return. Also, questionnaires were administered on the cluster sampled retirees from Obafemi Awolowo University (OAU), Ile-Ife. From total respondents of 239 retirees in the cluster sampled records show that 175 academic retirees were alive while 64 retirees have died. One hundred and sixty-one (161) respondents that were available represent 92% rate of return. Sample data size from both UI and OAU academic retirees is 529.

In Appendix I, we observed that population of male respondents is 80.3%. The outcomes of UI and OAU results show that universities academic staff members are predominantly male. Twenty one percent (21.0%) of the respondents have died according to the administrative records and 79.0% respondents were alive. Among the total respondents that were alive, 52.2% of them are with their spouse and 47.8% are widow. Those who retired as a result of length of service (35 years) are 49.5% and age limit (65 years) is 50.5%. The highest proportion of the respondents 68.2% retired on Cadre of Professor and the lowest proportion of the respondents 7.4% retired on Lecturer I Cadre. It is observed that the respondents that retired at age 60 years are 20.6%. In this case, the respondents might have served 35 years in the university system, which is one of the conditions for mandatory retirement.

IV. Application of Conventional Life Table Model and Derived Longevity Models to UI and OAU Academic Retirees

The factors affecting survival experience of UI and OAU retirees are presented in Table 8.3 below. We observed that the significant effect is found for; Age at appointment (AAA) (β = 1.923; p = 0.000), Age on retirement (AOR) (β = -1.774; p = 0.000), Reasons for retirement (RFR) (β = -0.526; p = 0.022) and Length of service (LOS) (β = 1.869; p = 0.000). We utilized explanatory variables (AAA, AOR, RFR and LOS) as estimate of DLM using R program.

International Journal of Applied Science and Mathematics
Volume 2, Issue 3, ISSN (Online): 2394-2894

Table 1: Academic Factors affecting survival of OAU and UI academic retirees (n=529)

Var	B	SE	Wald	d.f.	Sig. (p)	Exp(B)	95.0% CI for Exp(B) Lower	Upper
SEX	0.106	0.175	0.366	1	0.545	1.112	0.79	1.57
AAA	1.923	0.467	16.931	1	0.000	6.840	2.74	17.10
AOR	-1.774	0.454	15.254	1	0.000	0.170	0.07	0.41
RFR	-0.526	0.230	5.241	1	0.022	0.591	0.38	0.93
LOS	1.869	0.472	15.699	1	0.000	6.484	2.57	16.35
LAR	-0.090	0.081	1.246	1	0.264	0.914	0.78	1.07

Application of DLMs using Exponential, Weibull and Gompertz models to OAU and UI academic retirees' data and results were displayed in Appendix II and Appendix III using R package. In the analysis, Age at appointment (AAA), Age on retirement (AOR), Reasons for retirement (RFR) and Length of service (LOS) are utilized as an explanatory variables (Ajayi et al., 2015).

Appendixes II and III show the estimates of DLMs using Exponential, Weibull and Gompertz models for OAU and UI retirees' data with and without the effect of explanatory variables. At first year after retirement of the retirees is expected to live up to a total of 22.66 years on average from DLM using Exponential model (Appendix II). But with the effect of explanatory variables, an individual could survive up to a total of about 22.78 years (Appendix III). Furthermore, if we take the example of cohort at age 60 years he/she would expect to live up to about 82.66 years on average without effect of explanatory variables and 82.78 with the effect of explanatory variables (AAA, AOR, RFR and LOS). At the second year of retirement, we observed that the retirees will live up to 21.68 years with standard variance of 0.063518452 (DLM using Exponential model) and with explanatory variables, he/she will live up to 21.80 after second year of retirement with standard variance of 0.072224883 (Appendixes II and III).

On the report of DLM using Weibull based survival function model, the retirees will live up to 22.35 years without explanatory variables and about 22.82 years with explanatory variables at the first year after retirement (see Appendix II and III). At the second year after retirement, we can conclude that the cohort will live up to 21.35 years without explanatory variables and with explanatory variables he/she will live up to about 21.82 year (DLM using Weibull model) with variances of 0.0003342165 and 0.060836218 respectively.

After the first year of retirement, retirees will live up to 22.08 years and 22.48 years, with and without explanatory variables respectively on the report of DLM using Gompertz model (Appendixes II and III). At their second year, the retirees will survive up to about 21.08 years with standard variance of 0.060628986. Mores o, the retirees will live up to 21.48 with standard variance of 0.068542249 with effect of explanatory variable.

We observed from Appendixes II and III that post retirement years of both institutions academic retirees and standard variances are increasing and decreasing linearly simultaneously. The institutions academic retirees will survive up to 21.35 years with standard variance of 0.0003342165 without explanatory variable and 21.82 years when explanatory variables are considered with standard variance of 0.060836218 on the report of estimated DLM using Weibull model. The precision of DLM using Weibull based survival function model will be better for this cohort group retiring at age 60 years when explanatory variables are considered or not considered.

Figure 1 and 2 below display the estimate of DLMs without and with explanatory variables. The gap between lines of estimate of DLMs parametric models without (Figure 1) and with (Figure 2) explanatory variables are very close to each others. This shows that at each year after retirement the models give nearly the same estimate of longevity. The estimates of DLMs parametric both without and with explanatory variables come together on the same year. We observe that DLM with explanatory variables predicted more years after retirement than DLM without explanatory variables.

More so, looking at Figure 3 and 4, the bars of estimate of DLM using parametric models are of equal lengths and decreases exponentially for both with and without explanatory variables. We observe that DLM with explanatory variables proposed more post retirement years than without explanatory variables. Above all, DLMs using Weibull model is more suitable for estimate of longevity with and without explanatory variables.

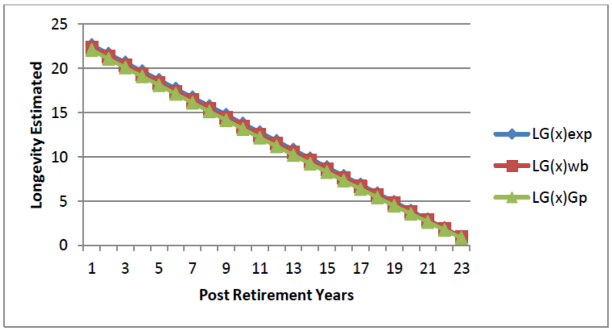

Fig.1. Line graph of estimate of DLMs and post retirement years using derived Longevity Exponential, Wiebull and Gompertz Models for OAU and UI academic retirees' data without explanatory variables.

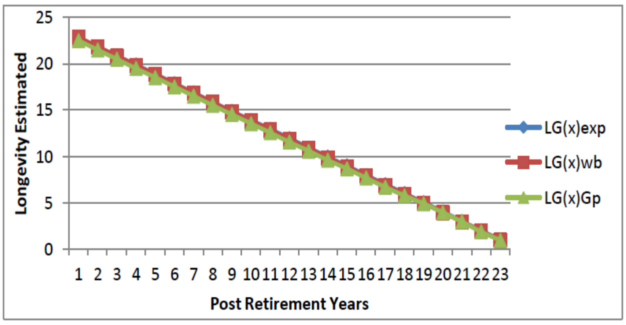

Fig.2. Line graph of estimate of DLMs post retirement years using Exponential, Wiebull and Gompertz Models for OAU and UI academic retirees' data without explanatory variables.

International Journal of Applied Science and Mathematics
Volume 2, Issue 3, ISSN (Online): 2394-2894

Fig.3. Bar chart of estimate of DLMs post retirement years using Exponential, Wiebull and Gompertz Models for OAU and UI academic retirees' data without explanatory variables.

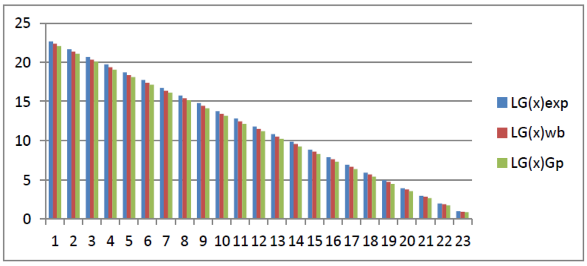

Fig.4. Bar chart of estimate of DLMs post retirement years using Exponential, Wiebull and Gompertz Models for OAU and UI academic retirees' data without explanatory variables.

V. Conclusion

The estimated mean of post retirement years for universities academic retirees from DLMs using Exponential Model without explanatory variables is about 22.66 years and with explanatory variables is about 22.78 years. Using Weibull model without covariates is about 22.35 years and with covariates is around 22.82 years. However, using Gompertz models, the estimate mean of post retirement years is nearly 22.08 years and with covariates is about 22.48 years.

The outcome of analyses from derived longevity using Weibull model is closely approximated to the outcome of derived longevity using Cox proportional model (2). We can recommend derived longevity using Weibull models for reliability, maintainability and safety (RMS) work. According to Reliwiki (7), Weibull distribution is a continuous probability distribution that can take on the characteristics of other types of distributions based on the value of the shape parameter.

Appendix I

Demographic Characteristics of cohort of both UI and OAU academic retirees

VARIABLES	FREQUENCY	PERCENTAGES
Gender		
Males	425	80.3
Females	104	19.7
Total	529	100.0
Current status		
Censor	418	79.0
Fail	111	21.0
Total	529	100.0
Marital Status		
With spouse	218	52.2
Widowed	200	47.8
Total	418	100
Reasons for ret.		
L. of service	262	49.5
Age Limit	267	50.5
Total	225	100
Level. at Ret.		
Prof	361	68.2
Ass. Prof	42	7.9
SL	87	16.4
L1	39	7.4
Total	529	100.00
Age at Retirement		
60	109	20.6
61	37	7.0
62	28	5.3
63	35	6.6
64	14	2.6
65	306	57.8
Total	529	100.0

APPENDIX II

Estimated Longevity using proposed Exponential, Weibull and Gompertz models for OAU and UI academic retirees

x	$LG_{(x)_{exp}}$	$std.var\ LG(x)_{exp}$	$LG_{(x)_{wb}}$	$std.var\ LG(x)_{wb}$	$LG_{(x)_{Gp}}$	$std.var\ LG(x)_{Gp}$
1	22.66	N/A	22.35	N/A	22.08	N/A
2	21.68	0.063518452	21.35	0.0003342165	21.08	0.060628986
3	20.69	0.063338791	20.36	0.0003332436	20.09	0.060447182
4	19.71	0.063161458	19.36	0.0003323145	19.09	0.060272599
5	18.72	0.062914498	18.37	0.0003310461	18.10	0.060033645
6	17.73	0.064301017	17.37	0.0003297965	17.11	0.059797917
7	16.75	0.062373999	16.39	0.0003283571	16.12	0.059526476
8	15.76	0.062043796	15.40	0.0003267513	15.13	0.059224235
9	14.78	0.062043796	14.42	0.0003249535	14.14	0.058886959
10	13.79	0.061669874	13.43	0.0003229342	13.15	0.058509867
11	12.81	0.061246424	12.46	0.0003206274	12.17	0.058081469
12	11.82	0.060760198	11.48	0.0003181999	11.19	0.057633301
13	10.84	0.060247477	10.51	0.0003156674	10.21	0.057168954
14	9.85	0.059713282	9.54	0.0003126418	9.24	0.056617883
15	8.87	0.059077887	8.57	0.0003087559	8.27	0.055914527
16	7.88	0.058267707	7.61	0.0003038072	7.31	0.055023413
17	6.90	0.057245673	6.65	0.0002966647	6.36	0.053742564
18	5.91	0.055790185	5.69	0.0002842999	5.41	0.051530253
19	4.93	0.053306782	4.73	0.0002647928	4.47	0.048041967
20	3.94	0.049443914	3.78	0.0002378064	3.56	0.043209195
21	2.96	0.044185842	2.83	0.0002006413	2.64	0.036528342
22	1.97	0.037072615	1.88	0.0001452487	1.74	0.026508976
23	0.99	0.0326672823	0.94	0.0000681883	0.86	0.045904285

APPENDIX III

Estimated Longevity using proposed Exponential, Weibull and Gompertz models for OAU and UI academic retirees with the effect of explanatory variable

x	$LG_{(x)_{exp}}$	$std.var\ LG(x)_{exp}$	$LG_{(x)_{wb}}$	$std.var\ LG(x)_{wb}$	$LG_{(x)_{Gp}}$	$std.var\ LG(x)_{Gp}$
1	22.78	N/A	22.82	N/A	22.48	N/A
2	21.80	0.07222488	21.82	0.060836218	21.48	0.068542249
3	20.81	0.072172536	20.82	0.060775557	20.49	0.06848401
4	19.88	0.071535252	19.82	0.060542917	19.50	0.062176511
5	18.86	0.070266578	18.82	0.060462406	18.50	0.062004534
6	17.86	0.069080762	17.82	0.060311861	17.50	0.061829066
7	16.88	0.068152801	16.82	0.060241168	16.51	0.061618239
8	15.88	0.067583671	15.83	0.06014406	15.52	0.061373733
9	14.88	0.067465415	14.83	0.05939217	14.53	0.061089006
10	13.90	0.063890173	13.83	0.05837828	13.53	0.060757505
11	12.94	0.063444461	12.85	0.057636027	12.57	0.060367531
12	11.93	0.062963018	11.85	0.056633901	11.57	0.059949633
13	10.92	0.062452221	10.86	0.057631397	10.57	0.059508391
14	9.95	0.061834013	9.87	0.056628034	9.61	0.058975277
15	8.99	0.06103862	8.89	0.055623237	8.67	0.058288577
16	7.96	0.06001611	7.90	0.054616457	7.69	0.057400813
17	6.97	0.058525899	6.90	0.053605487	6.69	0.055229165
18	6.01	0.055953903	5.93	0.052584749	5.77	0.053815362
19	5.04	0.051849352	4.95	0.051549077	4.90	0.050121892
20	4.05	0.046010378	3.96	0.049341453	3.95	0.044769693
21	3.01	0.037593966	2.97	0.048410875	2.98	0.036855431
22	2.00	0.024634802	1.97	0.033274081	1.88	0.024346963
23	1.01	0.032996953	0.98	0.009101560	0.96	0.011624695

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