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On the use of Probability with Generalized Linear Models for Evaluating Extreme Rainfall in Nigeria

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Abstract – **This study examined the variability of extreme rainfall (mm) in all states of Nigeria including Abuja (FCT) from 1971 to 2012. Generalized extreme value (GEV) theory was employed to extract out the monthly extreme rainfall as the response variable which possessed a bimodal structure and is best captured with beta distribution from existing literatures and the current study. Generalized linear models (GLMs) of beta distribution type therefore explored the variability in rainfall encountered by each state to explain the extreme rainfall model using betareg package in R. It was validated that Oyo, Cross River, Akwa Ibom, Imo ,Niger, Kwara, Kebbi, and Zamfara states contributed to the model at 5 percent significant level as affirmed by z-statistics and exploratory data analysis (EDA). This is an indication to the states validated by the extreme rainfall model to be prone to yearly flooding while the rest of the states not validated by the model are likely to encountered flooding whenever there is extreme rainfall which is a recent trend due to global warming. Therefore, structure and re-engineering solutions must be put in place in all parts of the country to avoid flooding.**

Keywords – **Beta Distribution, Extreme Rainfall Model, Generalized Linear Models, Rainfall Variability.**

I. Introduction

A change of climate which is attributed directly or indirectly to human activity that alters the composition of the global atmosphere in addition to natural climate variability observed over comparable periods of time [12], which has significantly contributed on the increase of global disaster cause by weather, climate and water related hazards as both developed and developing countries of the word are bearing the burden of repeated flood, droughts , temperature extremes and storms.

Reference [12] noted that Africa is one of the most vulnerable continents to climate change and climate variability, a situation aggravated by the interaction of multiple stresses, occurring at various levels and low adaptive capacity. Nigeria, a country located in the tropical zone of West Africa in continent of Africa between latitudes 4^0N to 14^0N and longitudes $2^02´$E to $14^030´$E which has a total area of 923,770km^2 with an estimated population of about 140 million is not left out in the recent trend of global disasters.

Climate change has been resultant effect of prolonged extreme rainfall across the globe that usually result to floods [11] whereas [21] preliminary report in West Ireland identified changing rainfall pattern as a possible cause of the increased flooding frequency.

Floods have been related to the occurrence and reoccurrence of prolonged extreme rainfall [1]-[2], [4], [18], [41], [51] and [52], however, the major cause of floods in Nigeria has been identified to excessive extreme rainfall [5]-[7] and [47].

Reference [24] identified flooding as the most common environmental hazard in Nigeria. In addition, [47] opened up on the torrential rainfall that ravaged Makurdi in Benue State and Babura in Jigawa State in a period of two days in the month of August, 2008 which he described very disastrous. Reference [24] reported how a Lagos flood forced Lagosians to relocate as a result of heavy rainfall of July, 2011 not knowing there was going to be a more devastating torrential rain that will result in more disastrous floods in Lagos metropolis in the following week [8] and [36].

Apart from Ogunpa flooding in 1980 that Ibadan in Oyo State experienced [4], flood disaster events are no longer a new phenomenon in the town as a whole and the state in general as it has become yearly occurrence for over two decades.

Furthermore, in the recent years, the rains came indeed, but in torrents, giving rise to deadly floods instead, causing harvests of pains. From Lagos, Ibadan, Abeokuta, Calabar, Port-Harcourt and Warri in the southern region through Ilorin , Abuja, Lokoja and Minna in the middle belt to Kano ,Kaduna, Jalingo, Maiduguri and Gombe in the north, the rains came down and floods came-up washing away streets, battering dams, collapsing bridges, submerging buildings, killing people, trapping some in their homes and separating thousands of others from their homes and belongings as nothing is spared by the marauding floods [3].

In practice, stochastic models of rainfall are well established and very useful tools for several engineering applications from short-term predication to long-term simulation of rainfall and rainfall-driven processes such as runoff [31].

In recent years, much work has been done on building stochastic and physical intuitive models for rainfall at single site [14] as the current study aimed at exploring and improving on generalized linear models (GLMs) to capture multi-sites to study the variability of rainfall among states of Nigeria with respect to climate change as to avoid future flood incidents.

II. Methodology

Consequently, all result in climate research have some degree of uncertainty attached to them [15]. This has implications for all who use the result either decision makers developing new policies or scientist seeking to develop further their understanding of the climate [44]. Therefore, it is useful to develop scientific methods that include some recognition of uncertainty and introduce statistical methods that deal with such uncertainty [42].

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III. Generalized Linear Models

Using probability models within the framework of generalized linear models (GLMs) profer solutions as suggested in recent papers [9], [14], [15], [17], [26], [30], [50] and [53]. Moreover, these methods also provide an appealing alternative as it offer an intuitive and flexible approach to such problems in multi-sites [15], [17], [48] and [53].

Nevertheless, an important statistical development of the last 30 years has been the advances in regression analysis provided by generalized linear models (GLMs), generalized additive models (GAMs) and generalized estimating equation (GEE) for interpreting variability [26]. For example, GLMs was applied in climate research based on the development of the first seminar publications by [20], also providing the link with practice through software availability are [13], [27]-[28], [34]-[35] and [38].

Mathematical extensions of linear models resulted to generalized linear models (GLMs) that do not force data into unnatural scales and thereby allow for non-linearity and non-constant variance structures in the data [27]. They are based on an assumed relationship (called a link function) between the mean of the response variable and the linear combination of the explanatory variables. Data may be assumed to be from several exponential family of distributions including Poisson, binomial, negative binomial, normal, lognormal, gamma, beta, etc suggested by exploratory data analysis as noted in [16] and [39], many of which better fit the non-normal error structures of most climate data.

Thus, GLMs are more flexible and better suited for analyzing climate relationships which can be poorly represented by classical Gaussian distribution [10]. As this study focuses on the monthly average of rainfall in Nigeria taking root in the work of [17], [19], [45] and [50] who fitted generalized linear models to daily rainfall variability data and [9] and [17] who also fitted generalized linear models to monthly rainfall variability data.

Generalized linear models (GLMs) extend the ideas underlying linear models to situations when the response variable has other distributions different from normal such as binomial, Poisson, beta, etc that belong to the exponential family of distributions. The marginal distribution of a response variable is assumed to follow a generalized linear models (GLMs) in which the variance is the function of the mean [22], [33] and [38].

IV. Exponential Families of Distributions

The response variable is a random variable Y whose pdf (continuous) or pmf (discrete) depends on parameters θ and ϕ that has the canonical representation of the form

$$f(y;\theta,\phi) = exp\{[y\theta - b(\theta)]/a(\phi) + c(y,\phi)\} \quad (1)$$

where a, b, c are known functions. This canonical representation is used in [32], [35] and [49]. In the above, $a(\phi) = \phi/w$ where w is a weight, ϕ is the dispersion parameter or scale parameter, which for some distributions is known while others is unknown and for dispersion parameter $a(\phi) > 0$ is required. If ϕ is known, equation (1) defined a one-parameter exponential family of distributions and if ϕ is unknown, equation (1) defines an exponential dispersion family of distributions.

However, the canonical representation given in [23] is

$$f(y;\theta) = exp\{a(y)b(\theta) + c(\theta) + d(y)\} \quad (2)$$

where a, b, c, d are known functions. The form in equation (2) does not explicitly include a scale parameter, as the case in equation (1). The distributions $N(\mu,\sigma^2)$ where σ^2 is known, $BIN(n,\mu)$ and Poisson (μ) known, equation (1) defines a one-parameter canonical exponential family.

Given the pdf or pmf of Y by equation (1),

$$E(Y) = b'(\theta) \quad (3)$$

where $'$ denotes differentiation with respect to θ, indicating the mean. This is shown for continuous random variable as $f(y)$ is abbreviated for $f(y;\theta,\phi)$ and if Y is a discrete random variable, then, the integration is replaced by summation.

Differentiating both sides of $\int_{-\infty}^{\infty} f(y)dy = 1$ with respect to θ gives

$$\int_{-\infty}^{\infty} [y - b'(\theta)]/a(\phi)\, f(y)dy = 0. \quad (4)$$

Assuming $a(\phi) = 0$, then $\int_{-\infty}^{\infty} yf(y)dy - b'(\theta) = E(Y) - b'(\theta) = 0$, giving the result in equation (3).

Differentiating with respect to θ the expression in equation (4),

$$\int_{-\infty}^{\infty} \{[y - b'(\theta)]^2/a(\phi)\, f(y) - b''(\theta)f(y)\}\, dy = var(Y)a(\phi) - b''\theta = 0,$$ giving the result

$$var(Y) = b''(\theta)a(\phi) \quad (5)$$

which is the variance. (Note that $l(y) = logf(y)$ denotes the log-likelihood function. The function derivative of l with respect to θ is typically called the score function for which it is known that $E\{\partial^2/\partial\theta^2\, l(y)\} = -E\{\partial/\partial\theta\, l(y)\}^2$). From the results of equations (3) and (5), the variance of Y can be written as $V(\mu)a(\phi)$ or $(\mu)\,\phi/w$, where $V(\mu)$ is called the variance function and canonical links expressed as $g(\mu) = \theta$.

V. Estimation for Generalized Linear Models

The responses $Y_i, i = 1,2 \ldots, N$ are independent from a distribution with pdf or pmf given in equation (1) assumes a common parameter ϕ for all observations. Let $E(Y_i) = \mu_i$, $\beta = (\beta_1, \beta_2, \ldots, \beta_p)^T$, x_{ij} be the j^{th} element of x_i^T and the linear predictor for the i^{th} observation is $g(\mu_i) = \eta_i = \sum_j^p \beta_j x_{ij}$, that is,

$$g(\mu_i) = \eta_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_p x_{ip}. \quad (6)$$

For this study to be able to explain the flooding situations in Nigeria due to extreme rainfall, the response variable is chosen to be the monthly average of extreme rainfall in Nigeria with other predictor variables being the average monthly rainfall in all state of Nigeria including Abuja (FCT) from January, 1971 to December, 2012 as data were sourced from Nigeria Meteorological Agency (NIMET), Abuja.

In our estimation for GLMs, the method of maximum likelihood and deviance minimization shall be employed with iterative procedure of Fisher scoring method using version 3.1.1of R. The log-likelihood function denoted by l given data $y_1, y_2, \dots, y_N$ and the likelihood equations are

$$\partial l/\partial \beta_j = U_j = \sum_{i=1}^{N} ((y_i - \mu_i)/V_i)\,(d\mu_i/d\eta_i)x_{ij} = 0 \quad (7)$$

for $j = 1,2, \dots p$, where $V_i = var(Y_i)$.

Hence, equation (7) can be interpreted as sums of weighted residuals with covariance matrix of the parameters estimated as

$$\hat{V}ar(\hat{\beta}) = \left[X^T \widehat{W} X\right]^{-1} \hat{V}ar(U)[X^T \widehat{W} X]^{-1}. \quad (8)$$

The likelihood of equation (7) cannot be solved algebraically (except in the case of normally distributed errors) and so a numerical iterative procedure is required to be used [29] and [37]. This study examines Fisher's scoring iterative algorithms to profer solutions in R.

VI. Results and Discussion

The density plot for the response variable which is the monthly average of extreme rainfall in Nigeria as shown in Fig. 1 has a bimodal structure. Beta regression was identified by [40] called betareg, to be more accurate and efficient parameter estimates than ordinary least squares regression when the dependent variable follows a skewed underlying distribution with bimodal densities. Moreover, [46] also argued that beta regression is a highly flexible parametric distribution able to accommodate bimodal densities with varying severity skewness. Therefore, the current study is of the same line of thought with the afore mentioned authors as beta distribution allows specification of a probability model for continuous response variable with support over the closed interval of [0,1] [25] and [43].

Suitable transformation was carried out to make a closed interval of [0,1] as shown in Fig. 2 in order for betareg package to be applicable on the response variable with bimodal density for estimating the parameters of the extreme rainfall model for states of the country.

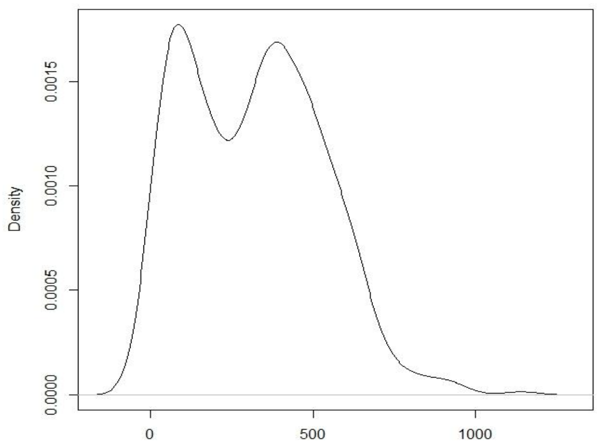

Fig.1. Density Plot of Average Monthly Extreme Rainfall in Nigeria from January 1971 to December 2012.

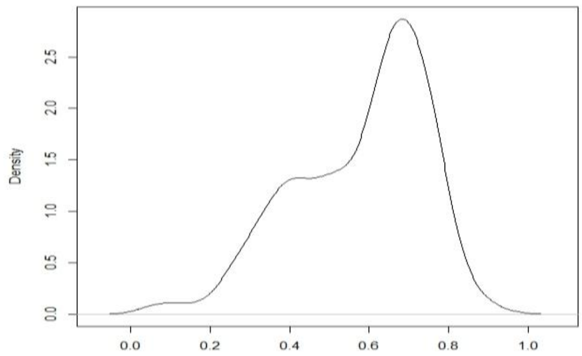

Fig.2. Transformed Beta Density Plot of Average Monthly Extreme Rainfall in Nigeria from January 1971 to December 2012.

The beta regression extreme rainfall model validated the following states; Oyo, Cross-River, Akwa Ibom, Imo, Niger, Kwara, Kebbi, and Zamfara at 5 percent level of significance (appendix 1) which cut across all the geo-political zones in the country to have contributed significantly to the extreme rainfall in Nigeria as attested to in Fig. 3 and 4 which is an indication that there is a high variability in rainfall among states of Nigeria identifying extreme rainfall as one of the major causes of flooding in all the states validated by the model. Other states not validated by the model may experience flooding due to structural, architectural and engineering problems.

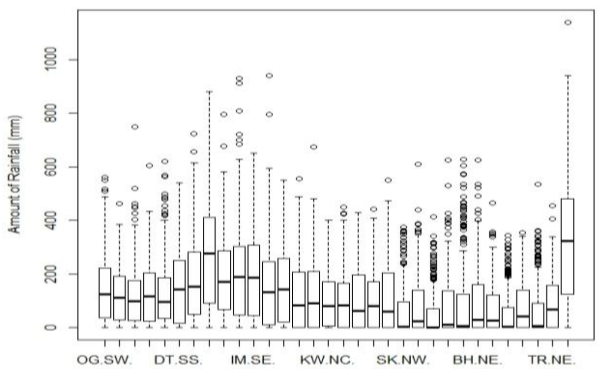

Fig.3. Box Plot of Rainfall in States of Nigeria from January 1971 to December 2012.

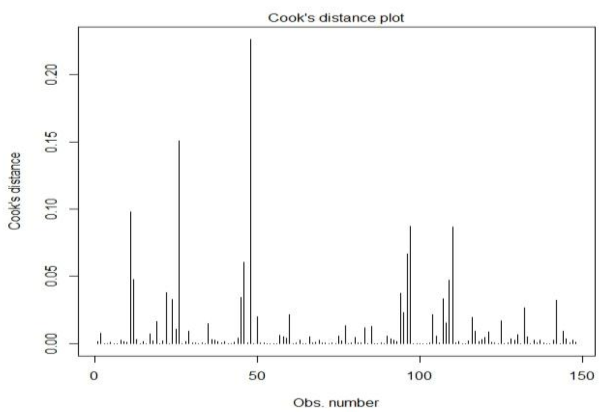

Fig.4. Cook's Distance Plot for Beta Regression Extreme Rainfall Model in Nigeria.

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It was noted that extreme rainfall in Nigeria from January 1971 to December 2012 followed a skewed underlying distribution with bimodal densities and modelled using beta distribution as discussed in [40], the graphical model evaluations and diagnostics in Fig. 5 and 6 also affirmed the efficiency and accuracy of beta distribution for the extreme rainfall model from January 1971 to December 2012 in Nigeria as no curvature was suspected.

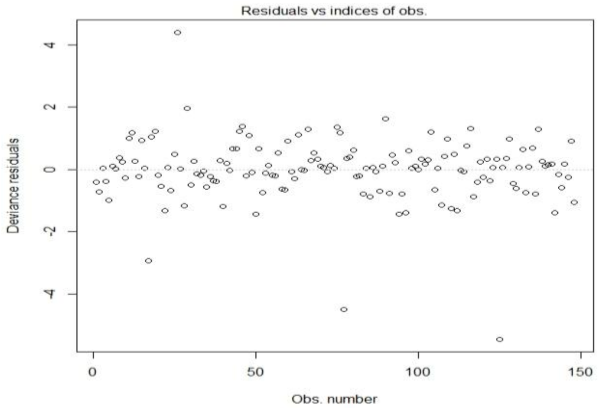

Fig.5. Residual and Indices of Observations for Beta Regression Extreme Rainfall Model in Nigeria.

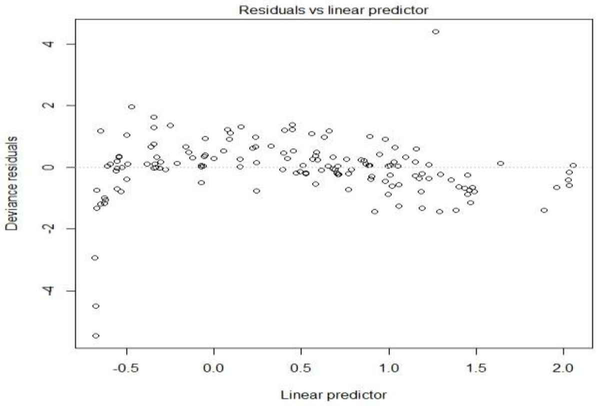

Fig.6. Residuals and Linear Predictor for Beta Regression Extreme Rainfall Model in Nigeria.

VII. Conclusion

The extreme rainfall as a major cause of flooding was investigated using generalized linear models (GLMs) of beta regression type to explain its variability in all states of Nigeria. The model affirmed that extreme rainfall causes flooding in Oyo, Cross-River, Akwa Ibom, Imo, Niger, Kwara, Kebbi and Zamfara states ($p < 0.05$) as the dispersion parameter of the beta regression extreme rainfall model was on the high side ($\phi = 60.909$). Moreover, beta regression type of GLMs asymptotically increases efficiency while dealing with a skewed underlying distribution with bimodal densities as possessed by the monthly average of extreme rainfall in Nigeria.

Appendix

Appendix 1: Generalized Linear Models (GLMs) of Beta Regression Type for Extreme Rainfall Model From 1971 to 2012 in Nigeria

Call:
betareg(formula=TRANS~OG.SW.+OD.SW.+OS.SW.+OY.SW.+LA.SW.+DT.SS.+ED.SS.+CR.SS.+RV.SS.+AK.SS.+IM.SE.+EN.SE.+AN.SE.+AB.NC.+NG.NC.+KW.NC.+KG.NC.+PT.NC.+BN.NC.+KD.NW.+SK.NW.+KB.NW.+KT.NW.+ZF.NW.+KN.NW.+BH.NE.+GM.NE.+BR.NE.+YB.NE.+AD.NE.+TR.NE.)
Standardized weighted residuals 2:
Min 1Q Median 3Q Max
-6.2919 -0.4605 0.0072 0.4429 6.0495
Coefficients (mean model with logit link):
States Estimate Std. Error z value
Pr(>|z|)
(Intercept) -6.754e-01 4.579e-02
-14.751 < 2e-16***(significant)
OG.SW. 1.255e-05 3.969e-04 0.032
0.97476
OD.SW. 5.394e-04 6.041e-04 0.893
0.37194
OS.SW. 2.828e-04 4.790e-04 0.590
0.55490
OY.SW. 1.115e-03 5.892e-04 1.893
0.04139*(significant)
LA.SW. 2.947e-04 3.543e-04 0.832
0.40552
DT.SS. -5.127e-04 4.308e-04 -1.190
0.23400
ED.SS. 2.052e-04 3.430e-04 0.598
0.54961
CR.SS. 1.757e-03 2.919e-04 6.017
1.77e-09***(significant)
RV.SS. 5.728e-04 3.676e-04 1.558
0.1191
AK.SS. 1.170e-03 2.602e-04 4.499
6.82e-06***(significant)
IM.SE. 7.786e-04 3.824e-04 2.036
0.04176*(significant)
EN.SE. -2.292e-04 5.062e-04 -0.453
0.65065
AN.SE. 5.298e-04 4.901e-04 1.081
0.27972
AB.NC. 2.870e-04 5.081e-04 0.565
0.57214
NG.NC. 8.868e-04 5.098e-04 1.740
0.04191*(significant)
KW.NC. -1.064e-03 6.123e-04 -1.737
0.04238*(significant)
KG.NC. -3.651e-04 5.812e-04 -0.628
0.52996
PT.NC. -7.518e-04 6.109e-04 -1.231
0.21845
BN.NC. -5.874e-04 6.653e-04 -0.883
0.37730
KD.NW. -5.416e-04 6.232e-04 -0.869

0.38480
SK.NW. -1.388e-04 8.706e-04 -0.159
0.87334
KB.NW. -1.223e-03 7.335e-04 -1.668
0.04237*(significant)
KT.NW. 5.342e-04 9.537e-04 0.560
0.57539
ZF.NW. 1.640e-03 6.270e-04 2.615
0.00892**(significant)
KN.NW. 3.870e-04 4.304e-04 0.899
0.36853
BH.NE. -1.573e-04 4.748e-04 -0.331
0.74043
GM.NE. 6.990e-04 6.800e-04 1.028
0.30400
BR.NE. -1.036e-03 8.680e-04 -1.194
0.23248
YB.NE. -4.807e-04 8.744e-04 -0.550
0.58249
AD.NE. -1.307e-03 8.420e-04 -1.552
0.12071
TR.NE. 2.743e-04 4.917e-04 0.558
0.57691

Phi coefficients (precision model with identity link):
Parameter Estimate Std. Error z value
Pr(>|z|)
(phi) 60.909 7.032 8.662
<2e-16***(significant)

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Type of estimator: ML (maximum likelihood)
Log-likelihood: 210.7 on 33 Df
Pseudo R-squared: 0.8543
Number of iterations: 43 (BFGS) + 2 (Fisher scoring)

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References

[1] Action Aid, "Climate Change, Urban Flooding and the Rights of the Poor Urban in Africa Cities". *A Report by Action Aid*, October 2006. Available: http://216.219.73.118/docs/urban.

[2] O. Adeaga, "Flood Hazard Mapping and Risk Management in Parts of Lagos N.E.", *Department of Geography, University of Lagos, Akoka, Lagos, Nigeria*, 2008. Available: http://gsdi.org//gsdi10page/Ts.13papar.pdf.

[3] S. Adedeji and P. Kuyoro, "Panorama: The day the Heaven opened up in Lagos". *The Tell Magazine*. No. 30. August 1, 2011, pp. 11-13.

[4] K. A. Aderogba, "Qualitative Studies of Recent Floods and Sustainable Growth and Development of Cities and Towns in Nigeria". *International Journal of Academic Research in Economics and Management Sciences*. No.3. Vol. 1. 2012, ISSN: 2226-3624.

[5] K. A. Aderogba, " Global Warming and Challenges of Floods in Lagos Metropolis, Nigeria". *Academic Research International*. Vol. 2. No. 1. 2012a, pp 448-468.

[6] K. A. Aderogba, "Qualitative Study of Recent Floods and Sustainable Growth and Development of Cities and Towns in Nigeria". *International Journal of Basic and Applied Sciences*. Insan Akademika Publications . October, 2012b, pp. 200-216.

[7] O. Akanni and L. Bilasanmi, "Flood; Lagos Residents Forced to Relocate…Drowning Teenager Rescued". *Vanguards Media*. Friday, July 10, 2011, p. 20.

[8] E. Amaize, "Flood Displaces 50 Villagers in Delta State". *Vanguard Media*. Monday, July 4, 2011, p. 9.

[9] C. Ambrosino, R. E. Chandler and M. C. Todd, "Southern African Monthly Rainfall Variability: An Analysis Based on Generalized Linear Models". *Journal of climate*. 24. 2011, pp. 4600-4617.

[10] M. P. Austin, "Models for the Analysis of Species Response to Environmental Gradients". *Vegetatio*. 69. 1987, pp. 35-45.

[11] P. Bariweni, C. Tawari and J. Abowei, "Some Environmental Effects of Flooding in the Niger Delta Region of Nigeria". *International Journal of Fisheries and Aquatic Sciences*. No. 1. Vol. 1. 2012, pp. 35-46.

[12] M. Boko, I. Niang, A. Nyong, C. Vogel, A. Githeko, M. Medany, B. Osman-Elasha, R. Tabo and P. Yanda, "Climate Change 2007: Impacts, Adaptation and Vulnerability". *Contribution of Working Group II to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change* (IPCC). M. L. Parry, O. F. Canziani, J. P. Palutikof, P. J. van der Linden and C. E. Hanson (Eds.). Cambridge University Press, Cambridge UK, 2007, pp. 433-467.

[13] J. M. Chambers, "Fitting Non-Linear Models : Numerical Techniques". *Biometrika*. 60. 1973, pp. 1-13.

[14] R. E. Chandler, "Advanced Analytical Methods for Climate Research". *Lecture Notes for Short Course at Institute of Atmospheric Physics*. Beijing, China. 12th – 14th June, 2001.

[15] R. E. Chandler, "On the Use of Generalized Linear Models for Interpreting Climate Variability". *Environmentrics*, 16. 2005, pp. 699-715.

[16] R. E. Chandler and E. M. Scott, *Statistical Methods for Trend Detection and Analysis in the Environmental Sciences*. John Wiley & Sons Ltd ., West Sussex, United Kingdom, 2011.

[17] R. E Chandler, and H. S. Wheater, "Analysis of Rainfall Variability Using Generalized Linear Models: A Case Study from the West of Ireland". *Water Resources Research*. 38(10). 2002, dio: 1029/2001 WR000906.

[18] R. W. Christopherson, *Goesytems: An Introduction to Physical Geography*. Prentice Hall, London Third Edition, 1997, pp. 423.

[19] R. Ceo and R. D. Stern, "Fitting Models to Daily Rainfall". *Journal of Applied Meteorology*. 21. 1982, pp. 1024-1031.

[20] D. R. Cox, "Notes on Some Aspects to Regression Analysis (with Discussion)". *Journal of Royal Statistical Society*. 49(B). 1968, pp. 1-39.

[21] D. Daly, *A Report on the Flooding in the Gort-Ardrahan Area, Report of Geological Survey of Ireland*, Dublin, 1992.

[22] P. Diggle, P. Heagerty, K. Y. Liang and S. L. Zeger, *Analysis of Longitudinal Data*. 2nd ed. Oxford University Press, Oxford, 2002.

[23] A. J. Dobson, *An Introduction to Generalized Linear Models*. 2nd ed. Chapman & Hall/CRC Press, New York, 2001.

[24] A. K. Etuonovbe, "Devastating Effect of Flooding in Nigeria". *A paper presented at FIG Working Week*. 2011.

[25] S. L. P. Ferrari and F. Cribari- Neto, "Beta Regression for Modelling Rates and Proportions". *Journal of Applied Statistics*. 31(7). 2004, pp. 799-815.

[26] A. Guissan, C. Thomas, Jr. Edwards and T. Hastie, "Generalized Linear and Generalized Additive Models in Studies of Species Distributions: Setting the Scene". *Ecological Modelling*. 157. 2002, pp. 89-100.

[27] T. J. Hastie and R. J. Tibshirani, "Generalized Additive Models". *Statistical Science*. 1. 1986, pp. 297-318.

[28] T. J. Hastie and R. J. Tibshirani, *Generalized Additive Models*. Chapman & Hall, London, 1990.

[29] C. Hennig, "Statistical Models and Data Analysis". *Lecture Notes for STATG001*. University College London, London, 2007.

[30] L. Ingsriawang, S. Ingrsriawang, P. Luenam, P. Trisaranuwatana, S. Klinpratoom, P. Aungsuratana and W. Khantiyanan, "Application of Statistical Method for Rainfall Predication over the Eastern Thailand". *Proceeding of the International MultiConference of Engineers and computer Scientists, (IMECS).* Volume III. 17th – 19th March, 2010. HongKong.

[31] D. Koutsoyiannis and D. Pachakis, "Deterministic Chaos Versus Stochasticity in Analysis and Modelling of Point Rainfall Series". *Journal of Geophysical Research: Atmospheres*. Vol.101. Issue D21. 1996, pp. 26441-26451.

[32] W. J. Krzanowski, *An Introduction to Statistical Modelling*. Arnold, London, 1998.

[33] K. Y. Liang and S. Zeger, "Longitudinal Data Analysis Using Generalized Linear Models". *Biometrika*. 73(1). 1986, pp. 13-22.

[34] P. McCullagh and J. Nelder, *Generalized Linear Models.* 1st ed. Chapman & Hall, London, 1983.

[35] P. McCullagh and J. Nelder, *Generalized Linear Models*. 2nd ed. Chapman & Hall, London, 1989.

[36] R. Mordi, "The Tsunami in the Making". *The Tell Magazine*. No 29 July, 25. 2011, pp. 54-63.

[37] M. Muller, "Generalized Linear Models". *Handbook of Computational Statistics 1: Concepts and Fundamentals*. J. Gentle W. Hardle and Y. Mori, Eds. Springer-Verlag, Heidelberg, 2004.

[38] J. A. Nelder and R. W. M. Wedderburn, "Generalized Linear Models". *Journal of Royal Statistical Society*. 135(A). 1972, pp. 370-384.

[39] K. O. Obisesan, "Change Point Detection in Time Series with Hydrological Applications". *M. Phil. Thesis*. University College London, London, 2011.

[40] P. Palino, "Maximum Likelihood Estimation of Models with Beta Distributed Dependent Variables". *Political Anal*. 9 (4). 2001, pp. 325-346.

[41] D. H. Pilgrim and F. S. Cordery, "Flood Runoff". *Handbook of Hydrology*. D. R. Maidment, Ed. New York. McGraw-Hill Inc. 1993, pp. 9.1-9.42.

[42] M. A. Saunders, "Earth's Future Climate". *Philosophical Transactions of the Royal Society London*. 357(series A). 1999, pp. 3459-3480.

[43] M. Smithson and J. Verkuilen, "A Better Lemon Squeezer? Maximum Likelihood Regression with Beta Distributed Dependent variables". *Psychological Methods*. 11 (1). 2006.pp. 54-71.

[44] S. Solomon, D. Qin, M. Manning, Z. Chen, M. Marquis, K. B. Averyt, M. Tignor and HLM, Eds. *Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the IPCC*. Cambridge University Press, Cambridge, 2007.

[45] R. D. Stern and R. Coe, "A Model Fitting Analysis of Rainfall Data (with Discussion)". *Journal of Royal Statistical Society*. 147 (A). 1984, pp. 1-34.

[46] C. Swearingen, B. Tilley, R. Adams, Z. Rumboldt, J. Nicholas, D. Bandyopadhyay and R. Woolson, "Application of Beta Regression to Analyze Ischemic Stroke Volume in NNDS rt-PA Clinical Trials". *Methods in Neuroepidemiology*. 37. 2011, pp. 73-82.

[47] O. Taiwo, "Flood Sacks 500 in Babura". Thisday Media. Vol. 13. No. 4867. 2008, p. 18.

[48] S. M. Uppala, et al., "The ERA-40 Re-Analysis". *Quarterly Journal of the Royal Meteorological Society*. 131. 2005, pp. 2961-3012.

[49] W. N. Venables and B. D. Ripley, *Modern Applied Statistics with S.* 4th ed. Springer, New York, 2002.

[50] Y. Wang, W. Yan and R. E. Chandler, "An Analysis of Mid-Summer Rainfall Occurrence in Eastern China and its Relationship with Large-Scale Warming Using Generalized Linear Models". *International Journal of Climatology*. Wiley InterScience. 2009. dio:10.1002/joc.2018.

[51] H. Welch, P. Symons and D. Narver, "Some Effects of Potato Farming and Forest Clear Cutting on New Brunswick Streams". *Fisheries and Marine Service. Environ. Can. Technical Report*. No. 745. St Andrews New Brunswick, 1977.

[52] T. Wright, "Waterlogged: Pakistani Children Push a Motorbike Through Flooded Streets after Rain in Lahorerin". *The Wall Street Journal*. London, 2011.

[53] Z. Yan, S. Bate, R. E. Chandler, V. Isham and H. Wheater, "Analysis of Daily Maximum Windspeed in Northwestern Europe Using Generalized Linear Models". *Journal of Climate*, 15(15). 2002, pp. 2073-2088.

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