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Measurement of Relative Efficiency of Senior High Schools Using Data Envolpement Analysis

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***Abstract*—The purpose of this research is to utilize data envelopment analysis (DEA) to measure academic efficiency of four Senior High Schools in Ghana. The proposed approach is deployed based on empirical data collected from the four schools on an efficiency scale of 0-1.0. DEA analysis assesses the relative efficiency of each of these selected schools relative to the rest of the schools in terms of academic performance. The 2012/13 academic year data from the selected schools were used for the analysis. Four input variables and five output variables were identified. The input variables were teacher to student ratio, trained to non trained teachers ratio, library facilities and contact hours per day. Output variables were estimated as grades obtained, that is 8 passes, 7 passes, 6 passes, 5 passes and below. Three schools formed the efficiency frontier and the fourth school was found inefficient for the academic year. There was an indication that reduction in contact hours per day has a larger effect on efficiency of fourth campus than does library facilities and trained to non trained teachers ratio.** **The purpose of this research is to utilize data envelopment analysis (DEA) to measure academic efficiency of four Senior High Schools in Ghana. The proposed approach is deployed based on empirical data collected from the four schools on an efficiency scale of 0-1.0. DEA analysis assesses the relative efficiency of each of these selected schools relative to the rest of the schools in terms of academic performance. The 2012/13 academic year data from the selected schools were used for the analysis. Four input variables and five output variables were identified. The input variables were teacher to student ratio, trained to non trained teachers ratio, library facilities and contact hours per day. Output variables were estimated as grades obtained, that is 8 passes, 7 passes, 6 passes, 5 passes and below. Three schools formed the efficiency frontier and the fourth school was found inefficient for the academic year. There was an indication that reduction in contact hours per day has a larger effect on efficiency of fourth campus than does library facilities and trained to non trained teachers ratio.**

***Keywords* — Data Envelopment Analysis, Senior High School, Charnes-Cooper-Rhodes (CCR) model**

I. INTRODUCTION

Christian organizations (churches) have established educational institutions to augment government efforts in the provision of quality education to the Ghanaian populace, but the problem is that, the assessment of the performance of these institutions in order to ensure their continued existence has been minored upon. The collapse of some of these institutions is eminent if no work is done to evaluate their efficiency.

This paper describes the use of Data Envelopment Analysis (DEA) to assess academic efficiency of four Seventh Day Adventist Senior High Schools namely Seventh Day Adventist Senior High School (Tamale), Seventh Day Adventist Senior High School (Bantama) Seventh Day Adventist Senior High School (Kenyase), Seventh Day Adventist Senior High School (Bekwai) according to data of the year 2012/2013 West African Senior Secondary School Certificate Examination (WASSCE).

Adventist education dates back no further than the advent of the Adventist faith in Ghana. In 1953, the premier Adventist educational institution, the Bekwai S.D.A college was established. On October 8, 1974, the Adventist Girls' Vocational Institute was opened in Techiman, Brong Ahafo region, Ghana, as a day school by action of the Ghana Conference and the West African Union Mission. The church has also established some additional institutions including second cycle schools, of which four of these is captured for this study.

The results of DEA model are sensitive to the inputs and outputs factors. Indeed, an accurate selection of the input and output indicators, which are best adapted to the objective of the analysis, is critical to the success of the study. The inputs variables are units of measurement, which represent the factors used to carry out the delivery of services. The identification and measurement of these factors is crucial in a fair evaluation of the economy and efficiency in the programs and services management. Previous studies on other performance models [1],[2],[3] have shown that inputs of educational institutions can be categorized in various ways. In our case, we classify the inputs used by the four Senior High Schools as follows: trained to non trained teachers ratio, teacher to student ratio, contact hours per day and library facilities.

Output variables measure the yield or the level of activity of programmes and services. The output indicator used in the study are the passes obtained: 8 passes, 7 passes, 6 passes, 5 passes and below.

In order to evaluate the activities embedded in some organizations that do not place emphasis on profit making, the traditional use of measuring the effectiveness and efficiency of a product in terms of profitability cannot be employed since the emphasis here is on the efficiency of an institution not a product. Here performance and efficiency measurement considers inputs or resources used by the institution and the outputs that are the result of input utilization. The whole idea of efficiency measurement relies on production theory, which sees a firm as a production system where inputs are the resources that are utilized by the firm or the organization and are transformed into desirable outputs.

II. THE MODELS

DEA is a flexible, mathematical programming approach for the assessment of efficiency, where efficiency is defined as the maximum of a ratio of weighted outputs to weighted inputs subject to the condition that the similar ratios for every Decision Making Units (DMU) be less than or equal to unity . In DEA modeling (Charnes-Cooper-Rhodes model), we assume that there are number 'n' DMUs, each of which has 'm' inputs and 'r' outputs of common types. All inputs and outputs are assumed to be nonnegative, but at least one input and one output are positive [4],[5], [6]. The following notations were employed in this study.

Indices:
$i = 1, 2, \cdots, n$
$J = 1, 2, \cdots, m$
$K = 1, 2, \cdots, r$

where DMU_i is the i^{th} DMU,
DMU_o is the target DMU,
X_{ji} is the amount of input j consumed by DMU_i,
$X_i = (x_{ji})m \times 1$ is the column vector of inputs consumed by DMU_i, $X_o = (x_{jo})m \times 1$ is the column vector of inputs consumed by the target DMU,
$X = (x_{ji})m \times n$ is the matrix of inputs,
yki is the amount of output k produce by DMU_i,
$yi = (yki)r \times 1$ is the column vector of outputs produced by DMU_i,
$yo = (xko)r \times 1$ is the column vector of outputs produced by the target DMU,
$Y = (yki)r \times n$ is the matrix of outputs,
uj is the weight of input j,
$U = (uj)m \times 1$ is the column vector of input weights,
vk is the weight of output k,
$V = (vk)r \times 1$ is the column vector of output weights,
$\lambda = (i)_{n \times 1}$ is the matrix of outputs, £Rn is the column vector of a linear combination of n DMUs,
Θ is the objective value (efficiency) of the Charnes-Cooper-Rhodes (CCR) model.

In the CCR model, the multiple-inputs and multiple-outputs of each DMU are aggregated into a single virtual input and a single virtual output, respectively. The input-oriented CCR model for target DMUO can be expressed by the following fractional programming model:

$$\begin{aligned} Max\theta &= \frac{\sum_{r=1}^{s} V_r Y_{ro}}{\sum_{m=1}^{n} U_m X_{mo}} \\ \text{s.t. } & \frac{\sum_{r=1}^{s} V_r Y_{ri}}{\sum_{m=1}^{n} U_m X_{mi}} \leq 1, \\ & i = 1, \cdots, n \\ & u_1, u_2, \cdots u_m \geq 0 \\ & v_1, v_2, \cdots v_r \geq 0 \end{aligned} \tag{1}$$

Let Θ^*, $u*$ and $v*$ be the optimal objective value (efficiency value), the optimal input weights and the optimal output weights, respectively. The objective of this model is to determine the input weights and output weights that maximizes the ratio of a virtual output for DMUo. The constraints restrict the ratio of the vitual outputs to the virtual inputs for every DMU to be less than or equal to one (1). This implies that the maximum; efficiency, Θ^*, is at most one (1). In the input-oriented CCR model, a DMU is inefficient if it is possible to reduce any input without increasing any other inputs and achieve the same level of output. Under the assumption that all outputs and inputs have non-zero worth, DMUo in the above model will be efficient if Θ^* is equal to 1. If $\Theta^* < 1$, it is possible to produce the given output $(y_{10}, y_{20}, \cdots, y_{r0})$ using a smaller vector of inputs which may be obtained as a linear combination of the input vectors of other DMUs. The efficiencies of all DMUs are obtained by solving model (3, 1) n times, once for each DMU as the target DMU: Charnes and Cooper developed a transformation from a linear fractional programming problem to an equivalent linear programming problem. By using His transformation; the fractional CCR model (3,1) can be transformed into the following linear programming model:

$$\begin{aligned} Max\Theta &= v_1 y_{10} + v_2 y_{20} + \cdots + v_r y_{ro} \\ s.t\ u_1 x_{10} + u_2 x_{20} + \cdots + u_m x_{mo} &= 1 \\ v_1 y_{1i} + v_2 y_{2i} + \cdots + v_r y_{ri} &\leq u_1 x_{1i} + u_2 x_{2i} + \cdots + u_m x_{mi}, \\ i = 1, \cdots, n u_1, u_2, \cdots, u_m &\geq 0 \\ v_1, v_2, \cdots, u_r &\geq 0 \end{aligned} \tag{2}$$

the above linear CCR model and its dual can be written in the following vector-matrix form:

$$\begin{aligned}(CCR)\ \max\ v^T y_o \\ s.t.\ u^T x_o = 1 \\ -u^T X + v^T Y \leq 0 \\ u \leq 0 \\ v \leq 0\end{aligned} \tag{3}$$

$$\begin{aligned}(DCCR) \min \Theta \\ s.t\ \Theta x_0 - X\lambda \geq 0 \\ Y\lambda \geq 0 \\ \lambda \geq 0\end{aligned} \tag{4}$$

Note that the Dual Charnes, Cooper and Rhodes (DCCR) model has a feasible solution, $\Theta = 1, \lambda_i = 0$ for $i \neq o$ and $\lambda_0 = 1$. Therefore, the optimal value Θ^* of the DCCR model is not greater than the constraint $Y\lambda \geq y_o$ forces λ to be a non zero vector. This along with $\Theta x_0 - X\lambda \geq 0$ implies that $\Theta^* > 0$.

Therefore, $0 < \Theta^* \leq 1$. Thus, the DCCR model has an optimal solution. From the strong duality theorem of linear programming, the CCR models are equal.

The target DMU (DMUo) is being compared with a linear combination of other DMUs. The objective of the CCR model is to find a vector of weights such that the efficiency of DMU, relative to other DMUs is maximized, provided that no other DMUs or linear combination of other DMUs could achieve the same output levels with smaller amount of any input.

DMUo is efficient if no linear combination of other DMUs can produce the same or higher output levels using less of all inputs. Θ Indicates a possible proportional reduction in inputs (x_o). Reduction in inputs x_o can be viewed as a radial movement from (x_o, y_o) toward the production frontier.

$\Theta* = 1$ implies that no linear combination of other DMUs has $X\lambda < x_o$ and $Y\lambda \geq y_o$. Otherwise, we can further reduce $\Theta*$ while $X\lambda \leq \Theta * x_o$ still holds. Thus, $\Theta*$ is not an optimal solution because we can find $\Theta < \Theta*$ that satisfies all the constraints. On the other hand, $\Theta* < 1$ indicates that the resulting linear combination of DMUs acts as a benchmark for DMUo. $\Theta*$ can also be interpreted as the largest ratio of x_0 to $X\lambda$ which outputs are at least equalized, ie, $Y\lambda \geq y_o$.

A DMU is a Pareto-Koopmans efficient if and only if it is impossible to improve any input or output without worsening some other inputs or outputs. From the above definition, the DMUo with $\Theta* = 1$ may not be Pareto-Koopmans.

It is efficient if it is possible to make additional improvement (lower input or higher output) without worsening any other input or output. Therefore, we introduce a vector of input excesses (s-) and output shortfalls (s+) as follows:

$$s- = \Theta x_o - X\lambda, \text{ and } s^+ = Y\lambda - y_o$$

where $s^- \geq 0, s^+ \geq 0$ are defined as slack vectors for any feasible solution $(\Theta, \quad)$ of the DCCR model 4.

Based on the slack vectors, a DMU is Pareto-Koopmans efficient if is satisfies the following two conditions:

1) $\Theta* = 1$
2) $S^- = 0$ and $s^+ = 0$

The first condition is referred to as a weak efficiency, technical efficiency of "Farrell Efficiency" [7]. For the CCR model, the Pareto-Koopmans efficiency is called the CCR efficiency.

A summary of CCR efficiency conditions for a DMU is as follows:

- If $\Theta* < 1$, then the DMU is CCR-efficiency.
- If $\Theta* = 1$, and there is nonzero slacks, i.e., $s^{-*} = 0$, then the DMU is CCR-inefficient. From the complementary slackness conditions of linear programming, the elements of the vectors u* and v* corresponding to the positive slacks must be zero. Thus, the DMU with $\Theta* = 1$ is CCR-efficient if there is not at least one optimal u* and v* such that $u* > 0$ and $v* > 0$.
- If $\Theta* = 1$ with zero slack, then the DMU is CCR-efficient. From the strong theorem of complementarity, there exist optimal u* and v* such that $u* > 0$ and $v* > 0$.

The inefficiency that occurs from the slack variable is called the "mix inefficiency". To determine the efficiency of a DMU, we have to solve the following two-phase linear programming problem:

Phase 1: Solve the DCCR model 4. $\Theta*$ is equal to the optimal objective value $(u*^T y_o)$ of the CCR model 3.

Phase 2: Use $\Theta*$ from phase 1 to solve the following LP with (λ, s^-, s^+) as variables.

$$\begin{aligned}\text{Max} e^T s^- + e^T s^+ \\ s.t s^- = \Theta * x_o - x\lambda \\ s^+ = Y\lambda - y_0 \\ \lambda \leq 0 \\ s^- \geq 0, \quad s^+ \geq 0\end{aligned}$$

Where $= (1, \cdots, 1)^T$, $e^T s^- = \sum_j^m = 1s^- j$ and $e^T s^+ = \sum_k^r = 1s_k^-, s_j^-$ is the input excess of the j^{th} input, and s_k^- is the output shortfall of the k^{th} output. An optimal solution $(\lambda*, s-*, s+*)$ of phase 2 is called the max-slack solution.

If the max-slack solution satisfies s-* = 0 and s+* = 0, then it is called zero slack.

Phase 2 finds an optimal solution that maximizes the sum of input excesses and output shortfalls obtainable with $\Theta*$ from phase 1. If a DMU has $\Theta* = 1, s-* = 0$ and $s+* = 0$, it is CCR-efficient.

For an efficient DMUo, a "reference set", Eo, is defined based on the max-slack solution as follows:

$$Eo = \{i1\lambda i* > 0,\ I = 1, \cdots, n\}.$$

The linear combination of the reference set is the projected point on the efficient frontier of the inefficient DMUo. The relationship between the optimal solution of DMUo and its reference set can be given as:
$\Theta * x_0 = \Sigma_{i\pounds EO} x_i \lambda_{i*} + s^{-*}$
$y_0 = \Sigma_{i\pounds EO} y_i \lambda_{i*} + s^{-*}$

From this relationship, the efficiency of the DMUo with (x_o, y_o) can be improved by reducing the input values x_o radially by the ratio $\Theta*$ and then reducing the remaining input excesses by $s - *$. From the output viewpoint, the efficiency can be improved by increasing the outputs y_o by the output shortfalls, $s + *$. The CCR model 3 is developed on the assumption of "constant return to scale" of DMUs. For the long-run analysis, the scale of firm's operations should be considered.

The amount of increased outputs associated with increased inputs is fundamental to the long-run nature of the firm's production process. From the economic theory, there are three types of "return to scale": If output increases by that same proportional change then there are constant returns to scale (CRS). If output increases by less than that proportional change, there are decreasing returns to scale (DRS). If output increases by more than that proportional change, there are increasing returns to scale (IRS).

III. ANALYSIS AND RESULTS

The basic DEA model for a given campus system can be formulated as follows:

Target DMU (Max Θ) = $v_1 y_{1o}$ + $v_2 y_{2o}$+ …………..+ $v_r y_{ro}$

s.t $u_1 x_{10}$ + $u_2 x_{20}$ + …………………. + = 1,

$v_1 y_{1i}$ + $v_2 y_{2i}$ + … + $v_r y_{ri} \leq u_1 x_{1i}$ + $u_2 i_{20}$ + … + $u_m x_{mi}$, i = 1 $\cdots$ n

$V_1, V_2, V_3, V_4, V_5, U_1, U_2, U_3, U_3, U_4 \geq 0$

y_r = amount of output r

v_r = weigut assigned to output r

x_i = amount of input i

u_i = weight assigned to input i

MAX : TAMALE = $99V_1 + 21V_1$

Subject to $0.1042U_1 + 0.0462U_2 + U_3 + 8U_4 = 1$

$99V_1 + 21V_2 - (0.1042U_1 + 0.0462U_2 + U_3 + 8U_4) \leq 0$

$422V_1 + 141V_2 + V_3 + V_4 - (0.050U_1 + 0.0462U_2 + U_3 + 7U_4) \leq 0$

$421V_1 + 177V_2 + 45V_3 + 5V_4 + 4V_5 - (0.0852U_1 + 0.0714U_2 + 2U_3 + 8U_4) \leq 0$

$455V_1 + 74V_2 + V_3 - (0.04U_1 + 0.05U_2 + U_3 + 7U_4) \leq 0$

$V_1, V_2, V_3, V_4, V_5, U_1, U_2, U_3, U_3, U_4 \geq 0$

MAX: BANTAMA = $422V_1 + 141V_2 + V_3 + V_4$

Subject to $0.0502U_1 + 0.0462U_2 + U_3 + 7U_4 = 1$

$99V_1 + 21V_2 - (0.1042U_1 + 0.0462U_2 + U_3 + 8U_4) \leq 0$

$422V_1 + 141V_2 + V_3 + V_4 - (0.050U_1 + 0.0462U_2 + U_3 + 7U_4) \leq 0$

$421V_1 + 177V_2 + 45V_3 + 5V_4 + 4V_5 - (0.0852U_1 + 0.0714U_2 + 2U_3 + 8U_4) \leq 0$

$455V_1 + 74V_2 + V_3(0.04U_1 + 0.05U_2 + U_3 + 7U_4) \leq 0$

$V1, V2, V3, V4, V5, U1, U2, U3, U3, U4 \geq 0$

MAX KVNYASE = $421V_1 + 177V_2 + 45E_3 + 5V_4 + 4V_5$

Subject to: $0.0852U_1 + 0.0714U_2 + 2U_3 + 8U_4 = 1$

$99V_1 + 21V_2 - (0.1042U_1 + 0.0462U_2 + U_3 + 8U_4) \leq 0$

$422V_1 + 141V_2 + V_3 + V_4 - (0.050U_1 + 0.0462U_2 + U_3 + 7U_4) \leq 0$

$421V_1 + 177V_2 + 45V_3 + 5V_4 + 4V_5 - (0.0852U_1 + 0.0714U_2 + 2U_3 + 8U_4) \leq 0$

$455V_1 + 74V_2 + V_3(0.04U_1 + 0.05U_2 + U_3 + 7U_4) \leq 0$

$V1, V2, V3, V4, V5, U1, U2, U3, U3, U4 \geq 0$

MAX BEKWAI = $455V_1 + 74V_2 + V_3$

Subject to: $0.04U_1 + 0.05U_2 + U_3 + 7U_4 = 1$

$21V_2 - (0.1042U_1 + 0.0462U_2 + U_3 + 8U_4) \leq 0$

$422V_1 + 141V_2 + V_3 + U_4 - (0.050U_1 + 0.0462V_2 + U_3 + 7U_4) \leq 0$

$421V_1 + 177V_2 + 45V_3 + 5V_4 + 4V_5 - (0.0852U_1 + 0.0714U_2 + 2U_3 + 8U_4) \leq 0$

$455V_1 + 74V_2 + V_3(0.04U_1 + 0.05U_2 + U_3 + 7U_4) \leq 0$

$V_1, V_2, V_3, V_4, V_5, U_1, U_2, U_3, U_4 \geq 0$

TABLE I. DESCRIPTIVE STATISTICS FOR DEA RESULTS

Items	Scores
Total number of DMU_S	4.00
Number of efficient DMU_S	3.00
Number of inefficient DMU_S	1.00
Maximum efficiency	1.00
Minimum efficiency	0.22
Average efficiency	0.76

DEA solver software is used to run the CCR model. Table I summarizes the descriptive statistics of the results. The maximum efficiency score is 1.00, while the minimum efficiency score is 0.22. The efficiency score average is 0.76. The means that the input for an average unit could be reduced by 24%.

TABLE II. EFFICIENCY SCORES OF THE SCHOOLS

Schools	Efficiency
Tamale	0.22
Bantama	1.00
Kenyase	1.00
Bekwai	1.00

Table II shows the scores of the four schools obtained from DEA using CCR model.
These efficiency scores were under the following conditions:

- All data and all weights are positive
- Efficiency scores must lie between zero and unity
- The same weights for the target school are applied to all schools

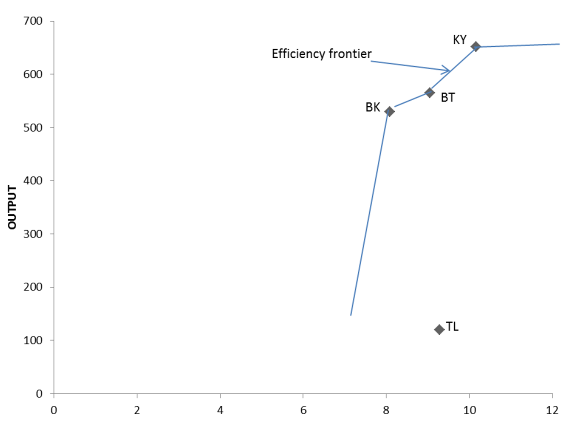

Fig. 1. Efficiency Frontier For The Schools

TABLE III. TARGET CELL (MAX) - BEKWAI

Cell	Name	Original Value	Final Value
$K9$	BEKWAI weighted output	530	1

TABLE IV. ADJUSTABLE CELLS (BEKWAI)

Cell	Name	Original Value	Final Value
$B11$	weight (V1)8 PASSES	1.0000	0.0022
$C11$	weight (V2)7 PASSES	1.0000	0.0000
$D11$	weight (V3)6 PASSES	1.0000	0.0000
$E11$	weight (V4)5 PASSES	1.0000	0.0000
$F11$	weight (V5)4 PASSES AND BELOW	1.0000	0.0000
$G11$	weight (U1)Teacher to student ratio	1.0000	0.0000
$H11$	weight (U2)TRAINED TO NON- T RATIO	1.0000	0.0019
$I11$	weight (U3)LIBRARY FACILITIES	1.0000	0.0000
$J11$	weight (U4)CONTACT HOURS PER DAY	1.0000	0.1429

From Fig. 1 it can be seen that Bekwai S.D.A Senior High School(BK), Bantama S.D.A Senior High School(BT) and Kenyase S.D.A Senior High School(KY) are on the efficiency frontier whiles Tamale S.D.A Senior High School(TL) is not on the efficiency frontier which confirms its inefficiency.

The following schools (Bekwai , Bantama, Kenyase) are efficient and are considered to have better academic performance. The efficient schools have an efficiency score equal to one (1.00). They are on the efficient frontier. These schools are more efficient in converting the inputs into better academic performance of students as compared to Tamale S.D.A S.H.S (0.22), which is inefficient.

A. Bekwai as a target DMU

From tables III and IV, the optimal solution to the LP has the value of 1 and the best input and output weights are $U_1 = 0$ $U_2 = 0.0119,$ $U_3 = 0$, $U_4 = 0.1429$

$$V_1 = 0.0022,\ V_2 = 0,\ V_3 = 0,\ V_4 = 0,\ V_5 = 0$$

We now observe the difference between optimal weights $U_2 = 0.0119$, and $U_4 = 0.1429$. The ratio $U_4/U_2 = 12$ suggests that it is advantageous for, Tamale to weight input(contact hours per day 12 times more than input(trained to non trained teachers ratio) in order to maximize efficiency. This means that a reduction in contact hours per day has a bigger effect on efficiency than trained to non trained teachers ratio.

Table V, also indicates that the three working constraints (Tamale, Kenyase, Bantama) with non zero slack values are said to be non binding they are not satisfied with equality at the LP optimal.

1) Sensitivity Analysis: From table VI

$$U_1 = 0\ U_2 = 0.0119,\ U_3 = 0,\ U_4 = 0.1429$$

$$V_1 = 0.0022,\ V_2 = 0,\ V_3 = 0,\ V_4 = 0,\ V_5 = 0.$$

Suppose we vary the coefficient of V_1 in the objective function. The solution value for for V_1 is 0.0022 and the objective function value is 455. The allowable increase or decrease suggests that provided the coefficient of V_1 in the objective function lies between 455+ 0 =455 and 455- 0 =455, the values of the variables in the optimal LP will remain unchanged. Similar conclusions can be drawn for other weights.

From table VII we can study the effect of changing the right hand side of the Bekwai constraint. If the right hand side of the bekwai constraint lies between 0 + 0.078199053 = 0.078199053 and 0 - 1= -1, the objective function change will be exactly 0.

B. Bantama as a target DMU

From tables VIII and IX, we can see that the optimal solution to the LP has the value of 1 and the best input and output weights are

$$U_1 = 0 \; U_2 = 0, \; U_3 = 0.008, \; U_4 = 0.1429$$

$$V_1 = 0.0022, V_2 = 0.001, V_3 = 0, V_4 = 0, V_5 = 0$$

We now observe the difference between optimal weights $U_3 = 0.008$, and $U_4 = 0.1429$. The ratio $U_4/U_3 = 18$ suggests that it is advantageous for, Tamale to weight input(contact hours per day 18 times more than input(library facilities) in order to maximize efficiency. This means that a reduction in contact hours per day has a bigger effect on efficiency than library facility.

Table X , also indicates that the three working constraints (Tamale, Kenyase,) with non zero slack values are said to be non binding they are not satisfied with equality at the LP optimal.

1) Sensitivity Analysis: From table X

$$U_1 = 0 \; U_2 = 0.0119, \; U_3 = 0, \; U_4 = 0.1429$$

$$V_1 = 0.0022, \; V_2 = 0, \; V_3 = 0, \; V_4 = 0, \; V_5 = 0.$$

Suppose we vary the coefficient of V_1 in the objective function. The solution value for for V_1 is 0.0022 and the objective function value is 422. The allowable increase or decrease suggests that provided the coefficient of V_1 in the objective function lies between 422+ 0 =422 and 422- 0 =422, the values of the variables in the optimal LP will remain unchanged. Similar conclusions can be drawn for other weights.

From table XII we can study the effect of changing the right hand side of the Bantama constraint.If the right hand side of the Bantama constraint lies between 0 + 0.072555378 = 0.072555378 and 0 - 0.072527473 = -0.072527473., the objective function change will be exactly 0.

C. Kenyase as a target DMU

From tables XIII and XIV, we can see that the optimal solution to the LP has the value of 1 and the best input and output weights are

$$U_1 = 0.006 \; U_2 = 0, \; U_3 = 0.000, \; U_4 = 0.1250$$

$$V_1 = 0.0018, \; V_2 = 0.009, \; V_3 = 0.022, \; V_4 = 0, \; V_5 = 0$$

We now observe the difference between optimal weights $U_1 = 0.006$, and $U_4 = 0.1250$. The ratio $U_4/U_1 = 21$ suggests that it is advantageous for, Tamale to weight input(contact hours per day 21 times more

TABLE V. CONSTRAINTS OF THE MODEL (BEKWAI)

Cell	Name	Cell Value	Formula	Status	Slack
$N6$	TAMALE working	-0.9253	$N6{<=}0$	Not Binding	0.92527
$N7$	BANTAMA working	-0.0725	$N7{<=}0$	Not Binding	0.07253
$N8$	KENYASE working	-0.2176	$N8{<=}0$	Not Binding	0.21758
$N9$	BEKWAI working	0	$N9{<=}0$	Binding	0
$L9$	BEKWAI weighted input	1	$L9$=1	Not Binding	0

TABLE VI. ADJUSTABLE CELLS (BEKWAI)

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B11$	weight (V1)8 PASSES	0.0022	0	455	0	0
$C11$	weight (V2)7 PASSES	0	0	74	0	1E+30
$D11$	weight (V3)6 PASSES	0	0	1	0	1E+30
$E11$	weight (V4)5 PASSES	0	0	0	0	1E+30
$F11$	weight (V5)4 PASSES AND BELOW	0	0	0	0	1E+30
$G11$	weight (U1)Teacher to student ratio	0.0119	0	0	0	1E+30
$H11$	weight (U2)TRAINED TO NON- T RATIO	0	0	0	0	1E+30
$I11$	weight (U3)LIBRARY FACILITIES	0	0	0	0	1E+30
$J11$	weight (U4)CONTACT HOURS PER DAY	0.1429	0	0	0	0

TABLE VII. CONSTRAINTS OF THE MODEL (BEKWAI)

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$N6$	TAMALE working	-0.9253	0	0	1E+30	0.92527
$N7$	BANTAMA working	-0.0725	0	0	1E+30	0.07253
$N8$	KENYASE working	-0.2176	0	0	1E+30	0.21758
$N9$	BEKWAI working	0	1	0	0.0782	1
$L9$	BEKWAI weighted input	1	1	1	1E+30	1

TABLE VIII. TARGET CELL (MAX) - BANTAMA

Cell	Name	Original Value	Final Value
$K7$	BANTAMA weighted output	565	1

TABLE IX. ADJUSTABLE CELLS (BANTAMA)

Cell	Name	Original Value	Final Value
$B11$	weight (V1)8 PASSES	1	0.002
$C11$	weight (V2)7 PASSES	1	0.001
$D11$	weight (V3)6 PASSES	1	0
$E11$	weight (V4)5 PASSES	1	0
$F11$	weight (V5)4 PASSES AND BELOW	1	0
$G11$	weight (U1)Teacher to student ratio	1	0
$H11$	weight (U2)TRAINED TO NON- T RATIO	1	0
$I11$	weight (U3)LIBRARY FACILITIES	1	0.008
$J11$	weight (U4)CONTACT HOURS PER DAY	1	0.1429

than input(teacher to student ratio) in order to maximize efficiency. This means that a reduction in contact hours per day has a bigger effect on efficiency than teacher to student ratio

Table XV, also indicates that the three working constraint (Tamale,) with non zero slack values are said to be non binding. They are not satisfied with equality at the LP optimal.

1) Sensitivity Analysis: From table XVI

$$U_1 = 0\ U_2 = 0.00,\ U_3 = 0,\ U_4 = 0.1250$$

$$V_1 = 0.0018,\ V_2 = 0.009\ V_3 = 0.0022,\ V_4 = 0,\ V_5 = 0.$$

Suppose we vary the coefficient of V_2 in the objective function. The solution value for for V_2 is 0.009 and the objective function value is 177.The allowable increase or decrease suggests that provided the coefficient of V_2 in the objective function lies between 177+ 0 =177 and 177- 0 =177, the values of the variables in the optimal LP will remain unchanged. Similar conclusions can be drawn for other weights.

From table XVII we can study the effect of changing the right hand side of the Kenyase constraint.If the right hand side of the constraint Kenyase constraint lies between 0 + 38.374996 = 38.374996 and 0 - 0.095210617 = -0.0952106., the objective function change will be exactly 0.

TABLE X. CONSTRAINTS OF THE MODEL (BANTAMA)

Cell	Name	Cell Value	Formula	Status	Slack
$N6$	TAMALE working	-0.9204	$N6$<=0	Not Binding	0.92036
$N7$	BANTAMA working	0	$N7$<= 0	Binding	0
$N8$	KENYASE working	-0.1088	$N8$<= 0	Not Binding	0.10881
$N9$	BEKWAI working	0	$N9$<= 0	Binding	0
$L7$	BANTAMA weighted input	1	$L7$=1	Not Binding	0

TABLE XI. ADJUSTABLE CELLS (BANTAMA)

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B11$	weight (V1)8 PASSES	0.002	0	422	0	0
$C11$	weight (V2)7 PASSES	0.001	0	141	0	0
$D11$	weight (V3)6 PASSES	0	0	1	0	1E+30
$E11$	weight (V4)5 PASSES	0	0	1	0	1E+30
$F11$	weight (V5)4 PASSES AND BELOW	0	0	0	0	1E+30
$G11$	weight (U1) Teacher to student ratio	0	0	0	0	1E+30
$H11$	weight (U2) TRAINED TO NON- T RATIO	0	0	0	0	1E+30
$I11$	weight (U3) LIBRARY FACILITIES	0	0	0	0	1E+30
$J11$	weight (U4) CONTACT HOURS PER DAY	0.1429	0	0	1E+30	0

TABLE XII. CONSTRAINTS OF THE MODEL (BANTAMA)

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$N6$	TAMALE working	-0.9204	0	0	1E+30	0.92036
$N7$	BANTAMA working	0	1	0	0.07256	0.07253
$N8$	KENYASE working	-0.1088	0	0	1E+30	0.10881
$N9$	BEKWAI working	0	0	0	0.0782	0.23367
$L7$	BANTAMA weighted input	1	1	1	1E+30	1

IV. CONCLUSION

In this paper we assessed the academic efficiency of students performance in this four S.D.A Senior High Schools using DEA. When considering this analysis as a whole, one must also give consideration to the variables selected as outputs and inputs. Grades obtained were selected as outputs and teacher to student ratio, trained to non trained teachers ratio, library facilities and contact hours per day were selected as inputs. They were selected in an attempt to show the most important attributes pertinent to the problem at hand. This paper contributes a DEA approach for academic performance of students. A point of departure for the DEA approach compared to existing methods is the input-output framework. Compared to each other, DEA measures the efficiency of academic performance of students in utilizing, teacher to student ratio, contact hours per day and staff to maximize the grades obtained by students. Therefore, the DEA approach relates resources expended on students to academic performance. The analysis identifies Bekwai, Kenyase and Bantama

TABLE XIII. TARGET CELL(MAX)- KENYASE

Cell	Name	Original Value	Final Value
$K8$	KENYASE weighted output	652	1

TABLE XIV. ADJUSTABLE CELLS (KENYASE)

Cell	Name	Original Value	Final Value
$B11$	weight (V1)8 PASSES	1	0.0018
$C11$	weight (V2)7 PASSES	1	0.0009
$D11$	weight (V3)6 PASSES	1	0.0022
$E11$	weight (V4)5 PASSES	1	0
$F11$	weight (V5)4 PASSES AND BELOW	1	0
$G11$	weight (U1)Teacher to student ratio	1	0.006
$H11$	weight (U2)TRAINED TO NON- T RATIO	1	0
$I11$	weight (U3)LIBRARY FACILITIES	1	0
$J11$	**weight (U4)CONTACT HOURS PER DAY**	**1**	**0.125**

TABLE XV. CONSTRAINTS OF THE MODEL (KENYASE)

Cell	Name	Cell Value	Formula	Status	Slack
$L8$	KENYASE weighted input	1	$L8$=1	Not Binding	0
$N6$	TAMALE working	-0.8058	$N6<=0$	Not Binding	0.8058
$N7$	BANTAMA working	0	$N7<=0$	Binding	0
$N8$	KENYASE working	0	$N8<=0$	Binding	0
$N9$	BEKWAI working	0	$N9<=0$	Binding	0

S.D.A. S.H.S'S as efficient. They serve as the benchmark for the schools and can be utilized as role models to which inefficient School (Tamale S.D.A S.H.S) may adjust its resources in order to become efficient. There was an indication that when Bekawi S.D.A was set as target DMU for Tamale campus, the reduction in input, that is contact hours per day has a larger effect on efficiency of Tamale S.H.S than does trained to non trained teachers ratio. In otherwords, contact hours per day should be utilized effectively by management of Tamale S.D.A Senior High School.

TABLE XVI. ADJUSTABLE CELLS (KENYASE)

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B11$	weight (V1)8 PASSES	0.0018	0	421	0	0
$C11$	weight (V2)7 PASSES	0.0009	0	177	0	0
$D11$	weight (V3)6 PASSES	0.0022	0	45	0	0
$E11$	weight (V4)5 PASSES	0	0	5	0	1E+30
$F11$	weight (V5)4 PASSES AND BELOW	0	0	4	0	1E+30
$G11$	weight (U1)Teacher to student ratio	0	0	0	0	1E+30
$H11$	weight (U2)TRAINED TO NON- T RATIO	0	0	0	0	1E+30
$I11$	weight (U3)LIBRARY FACILITIES	0	0	0	0	1E+30
$J11$	weight (U4)CONTACT HOURS PER DAY	0.125	0	0	1E+30	0

TABLE XVII. CONSTRAINTS OF THE MODEL (KENYASE)

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$L8$	KENYASE weighted input	1	1	1	1E+30	1
$N6$	TAMALE working	-0.8058	0	0	1E+30	0.8058
$N7$	BANTAMA working	0	0	0	0.06349	0.06315
$N8$	KENYASE working	0	1	0	38.375	0.09521
$N9$	BEKWAI working	0	0	0	0.0682	0.20446

Also there was an indication that when Bantama S.D.A was set as target DMU for Tamale campus, the reduction in input that is contact hours per day has a larger effect on efficiency of Tamale S.D.A S.H.S than library facilities. This confirms once more that contact hours for Tamale S.D.A should be effectively utilized by management.

There was also an indication that, Tamale constraint was not binding because it was not satisfied with equality at the LP optimal. This is true for both Bantama and Bekwai as target DMU'S.

Again, when Kenyase S.H.S was set as a target DMU for Tamale S.D.A Senior High School, it indicated that in order to achieve Tamale S.H.S as efficient, it is better not to decrease contact hours per day. In other words, management should effectively utilize contact hours per day on students in order to be at the frontier.

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