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Blow-up of Solutions for Some Nonlinear Schrodingher Systems

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***Abstract* - This paper concerns the theoretical study for the following initial-boundary value problem:**

$$\begin{cases} u_t = iaLu - \gamma f(|v|) \ \ in \ \ \Omega\times(0,T), \\ v_t = iaLv - \gamma g(|u|) \ \ in \ \ \Omega\times(0,T), \\ u(x,t) = 0 \quad on \quad \partial\Omega\times(0,T), \\ v(x,t) = 0 \quad on \quad \partial\Omega\times(0,T), \\ u(x,0) = u_0(x) \quad in \ \ \Omega, \\ v(x,0) = v_0(x) \quad in \ \ \Omega, \end{cases}$$

where Ω is a bounded domain in R^N with smooth boundary ∂Ω, a ∈ R, a ≠ 0, γ is a complex number, *f*(s), *g*(s) are positive, increasing and convex functions for positive values of s, L is an elliptic operator.
We find some conditions under which any solution (*u*, *v*) of the above problem blows up in a finite time and estimate its blow-up time.

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I. Introduction

Let Ω be a bounded domain in R^N with smooth boundary ∂Ω. Consider the following initial-boundary value problem

$$u_t = iaLu - \gamma f(|v|) \ \ in \ \ \Omega\times(0,T), \tag{1}$$

$$v_t = iaLv - \gamma g(|u|) \ \ in \ \ \Omega\times(0,T), \tag{2}$$

$$u(x,t) = 0 \quad on \quad \partial\Omega\times(0,T), \tag{3}$$

$$v(x,t) = 0 \quad on \quad \partial\Omega\times(0,T), \tag{4}$$

$$u(x,0) = u_0(x) \quad in \ \ \Omega, \tag{5}$$

$$v(x,0) = v_0(x) \quad in \ \ \Omega, \tag{6}$$

where a ∈ R, a ≠ 0, γ is a complex number, f(s), g(s) are positive, increasing and convex functions for positive values of s,

$$Lu = \sum_{i,j=1}^{N} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_i}\right) + \sum_{i=1}^{N}\left(b_i(x)\frac{\partial u}{\partial x_i}\right).$$

The coefficients $a_{ij}(x) \in C^1(\Omega)$ satisfy the following inequality

$$\sum_{i,j=1}^{N} a_{ij}(x)\xi_i\xi_j \geq C|\xi|^2 \ \ for \ \ \xi \in R^N, \ \ C > 0,$$

$$a_{ij}(x) = a_{ji}(x) \in R, \ \ b_i(x) \in C^1(\Omega), b_i(x) \in R.$$

The initial data (u_0,v_0) is such that u_0 and v_0 are continuous functions in Ω and $u_0 = 0$ on ∂Ω. Here $(0,T)$ is the maximal time interval of existence for the solution (*u*, *v*). The time *T* may be finite or infinity. When *T* is infinity, we say that the solution (*u*,*v*) is global. When *T* is finite, the solution (*u; v*) develops a singularity in a finite time, namely

$$\lim_{t\to T}\left(\|u(x,t)\|_\infty + \|v(x,t)\|_\infty\right) = +\infty,$$

where $\|u(x,t)\|_\infty = \sup_{x\in\Omega}|u(x,t)|$. In this case, we say that the solution (*u*, *v*) of (1)-(3) blows up in a finite time and the time *T* is called the blow-up time of the solution (*u*,*v*). Nonlinear Schrödinger equations and systems appear in a lot of models of nonlinear optics, energy transfer in molecular systems, quantum mechanics, seismology, plasma physics, see [3], [5], [24], [27] to cite only a few cases. Solutions of nonlinear Schrödinger systems which blow up in a finite time have been the subject of investigation of many authors (see [1], [4], [13], [21], [22] and the reference cited therein). Generally the authors have considered the problem (1)-(6) in the case where the operator *L* is replaced by the Laplacian and $f(|v|)$, $g(|u|)$ are replaced by odd functions. In this case, one shows that the energy of the system is conserved and the method used to show blow-up solutions is the method of energy. In this paper, we use a modification of the method of Kaplan (see [16]) to prove the blow-up solutions. In [8], [12], [14], [15], [17]-[19], [23], [25], [26], one may find some results about blow-up solutions for nonlinear Schrödinger equations.
Our paper is written in the following manner. In the next section, under some assumptions, we show that any solution (*u*,*v*) of (1)-(6) blows up in a finite time and estimate its blow-up time.

II. Blow-up Solutions

In this section under some assumptions, we show that any solution (*u*, *v*) of (1)-(6) blows up in a finite time and estimate its blow-up time.

Before starting, let us recall a well known result proved by Kondratiev in [20].

Consider the following eigen value problem

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$$-L^*\varphi = \lambda\varphi \quad in \ \Omega, \tag{7}$$
$$\varphi = 0 \quad on \ \partial\Omega, \tag{8}$$
$$\varphi > 0 \quad in \ \Omega, \tag{9}$$
where
$$L^*\varphi = \sum_{i,j=1}^{N} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial \varphi}{\partial x_i}\right) - \sum_{i=1}^{N}\frac{\partial}{\partial x_i}\left(b_i(x)\varphi\right).$$

The above problem has a solution (φ,λ) such that $\lambda > 0$. We can normalize φ so that $\int_\Omega \varphi(x)dx = 1$.

Another result which will be useful later is the following

Lemma 2.1: If $u \in H_0^1(\Omega)$, then the following estimate holds
$$\int_\Omega |\nabla u| dx \geq \left(\int_\Omega |u|^{\frac{n}{n-1}} dx\right)^{\frac{n}{n-1}},$$
where $n \geq 2$.

The above result is well known. It may be found in [9]. Now, let us state our first result on the blow-up.

Theorem 2.1 Let
$$A = \int_\Omega \phi(x)\operatorname{Re}(u_0(x))dx > 0,\ B = \int_\Omega \phi(x)\operatorname{Re}(v_0(x))dx > 0,$$
$$G(s) = \int_0^s g(\sigma)d\sigma, \quad F(s) = \int_0^s f(\sigma)d\sigma.$$

i) If $\gamma = -c$ and
$$\frac{a\lambda}{c}\int_B^{+\infty}\frac{ds}{g(G^{-1}(F(s)) + G(A) - F(B))} < \frac{1}{2},$$ then any solution (u,v) of (1)-(6) blows up in a finite time and its blow-up time T is estimated as follows
$$T \leq \frac{1}{a\lambda}\arcsin\left(\frac{a\lambda}{c}\int_B^{+\infty}\frac{ds}{g(G^{-1}(F(s)) + G(A) - F(B))}\right), \tag{10}$$
where G^{-1} is the inverse of the function G

ii) If $\gamma = ic$ and
$$\frac{a\lambda}{c}\int_B^{+\infty}\frac{ds}{g(G^{-1}(F(s)) + G(A) - F(B))} < 1,$$ then any solution (u,v) of (1)-(6) blows up in a finite time and its blow-up time T satisfies the following inequality
$$T \leq \frac{1}{a\lambda}\arccos\left(1 - \frac{a\lambda}{c}\int_B^{+\infty}\frac{ds}{g(G^{-1}(F(s)) + G(A) - F(B))}\right). \tag{11}$$

Proof. Since $(0, T)$ is the maximal time interval on which the solution (u, v) exists, our aim is to show that T is finite and satisfies the above inequality.

Introduce the functions
$$w_1(t) = \int_\Omega u\phi dx \ and \ z_1(t) = \int_\Omega \bar{u}\phi dx \ for \ t \in (0,T).$$
Take the derivative of w_1 in t and use (1) to obtain
$$w'_1(t) = ia\int_\Omega \varphi L u dx - \gamma\int_\Omega f(|v|)\varphi dx.$$
Applying Green's formula, we arrive at
$$w'_1(t) = ia\int_\Omega u L^*\varphi dx - \gamma\int_\Omega f(|v|)\varphi dx.$$
It follows from (7) that
$$w'_1(t) = -ia\lambda w_1(t) - \gamma\int_\Omega f(|v|)\varphi dx,$$
which implies that
$$\left(e^{ia\lambda t}w_1\right)' = -\gamma^{ia\lambda t}\int_\Omega f(|v|)\varphi dx.$$
Take again the derivative of z_1 in t and use (1) to obtain
$$z'_1(t) = -ia\int_\Omega \bar{u} L\varphi dx - \bar{\gamma}\int_\Omega f(|v|)\varphi dx.$$
It follows from Green's formula and (7) that
$$\left(e^{-ia\lambda t}z_1\right)' = -\bar{\gamma}e^{-ia\lambda t}\int_\Omega f(|v|)\varphi dx.$$
Setting
$$J_1(t) = \frac{e^{ia\lambda t}w_1(t) + e^{-ia\lambda t}z_1(t)}{2}, \tag{12}$$
we get
$$J_1'(t) = \left(\frac{-\gamma e^{ia\lambda t} - \bar{\gamma}e^{-ia\lambda t}}{2}\right)\int_\Omega f(|v|)\varphi dx,$$
which implies that
$$J_1'(t) = -\operatorname{Re}(\gamma e^{ia\lambda t})\int_\Omega f(|v|)\varphi dx.$$
Hence, we have
$$J_1'(t) = c\cos(a\lambda t)\int_\Omega f(|v|)\varphi dx \quad if \ \gamma = -c, \tag{13}$$
And
$$J_1'(t) = c\sin(a\lambda t)\int_\Omega f(|v|)\varphi dx \quad if \ \gamma = ic. \tag{14}$$
Now, introduce the functions
$$w_2(t) = \int_\Omega v\varphi dx \quad and \quad z_2(t) = \int_\Omega \bar{v}\varphi dx.$$
By the same way, setting
$$J_2(t) = \frac{e^{ia\lambda t}w_2(t) + e^{-ia\lambda t}z_2(t)}{2}, \tag{15}$$
we find that
$$J_2'(t) = c\cos(a\lambda t)\int_\Omega g(|u|)\varphi dx \quad if \ \gamma = -c,$$
and
$$J_2'(t) = c\sin(a\lambda t)\int_\Omega g(|u|)\varphi dx \quad if \ \gamma = ic.$$
Applying Jensen's inequality, we get
$$\int_\Omega f(|v|)\varphi dx \geq f\left(\int_\Omega |v|\varphi dx\right),$$
$$\int_\Omega g(|u|)\varphi dx \geq g\left(\int_\Omega |u|\varphi dx\right).$$
Obviously, we have
$$|J_1(t)| \leq \int_\Omega |u|\varphi dx \quad and \quad |J_2(t)| \leq \int_\Omega |v|\varphi dx.$$
We deduce that
$$\int_\Omega f(|v|)\phi dx \geq f(|J_2(t)|) \ and \ \int_\Omega g(|u|)\phi dx \geq g(|J_1(t)|).$$

On the other hand, since

$$\cos(a\lambda t)\geq 0 \ \ for\ t\in\left(0,\frac{\pi}{2a\lambda}\right) \ and$$

$$\sin(a\lambda t)\geq 0 \ \ for\ t\in\left(0,\frac{\pi}{a\lambda}\right),$$

we discover that

$$J_1'(t)\geq c\cos(a\lambda t)f(|J_2(t)|)\geq 0 \quad for\ t\in(0,T_1)\ if\ \ \gamma=-c, \quad (16)$$

$$J_2'(t)\geq c\cos(a\lambda t)g(|J_1(t)|)\geq 0 \quad for\ t\in(0,T_1)\ if\ \ \gamma=-c, \quad (17)$$

$$J_2'(t)\geq c\sin(a\lambda t)f(|J_2(t)|)\geq 0 \quad for\ t\in(0,T_2)\ if\ \ \gamma=ic, \quad (18)$$

$$J_2'(t)\geq c\sin(a\lambda t)g(|J_1(t)|)\geq 0 \quad for\ t\in(0,T_2)\ if\ \ \gamma=ic, \quad (19)$$

where $T_1=\min\left\{T,\frac{\pi}{2a\lambda}\right\}$, $T_2=\min\left\{T,\frac{\pi}{a\lambda}\right\}$.

Suppose that $\gamma=-c$. It is not hard to see that $J_1(t)$ and $J_2(t)$ are nondecreasing functions for $t\in\left(0,\frac{\pi}{2a\lambda}\right)$, which implies that

$$J_1'(t)\geq c\cos(a\lambda t)f(J_2(t))\geq 0 \ \ for\ t\in(0,T_1), \quad (20)$$

$$J_2'(t)\geq c\cos(a\lambda t)g(J_1(t))\geq 0 \ \ for\ t\in(0,T_1), \quad (21)$$

because $f(s)$ and $g(s)$ are positive for the positive values of s, $J_1(0)=A>0$, $J_2(0)=B>0$.

Let $(\alpha(t),\beta(t))$ be the solution of the following differential system

$$\alpha'(t)=c\cos(a\lambda t)f(\beta(t)) \ \ for\ t\in(0,T_0), \quad (22)$$

$$\beta'(t)=c\cos(a\lambda t)g(\alpha(t)) \ \ for\ t\in(0,T_0), \quad (23)$$

$$\alpha(0)=J_1(0), \quad (24)$$

$$\beta(0)=J_2(0), \quad (25)$$

where $(0,T_0)$ is the maximal time interval of existence of the solution $(\alpha(t),\beta(t))$.

We observe that

$$\frac{d\alpha}{d\beta}=\frac{f(\beta)}{g(\alpha)},$$

which implies that $(G(\alpha))_t=(F(\beta))_t$. Integrate the above equality over (0, t) to obtain

$$G(\alpha)-G(\alpha(0))=F(\beta)-F(\beta(0)),$$

which implies that

$$\alpha(t)=G^{-1}(F(\beta)+G(J_1(0))-F((J_2(0))).$$

Hence, we get

$$\beta'(t)=c\cos(a\lambda t)g(G^{-1}(F(\beta)+G(J_1(0))-F((J_2(0)))).$$

Therefore, we have

$$\frac{d\beta}{g(G^{-1}(F(\beta)+G(J_1(0))-F((J_2(0))))}=c\cos(a\lambda t)dt.$$

Integrate this equality over $(0,T_0)$ to obtain

$$\frac{c\sin(a\lambda T_0)}{a\lambda}=\int_{J_2(0)}^{+\infty}\frac{ds}{g(G^{-1}(F(s)+G(J_1(0))-F((J_2(0))))},$$

which implies that

$$T_0=\frac{1}{a\lambda}\arcsin\left(\frac{a\lambda}{c}\int_{J_2(0)}^{+\infty}\frac{ds}{g(G^{-1}(F(s)+G(J_1(0))-F((J_2(0))))}\right).$$

We easily see that $(\alpha(t),\beta(t))$ blows up at the time T_0. The maximum principle implies that

$$J_1(t)\geq\alpha(t) \ \ for\ t\in(0,T_*), \quad (26)$$

$$J_2(t)\geq\beta(t) \ \ for\ t\in(0,T_*), \quad (27)$$

where $T_*=\min\{T_0,T_1\}$.

We observe that $T_1\leq T_0$. Indeed suppose that $T_1>T_0$. From (26), we get $J_1(T_0)\geq\alpha(T_0)=+\infty$ which is a contradiction because $(0,T)$ is the maximal time existence of the solution (u,v). Hence

$$T_1\leq T_0=\frac{1}{a\lambda}\arcsin\left(\frac{a\lambda}{c}\int_{J_2(0)}^{+\infty}\frac{ds}{g(G^{-1}(F(s)+G(J_1(0))-F((J_2(0))))}\right).$$

Since $\frac{a\lambda}{c}\int_{J_2(0)}^{+\infty}\frac{ds}{g(G^{-1}(F(s)+G(J_1(0))-F((J_2(0))))}<\frac{1}{2}$

then $T_1\leq T_0\leq\frac{1}{a\lambda}\frac{\pi}{3}<\frac{\pi}{2a\lambda}$.

We deduce that $T_1=T$. Consequently, (u,v) blows up at the time T and

$$T\leq T_0<\frac{\pi}{2a\lambda}.$$

Now, suppose that $\gamma=-ic$. Then we have

$$J_1'(t)\geq c\sin(a\lambda t)f(J_2(t))\geq 0 \ \ for\ t\in(0,T_2),$$

$$J_2'(t)\geq c\sin(a\lambda t)g(J_1(t))\geq 0 \ \ for\ t\in(0,T_2),$$

Let $(\alpha_1(t),\beta_1(t))$ be the solution of the following differential system

$$\begin{cases}\alpha_1'(t)=c\sin(a\lambda t)f(\beta_1(t)) \ for\ t\in(0,T_1^0),\\ \beta_1'(t)=c\sin(a\lambda t)g(\alpha_1(t)) \ for\ t\in(0,T_1^0),\\ \alpha_1(0)=J_1(0),\\ \beta_1(0)=J_2(0),\end{cases}$$

where $(0,T_1^0)$ is the maximal time interval of existence of the solution $(\alpha_1(t),\beta_1(t))$.

Reasoning as above, we see that

$$\beta_1'(t)=c\sin(a\lambda t)g(G^{-1}(F(\beta_1)+G(J_1(0))-F(J_2(0)))),$$

and

$$\frac{c(1-\cos(a\lambda T_1^0))}{a\lambda}=\int_{J_2(0)}^{+\infty}\frac{ds}{g(G^{-1}(F(s)+G(J_1(0))-F((J_2(0))))},$$

which implies that

$$T_1^0=ar\cos(1-\frac{a\lambda}{c}\int_{J_2(0)}^{+\infty}\frac{ds}{g(G^{-1}(F(s)+G(J_1(0))-F((J_2(0))))}).$$

The maximum principle implies that $J_1(t)\geq\alpha_1(t)$ *for* $t\in(0,T_2^*)$ *and* $J_2(t)\geq\beta_1(t)$ *for* $t\in(0,T_2^*)$ *where* $T_2^*=\min\{T_2,T_1^0\}$. Reasoning as above, we discover that $T_2^*\leq T_1^0$. Since

$1-\frac{a\lambda}{c}\int_{J_2(0)}^{+\infty}\frac{ds}{g(G^{-1}(F(s)+G(J_1(0))-F((J_2(0))))}>0,$ when $T_1^0\leq\frac{1}{a\lambda}\frac{\pi}{2}<\frac{\pi}{a\lambda}$. It follows that $T_2^*=T$. Hence (u,v) blows up at time T. Use the fact that $J_1(0)=A$ and $J_2(0)=B$ to complete the rest of the proof.

Remark 2.1 Let

$f(s)=s^p$ *and* $g(s)=s^q$ *with* $p\geq1, q\geq1$. We have

$$F(s)=\frac{s^{p+1}}{p+1}, G(s)=\frac{s^{q+1}}{q+1}, G^{-1}(s)=((q+1)s)^{\frac{1}{q+1}},$$

$$G^{-1}(F(s))=(\frac{q+1}{p+1})^{\frac{1}{q+1}}s^{\frac{p+1}{q+1}},$$

$$g(G^{-1}(F(s)+G(A)-F(B)))=(q+1)^{\frac{q}{q+1}}(\frac{s^{p+1}}{p+1}+\frac{A^{q+1}}{q+1}-\frac{B^{p+1}}{p+1})^{\frac{q}{q+1}}$$

It is not hard to see that if $A>0, B>0$ and $pq>1$ then

$$\int_B^{+\infty}\frac{ds}{g(G^{-1}(F(s)+G(A)-F(B)))}<+\infty.$$

In addition if a is small enough then any solution (u, v) of (1)-(6) blows up in a finite time. We also have the blow-up of any solution of (1)-(6) if $\gamma=-c$ or $\gamma=ic$ and c is large enough.

Remark 2.2 Consider the following initial-boundary value problem

$$u_t=i\Delta u-\gamma|v|^p \quad in \ \Omega\times(0,T), \qquad (28)$$

$$v_t=i\Delta v-\gamma|u|^q \quad in \ \Omega\times(0,T), \qquad (29)$$

$$u(x,t)=0 \quad on \ \partial\Omega\times(0,T), \qquad (30)$$

$$v(x,t)=0 \quad on \ \partial\Omega\times(0,T), \qquad (31)$$

$$u(x,0)=u_0(x) \quad in \ \Omega, \qquad (32)$$

$$v(x,0)=v_0(x) \quad in \ \Omega, \qquad (33)$$

where $p\geq1$, $q\geq1$, $\gamma=+i$ or $\gamma=-1$.

Let

$$A=\int_\Omega\varphi(x)\mathrm{Re}(u_0(x))dx>0,$$

$$B=\int_\Omega\varphi(x)\mathrm{Re}(v_0(x))dx>0, \ and \ \frac{A^{q+1}}{q+1}\geq\frac{B^{p+1}}{p+1}.$$

Then any solution (u,v) of (28)-(33) blows up in a finite time if B is large enough and $pq>1$.

It is not hard to see that if $|v|^p$ is replaced by $|\nabla v|^p$ with $p>1$ and $n\geq2$, then (u,v) of (28)-(33) blows up in a finite time if B is large enough. In fact, we have

$$\int_\Omega|v|\varphi dx\leq(\int_\Omega|v|^{\frac{n}{n-1}}dx)^{\frac{n}{n-1}}(\int_\Omega\varphi^n dx)^{\frac{1}{n}},$$

$$\leq(\int_\Omega|\nabla v|dx)(\int_\Omega\varphi^n dx)^{\frac{1}{n}}.$$

On the other hand

$$\int_\Omega|\nabla v|dx\leq\int_\Omega|\nabla v|\varphi^{\frac{1}{p}}\varphi^{-\frac{1}{p}}dx,$$

$$\leq(\int_\Omega|\nabla v|^p\varphi dx)^{\frac{1}{p}}(\int_\Omega\varphi^{\frac{1}{1-p}}dx)^{\frac{p-1}{p}},$$

which implies that

$$\int_\Omega|v|\varphi dx\leq(\int_\Omega|\nabla v|^p\varphi dx)^{\frac{1}{p}}(\int_\Omega\varphi^n dx)^{\frac{1}{n}}(\int_\Omega\varphi^{\frac{1}{1-p}}dx)^{\frac{p-1}{p}}.$$

Therefore, there exists a constant $C>0$ such that

$$\int_\Omega|\nabla v|\varphi dx\geq C(\int_\Omega|v|\varphi dx)^p.$$

Now, proceeding as in the proof of the above theorem, we obtain the desired result.

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