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A FET of Galerkin Method for Unsteady Flow of a Conducting Viscous Fluid in a Porous Rectangular Duct

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***Abstract* – This problem is to investigate the effect of magnetic parameter and the aspect ratio on the flow velocity when an electrically conducting fluid flows in a porous rectangular duct in the presence of an imposed traverse magnetic field, using finite element technique. The velocity distributions are shown through graph. It is observed that the maximum velocity which occurs in the middle of the channel decreases with increasing in magnetic parameter 'M' and also, it is seen that the maximum velocity in the duct increases with decreasing 'd' this may be due to the increase in the height of the channel.**

***Keywords* – Rectangular Duct, Porous Media, Aspect Ratio, Hydrodynamics, Conducting Fluid, Magnetic Parameter.**

I. INTRODUCTION

It is desirable to extend various available hydrodynamic solutions to include the effect of the magnetic field for the cases when viscous fluid is electrically conducting. The liquid metal MHD are derive from conventional hydrodynamics of incompressible media and this is very important in the metallurgical industry, nuclear reactor sodium cooling system, energy storage etc. Hunt et al. (1976). In 1965 Snyder analyzed MHD flow in the entrance region of a rectangular channel. Hartmann (1937) has studied the problem of steady magneto hydrodynamic channel flow of conducting fluid under a uniform magnetic field transverse to an electrically insulated channel wall.

Hughes and young (1966) have considered the problem of flow through a rectangular channel bounded by walls which are perfectly insulating. The general problem of the rectangular channel flow has been discussed considering various cases of insulating conducting walls. Jagadeesan (1964) studied the hydrodynamic Couette flow between two conducting porous walls. Chandrasekhar (1975) studied the steady MHD flow of viscous incompressible fluid in a squeeze film bounded above by a porous lined plate. Chen and Chen (1972) and Hwang (1972) have considered the entry flow with arbitrary inlet velocity profile. These studies of entry flow in channels are very much essential for operational MHD devices like power generators and MHD accelerators.

Shercliff (1953) has examined the steady motion of an electrically conducting, viscous fluid along channels in the presence of an imposed transverse magnetic field when there is no conduction of currents through the walls. Gold (1962) has obtained an exact solution for incompressible laminar flows in circular pipes with non conducting walls. Fan and Chao (1965) have studied unsteady laminar incompressible fluid flow through rectangular duct with the pressure gradient varying with time. In (1990) Kumar and Singh have studied the unsteady flow of a dusty visco-elastic fluid through an inclined rectangular channel.

Sai and Nageswara Rao have studied Magneto hydrodynamic flow in rectangular duct with suction and injection (2000).In (2003) Tsangaris and Vlachakis obtained an exact solution of the Navier-stokes equations for a pulsating flow in a rectangular duct in order to explain the blood flow in fiber membranes which are used for the artificial kidney. The effects of side walls on axial flow in rectangular ducts with suction and injection have been investigated by Erdogan (2003). K.Avinash Reddy in 2007 investigated the effect of magnetic parameters M and aspect ratio d on the flow velocity when an electrically conducting fluid flows in a porus rectangular duct in the presence of an imposed transverse magnetic field. They have obtained analytic solution for the velocity when flow takes place under constant pressure and harmonically oscillating pressure gradient.

The aim of the present problem is to investigate the effect of magnetic parameter 'M' and the aspect ratio 'd' on the flow velocity when an electrically conducting fluid flows in a porous rectangular duct in the presence of an imposed transverse magnetic field. The Galarkin method of finite element technique to obtained a solution for the velocity when the flow takes place under constant pressure and harmonically oscillating pressure gradient.

II. MATHEMATICAL MODEL

Consider the unsteady flow of a viscous incompressible fluid (otherwise at rest) through a long porous duct of rectangular cross-section under the influence of a time dependent pressure gradient $f(t)$ down the duct length and a constant transverse magnetic field (Perpendicular to the one of the boundaries) of strength B_o.

Choose a rectangular Cartesian coordinate system $O(x, y, x)$ with the z -axis down the duct length and $\vec{B}$ is the applied magnetic field along the y -axis. Let the duct cross-Section given by $x=0$, $x=a$, $y=0$ and $y=b$. (Where, 'b' is the width and the 'a' is the height of the duct). Let the flow be given by the velocity field $U=(0,0,V(x, y, y))$ which evidently satisfies the continuity equation.

The following feasible assumptions are made in the analysis of the problem:

(a) The flow is laminar and fully developed.
(b) The boundary is impermeable.
(c) The induced magnetic field is neglected when compared with the applied magnetic field.
(d) The electric field is zero.

Basic Equations

The Navier-stokes equations for the fluid flow through porous media when coupled with the Maxwell field equations yield the basic equation of the problem under investigation.

$$\rho\frac{\partial \vec{U}}{\partial t} = -\nabla P - \frac{\mu}{k}\vec{U} + \frac{\mu}{\varepsilon}\nabla^2\vec{U} + \vec{J}\times\vec{B} \qquad (2.1)$$

Together with the equation of continuity

$$\nabla.\vec{U} = 0 \qquad (2.2)$$

From which we obtain the equation satisfied by the velocity function $V(x,y,t)$:

$$\frac{\partial V}{\partial t} = f(t) - \frac{\nu}{k}V + \frac{\nu}{\varepsilon}\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) - \frac{\sigma_e}{\rho}B_o^{\,2}V \qquad (2.3)$$

Where

$$f(t) = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

where ρ, p, V, σ_e, μ, ν, ε, k and B_0 are the density of the fluid, pressure, velocity component in z direction, an electrical conductivity ,viscosity, kinematic viscosity, porosity parameter, permeability and magnetic field in the y direction.

Boundary condition and the initial condition

$$V = 0 \ at \ x = 0, x = a \ and \ y = 0, y = b \qquad (2.4)$$
$$V = 0 \ at \ t \le 0$$

Non-Dimensionalisation of the Flow Quantitie

In terms of following non-dimensional quantities

$$V^* = \frac{V}{u_{av}}, x^* = \frac{x}{a}, y^* = \frac{y}{b}, P^* = \frac{P}{\rho u_{av}^2},$$

$$t^* = \frac{tu_{av}}{b}, z^* = \frac{z}{b}, q_m^* = aq_m, \ q_n^* = bq_n$$

The Basic equation (2.3) and the boundary and initial conditions (2.4) take the following form: after omitting the asterisks (*), we get

$$\frac{\partial V}{\partial t} = f(t) + \frac{1}{\varepsilon R_e}\left(d^2\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) - \left(\frac{1}{DaR_e} + M^2\right)V \qquad (2.5)$$

Where $R_e = \dfrac{u_{av}b}{\nu}$ (Reynolds number),

$Da = \dfrac{k}{b^2}$ (Darcy number),

$d = \dfrac{b}{a}$ (Aspect ratio),

And $M^2 = \dfrac{\sigma_e B_o^{\,2} b}{\rho u_{av}}$ (Magnetic parameter)

$$V = 0 \ at \ x = 0, \ x = 1 \ and \ y = 0, y = 1$$
$$V = 0 \ at \ t \le 0 \qquad (2.6)$$

III. Solution of the Problem

Assume f (t) = e^{wt}, V(x,y,t) = f(x,y)e^{wt} (3.1)

Using (3.1) into the equation (2.5), we get

$$1 + \frac{1}{\varepsilon \mathrm{Re}}\left(d^2\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{1}{\mathrm{Da\,Re}} + M^2 + w\right)f(x,y) = 0 \qquad (3.2)$$

The boundary conditions are

V = 0 at x = 0, x = 1 and y = 0, y = 1 (3.3)

The equation (3.2) is partial differential equation with boundary conditions (3.3). Through FET of Galerkin Method, the solution of f(x, y) is given by

$$f = (x - x^2)(y - y^2)A_4 \qquad (3.4)$$

The velocity distribution is given by

$$V = (x - x^2)(y - y^2)A_4 e^{wt} \qquad (3.5)$$

Where $A_4 = \dfrac{150}{60A_1 + 60A_2 + 6A_3}$

$A_1 = \dfrac{d^2}{\varepsilon R_e}$, $A_2 = \dfrac{1}{\varepsilon R_e}$,

$A_3 = \dfrac{1}{DaR_e} + M^2 + w$,

The numerically values of V(x, y, t) are carried out using Mat lab and C++ language

IV. Discussion of the Results

It is observed from Fig : (1) & Fig : (2) the velocity V increases with increasing 't' for a fixed values of Re = 2, M = 0,2, ε = 0.7, Da = 0.1, y = 0.5, d = 1 and w = 0.5. It is clear that the velocity of the fluid is higher in the absence of magnetic field. The maximum velocity occurs in the middle of channel.

It is observed from Fig: (3) the velocity V decreases with increasing 'M' for a fixed values of Re = 2, t = 0.3, ε = 0.7, Da = 0.1, y = 0.5, d = 1 and w = 0.5. The maximum velocity occurs in the middle of channel. It is observed from Fig: (4) & Fig: (5) the velocity V increases with decreasing 'd' for fixed values of Re = 2, M = 0, 2, ε = 0.7, Da = 0.1, y = 0.5, t = 0.3 and w = 0.5. It is clear that the velocity of the fluid is higher in the absence of magnetic field. The maximum velocity occurs in the middle of channel.

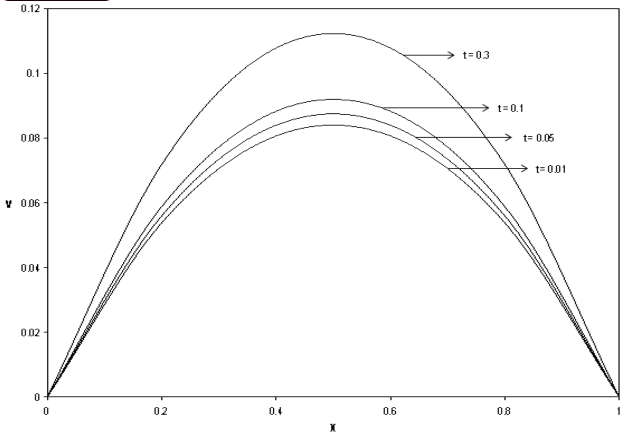

Fig.1. Velocity distribution versus X (M = 0)
Velocity under constant pressure gradient plotted against x for different - Values of 't' with $R_e = 2.0$, M = 0, ε = 0.7, Da = 0.1, $y = 0.5$ and $d = 1$

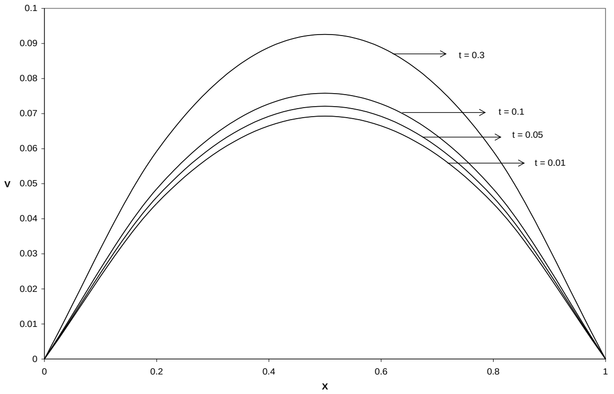

Fig.2. Velocity distribution versus X (M = 2)
Velocity under constant pressure gradient plotted against x for different - Values of 't' with $R_e = 2.0$, M = 2, ε = 0.7, Da = 0.1, $y = 0.5$ and $d = 1$.

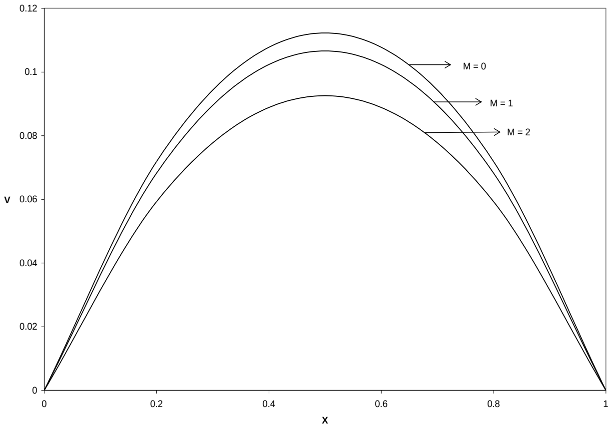

Fig.3. Velocity distribution versus X
Velocity under constant pressure gradient plotted against x for different - Values of 'M' with $R_e = 2.0$, t = 0.30, ε = 0.7, Da = 0.1, $y = 0.5$ and $d = 1$.

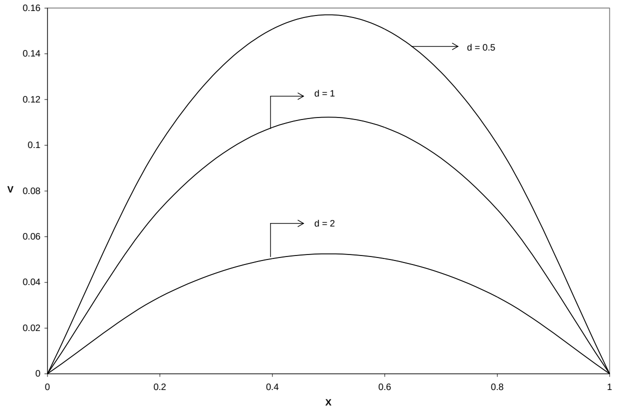

Figure 4: Velocity distribution versus X (M = 0)
Velocity under constant pressure gradient plotted against x for different - Values of 'd' with $R_e = 2$, M = 0, t = 0.30, ε = 0.7, Da = 0.1 and $y = 0.5$

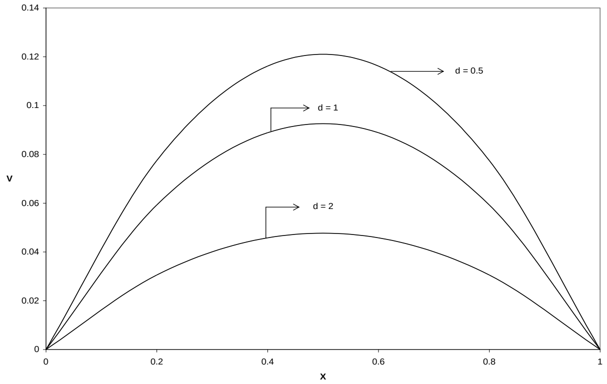

Fig.5. Velocity distribution versus X (M = 2)
Velocity under constant pressure gradient plotted against x for different - Values of 'd' with $R_e = 2$, M = 2, t = 0.30, ε = 0.7, Da = 0.1 and $y = 0.5$

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