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A Strong Convex Atmost k – Distance Dominating Sets In Graphs

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***Abstract* – Let *G* be a connected graph of order *p* ≥ 2. We study about the convex and dominating sets of *G*. We define strong convex sets and strong convex atmost k-distance dominating sets and we prove a theorem to develop new convex sets with domination number. Finally we present in this paper, various bounds and characterized the graphs, for which bounds are attained.**

***Keywords* – Convex Sets, Domination Number, Distance, Strong Convex Set.**

I. Introduction

By a graph $G = (V, E)$, we mean a finite, undirected, connected graph without loop or multiple edges. For a graph G, Let $V(G)$ and $E(G)$ denote is vertex and edge sets respectively. Let p and q be the number of vertices and edges respectively. For $S \subset V(G)$), the set $I[S]$ is the union of all sets $I[u, v]$. For $u, v \in S$ we say that a non-empty subset S is convex if $I[S] = S$ [5]. In this paper, we study about strong convex sets [2] and in this section, some basic definitions and important results on convex sets and domination number [3], [4] are presented.

Definition 1.1

A set S of vertices is called *geodesically convex, g-convex, or simply convex*, if $I[S] = S$. That is every pair $u, v \in S$, the interval $I[u, v] \subset S$. In any graph the empty set, the whole vertex set, every singleton, and every pair of adjacent vertices.

Definition 1.2

A set $S \subset V(G)$ is a *dominating set* for G if every vertex of G either belongs to S or is adjacent to a vertex of S.

II. Strong Convex Sets

In this section we define strong convex sets and strong convex atmost k-distance dominating set and prove some theorem.

Definition 2.1

A set $D \subseteq V$ is a *strong convex set* if $d_{<D>}(u,v) = d_{<G>}(u,v)$ for any two vertices u, v in D and the induced $<D>$ consists of all shortest path connecting every pair of vertices of D.

Definition 2.2

A set $S \subset V(G)$ is a *k-distance dominating set for G* if every vertex of G either belongs to S or is adjacent to *atmost k-distance* to a vertex of S.

Definition 2.3

A set $D \subseteq V$ is a *strong convex atmost k–distance dominating set* of G if every vertex in V-D is strongly dominated by atmost k–distance in D. The minimum cardinality of D is called the *strong convex atmost k-distance domination number of G* and it is denoted by $\gamma_{scd\leq k}(G)$.

Example : 2.4

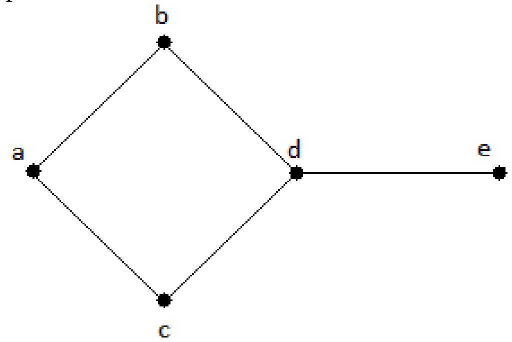

Fig. 1. Strong convex set of a graph

Here $\{d, e\}$is the strong convex set and $d(d, e)=1$, $d(e, b) = d(e, c) = 2$, $d(d, a) = 2$. So the vertices are dominated by atmost k-distance. Hence $\{d, e\}$ is the *strong convex atmost k-distance dominating set.*

Theorem 2.5

If G = P_n or C_n with $n \geq 3$,then

$$\gamma_{scd\leq k}(G) \leq \begin{cases} 2 & \text{if } n \leq 8 \\ n-6 & \text{if } n \geq 9 \end{cases}$$

Proof :

Case (i): $n \leq 8$.

Obviously $\{v_i, v_{i+1}\}$ is the strong convex set and also the strong convex atmost k-distance dominating set where $k \leq 6$.

Case (ii): $n \geq 9$.

If $P = 9$, the middle vertices $\{v_i, v_{i+1}, v_{i+2}\}$ is the minimum strong convex set and it dominates all other vertices atmost 3-distance, $\{v_i, v_{i+1}\}$ dominates all other vertices atmost k-distance, where $k > 3$. Hence $\gamma_{scd\leq k}(P_n) \leq 3$. If $P = 10$, the middle vertices $\{v_i, v_{i+1}, v_{i+2}, v_{i+3}\}$ dominates all other vertices atmost 3-distance. $\{v_i, v_{i+1}, v_{i+2}\}$ and $\{v_i, v_{i+1}\}$ dominates all other vertices atmost 4-distance and atmost $k \geq 5$. Hence $\gamma_{scd\leq k}(P_n) \leq 4$. Proceeding like this we get $\gamma_{scd\leq k}(G) \leq n - 6$. If C = 9, the vertices $\{v_i, v_{i+1}, v_{i+2}\}$ dominates all other vertices atmost 3-distance. If C = 10, the vertices $\{v_i, v_{i+1}, v_{i+2}, v_{i+3}\}$ dominates all other vertices atmost 3 – distance, proceeding like this we get in general $\{v_i, v_{i+1}\}$ dominates all other vertices atmost k-distance and hence $\gamma_{scd\leq k}(G) \leq n-6$. ■

We observe that for any connected graph G, $\gamma_{scd\leq k}(G) \leq diam(G)$, since the maximum eccentricity of a vertex is diam(G) . ■

III. Bounds in Terms of Degree and Diameter

Definition 3.1

For a vertex v in a connected graph G, the eccentricity $e(v)$ of v is the distance between v and a vertex farthest from v in G. The minimum eccentricity among the vertices of G is its *radius* and the maximum eccentricity is its *diameter*, which are denoted by $rad(G)$ and $diam(G)$.

Definition 3.2

The degree of a vertex v in a graph G is the number of edges incident with v and is denoted by $deg_G v$. The *minimum degree* of G is the minimum degree among the vertices of G and it is denoted by $\delta(G)$; the *maximum degree* of G is denoted by $\Delta(G)$.

Theorem 3.3

If $\gamma_{scd\leq k}(\bar{G}) \geq 3$ then $diam(G) \leq 2$

Proof: If G is not connected, we cannot conclude diam(G). The contra-positive of theorem determines $\gamma_{scd\leq k}(\bar{G})$ for any graph G having diameter atleast 3. ■

Theorem 3.4

If G has no isolated vertices and diam(G) ≥ 3 ; k ≥ 3 then $\gamma_{scd\leq k}(\bar{G}) = 2$.

Proof :

Let x and y be the vertices of G such that $d(x, y) = diam(G) \geq 3$. Obviously x and y dominate all vertices in $\bar{G}$. Since {x, y} is the strong convex set in $\bar{G}$. Hence {x, y} dominates all other vertices by atmost k-distance in $\bar{G}$ and the cardinality of {x, y} is 2. Therefore $\gamma_{scd\leq k}(\bar{G}) = 2$. ■

Theorem 3.5

For any connected graph G, $\left\lceil \frac{daim(G)+1}{3} \right\rceil \leq \gamma_{scd\leq k}(G)$.

Proof :

Let D be an atmost k-distance dominating set of a connected graph G. Consider an arbitrary path of length $diam(G)$ in G. Then there exist a diameteral path includes atmost two edges form induced subgraph $\langle N[v] \rangle$ for each $v \in D$. Furthermore, since D is a $\gamma_{scd\leq k}$ – set, the diameteral path includes atmost $\gamma_{scd\leq k}(G)-1$ edges joining the neighborhoods of the vertices of D. Hence $diam(G) \leq 2\gamma_{scd\leq k}(G) + \gamma_{scd\leq k}(G) = 3\,\gamma_{scd\leq k}(G) - 1$. ■

Theorem 3.6

If G is a graph with k $\leq \delta(G)$, $\gamma_{scd\leq k}(G) \leq kn / k + 1$

Proof :

Let D be a minimum strong convex atmost k-distance dominating set. The eccentricity value of atleast one vertex in D has (k + 1). Since k$\leq \delta(G)$, therefore $\delta(G) \geq$ k +1. We know that $\delta(G) \geq kn / \gamma_{scd\leq k}(G)$. Combining these two inequalities we get $\gamma_{scd\leq k}(G) \leq kn / k + 1$. ■

We observe that for any graph G,

$\gamma_{scd\leq k}(G) \leq kn / k + \Delta(G)$.

IV. Conclusion

In this paper we have studied the strong convex set of a finite, undirected, connected graph without loops or multiple edges, whose dominating sets are known. We have investigated strong convex sets strong convex atmost k - distance dominating sets and various bounds. Some results are useful to develop new convex dominating sets. Then we have presented various theorems to find strong convex atmost k - distance dominating sets and based on diameter, as well as degree.

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